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Population Growth in Chemostats – Lecture Notes

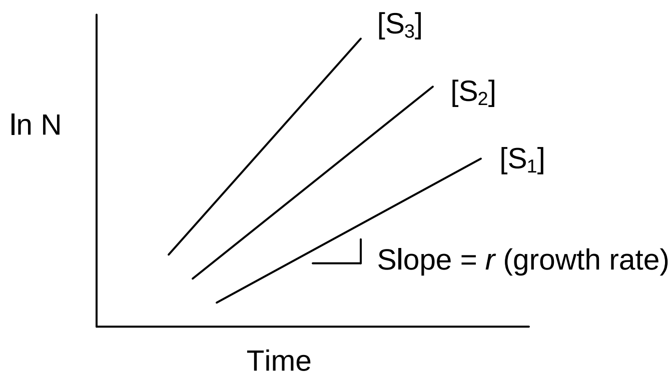
Chisholm

Steady State, resource-limited growth

Jaques Monod (1950s)

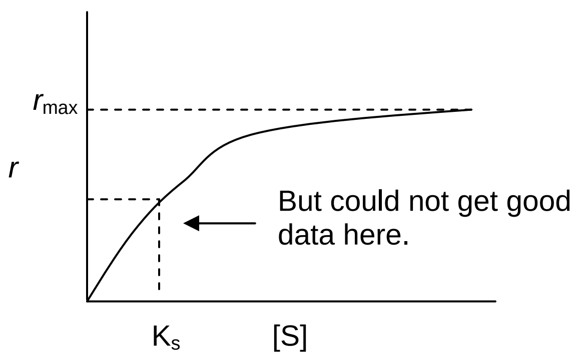
Wanted to look at the effects of a limited nutrient on the growth rate of bacteria

Noticed . . .



[S] = Substrate (Nutrient) Concentration

Hypothesized . . .

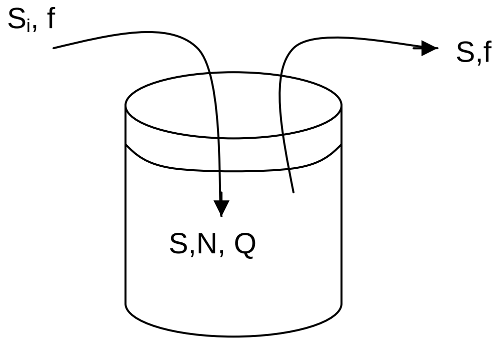


$$r = \frac{r_{\max} S}{K_s + S}$$

Needed system with constant <u>low</u> supply of nutrients

Chemostat Theory

System in which a fresh supply of nutrients is fed to a culture of constant volume at a fixed rate, and the contents of the culture are withdrawn at the same rate.



Assumptions

- Completely Mixed
- Inflow = Outflow
- One limiting Nutrient in Influent Medium

Constants & Variables	Units
N = Cell Concentration	cells /L
V = Volume of Culture	L
f = Flow Rate	L/hr
S_i = Influent Limiting Nutrient Concentration	μg/L
S = Residual Concentration in Vessel	μg/L
Q = Cell Quota	μg/cell
Y = Yield Coefficient = 1/Q	cells/μg
Dilution Rate D = f/V	hr^{-1}
Residence Time = 1/D	hr

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Analysis of a Chemostat

Change in Cell Concentration = Growth – Washout
"Births" – "Deaths"

$$\frac{dN}{dt} = rN - DN$$ Mass Balance

in steady-state . . .

$$\frac{dN}{dt} = 0 \quad \text{and} \quad r = D \qquad [\text{hr}^{-1}]$$

$$\frac{dS}{dt} = DS_i - DS - rQN$$

in steady-state . . .

$$\frac{dS}{dt} = 0 \quad \text{and} \quad N = \frac{S_i - S}{Q}$$

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and, by hypothesis and observation:

$$r = \frac{r_{\max} S}{K_s + S}$$

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Remember . . .

We have control over D and S_i.

Given (for steady-state assumption):

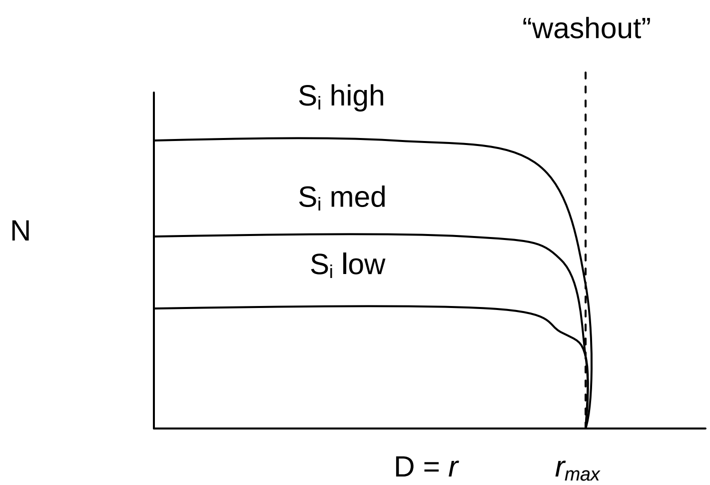
$$r = D$$

$$N = \frac{S_i - S}{Q}$$

$$r = \frac{r_{\max} S}{K_s + S}$$

What happens when we change D?

What happens when we change S_i?
(assuming a constant Q)
(knowing that D is fixed)
(knowing S is fixed for a given *r*)



Question:

What will the output of cells per unit time look like as a function of D for a given S_i?

Why does a chemostat always reach a steady-state?

If $r < D$
Cells will be washed out and
N ↓ S ↑, *r* ↑, until r = D

If $r > D$
Cells will get too dense and
N ↑, S ↓, *r* ↓ until r = D

Think about it carefully . . .

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