1.020 Ecology II: Engineering for Sustainability
Spring 2008

Lectures 08_16 & 08_17 Economic Optimization, Derived Demand, Irrigation

Motivation/Objective
Develop a model/optimization procedure to determine the most economically productive way to allocate limited resources (land and water) for a farm growing 2 crops.

Approach
1. Formulate allocation of limited resources as an optimization (quadratic programming) problem. Define objective (maxmize crop revenue, $), decision variables (land for each crop, ha), constraints (water and land limitations).
2. Put problem in a form suitable for solution in MATLAB. Construct all matrices required by MATLAB **quadprog** function.
3. Solve problem in MATLAB and evaluate sensitivities to resource constraints (also called shadow prices or Lagrange multipliers) for a range of water availabilities.
4. Consider how problem inputs (crop yield, crop water demand, crop prices, etc.) affect solution.

Concepts and Definitions Needed:
Resource allocation -- to obtain derived demand we focus on effect of resource limits on crop revenue.
General resource allocation optimization problem:

$\underset{x}{Maximize}\, F_{rev}(x) = \text{Net revenue}(x)$ ($)	Objective function
x = vector of quantities produced	Decision variables
Such that following constraints hold for each resource:	
Resource used (x) $\leq$ Resource available	Inequality constraints
Upper and lower bounds on x	Inequality constraints
Physical constraints (e.g. mass, energy balance)	Equality constraints

For 2 crop example this becomes:
Objective:

$$Maximize\ F_{rev}(x) = \sum_{i=1}^{2} p_i Y_i x_i\,,\ p_i = \text{ Price crop } i \text{ (\$ tonne}^{-1}) \quad Y_i = \text{Yield crop } i \text{ (tonne ha}^{-1} \text{ season}^{-1})$$

$F_{rev}(x) =$ revenue ($ season^{-1}) $\quad x = [x_1 \quad x_2]$, x_i = Area crop i (ha)

$Y_i = Y_{i0} - d_i x_i \quad Y_{i0}$ = nominal yield (tonne ha^{-1} season^{-1})

d_i = yield reduction coef (tonne ha^{-2} season^{-1})

Constraints: (MCM = 10^6 m^3) , w_i = Water rqmt crop i (MCM ha^{-1} season^{-1})

$$\text{Water: } \sum_{i=1}^{2} w_i x_i \leq Q = \text{water available (MCM season}^{-1})$$

$$\text{Land: } \sum_{i=1}^{2} x_i \leq L_{avail} = \text{land available (ha)}$$

Nonnegativity: $x_i \geq 0 \quad i = 1, 2$

Input Arrays for MATLAB (quadprog):
Quadprog format:

$$\underset{x}{Minimize}\; F_{rev}(x) = \frac{1}{2} x^T H x + f^T x \qquad \text{Find decision variables } x \text{ that minimize } F_{rev}(x)$$

$$such\ that:$$

$Ax \le b$	Inequality constraints
$A_{eq} x = b_{eq}$	Equality constraints
$x_{lb} \le x \le x_{ub}$	Lower and upper bound constraints

For 2 crop resource allocation problem (converted to minimization problem):

$$f = -[p_1 Y_{10} \quad p_2 Y_{20}] \quad H = 2\begin{bmatrix} p_1 d_1 & 0 \\ 0 & p_2 d_2 \end{bmatrix} \quad A = \begin{bmatrix} w_1 & w_2 \\ 1 & 1 \end{bmatrix} \quad b = \begin{bmatrix} Q \\ L_{avail} \end{bmatrix} \quad x_{lb} = [0 \;\; 0]$$

$A_{eq} = b_{eq} = x_{ub} = []$ (unused for this example)

Make sure that H is a symmetric matrix.

Shadow price of water = $\lambda\ (Q) = \dfrac{\partial F_{rev}(x)}{\partial Q}$, (\$ m^{-3})

Plot of $\lambda(Q)$ vs Q gives derived demand (a curve).

Crop Allocation Example Results
Crop 2 (lower water rqmt) preferred over Crop 1 (higher value per ha), especially when water is limited.
Plots show that revenue increases at a diminishing rate as available water increases
Demand for water decreases as available water increases
Results depend strongly on yield loss coefficient