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1.020 Ecology II: Engineering for Sustainability  
Spring 2008

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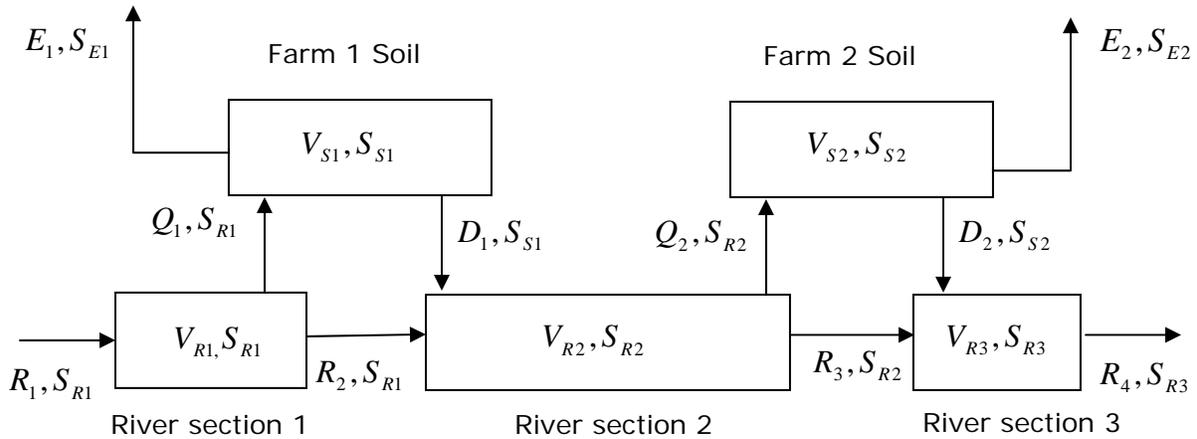
**MASSACHUSETTS INSTITUTE OF TECHNOLOGY**  
**Department of Civil and Environmental Engineering**

**1.020 Ecology II: Engineering for Sustainability**

**Problem Set 1 –Salination in Irrigated Agriculture**  
**Due: 5PM Tuesday Feb. 19, 2008**

**Problem Description**

This modeling exercise considers the problem of salt accumulation in irrigated agriculture. The system of interest includes a river and two farms that divert irrigation water from the river and discharge drainage water back to the river. Water and salt accumulate in the soil below each farm. Relevant mass flows and storages are shown in the figure below. Each box is a separate mass balance control volume.



- $V_{Ri}$  Volume of water in river section  $i$  ( $\text{m}^3$ )
- $S_{Ri}$  Salt concentration in river section  $i$  ( $\text{mg L}^{-1}$ )
- $R_i$  Flow into river section  $i$  and out of river section  $i-1$  ( $\text{m}^3 \text{sec}^{-1}$ )
- $S_{R1}$  Upstream salt concentration ( $\text{mg L}^{-1}$ )

- $V_{Si}$  Volume of water stored in soil at Farm  $i$  ( $\text{m}^3$ )
- $S_{Si}$  Salt concentration in soil at Farm  $i$  ( $\text{mg L}^{-1}$ )
- $E_i$  Evaporation rate from soil at Farm  $i$  ( $\text{m}^3 \text{sec}^{-1}$ )
- $S_{Ei}$  Salt concentration in water evaporated from soil at Farm  $i$  ( $\text{mg L}^{-1}$ ) -- **note  $S_{Ei} = 0$**

- $D_i$  Drainage rate from Farm  $i$  ( $\text{m}^3 \text{sec}^{-1}$ )
- $Q_i$  Irrigation diversion rate into Farm  $i$  ( $\text{m}^3 \text{sec}^{-1}$ )

**Model Specifications**

Construct a MATLAB model that simulates the salinity history in the soil at each farm and in each section of the river, over a 100 day period.

Assume:

- All water mass balances are in steady-state (time derivatives are zero) but salt balances are dynamic (time derivatives are not zero)
- River and farm soil salt concentrations at beginning of first day all = 30 mg/L
- River section volumes each = 3000000 m<sup>3</sup>
- Soil storage volumes each = 6000000 m<sup>3</sup>
- $S_{R1} = 30$  mg/L
- $R_1 = 15$  m<sup>3</sup>/sec
- Nominal values of  $E_1 = E_2 = 3.5$  m<sup>3</sup>/sec
- Nominal values of  $Q_1 = Q_2 = 7$  m<sup>3</sup>/s
- $S_{Ei} = 0$  for all  $i$  (plants do not evaporate salt)
- Use 1 day time steps and assume salt concentrations in flows between control volumes are equal to the concentration value computed at the beginning of the day, throughout the day.
- The maximum soil salinity  $\bar{S}_{si}$  must always be less than 230 mg/L or the crop will die. Make sure this requirement is met when you adjust  $Q_1$  and  $Q_2$ .
- In the model code, derive crop revenue for each farmer from the following yield equation:

$$N_i = \gamma E_i \exp\left[-\frac{\bar{S}_{si}^2}{2S_0^2}\right]$$

where  $\bar{S}_{si}$  is the maximum salinity over the 100 day growing period and  $S_0 = 36$  mg/L.

- $\gamma = 0.015$  \$/(m<sup>3</sup>/day)

## Using the Model

1. **Nominal case:** Plot the time histories of soil salt concentration at each farm and in each river section for the nominal values of  $E_1, E_2, Q_1$  and  $Q_2$  given above. What are the nominal farm revenues  $N_1$  and  $N_2$  for both farms?

2. **Alternative case:** Suppose that Farm 1 doubles its production by irrigating more land. Then its evaporation increases to  $E_1 = 7$  m<sup>3</sup>/s. Adjust the Farm 1 diversion  $Q_1$  to ensure that  $Q_1 \geq E_1$  and that  $\bar{S}_{s1} \leq 230$  mg/L. What are the new farm revenues for both farms when  $E_2$  and  $Q_2$  are held fixed at their nominal values? What is the penalty paid by Farm 2 when Farm 1 increases production? What do you think is a fair way to deal with this “externality”?

Submit via Stellar your MATLAB source file, the requested plots (as \*.fig files), and a text file with a few sentences summarizing your answers to the questions in the “Using the Model” section. Be sure to define all variables and to identify all major calculations in comments included in your MATLAB file (comments will be considered in the grade).