1.010 Uncertainty in Engineering
Fall 2008

Application Example 10
(Independent Random Variables)

RELIABILITY OF A BUILDING UNDER EXTREME WIND LOADS - CHOOSING THE DESIGN WIND SPEED

Consider the problem of designing a tall building for a certain level of reliability against wind loads. The building has a planned life of N years.

Let V_i be the maximum wind speed in year i at the site of the building (for simplicity we ignore wind direction). Although the values of wind speed at closely spaced times are probabilistically dependent random variables, yearly maximum values may be considered independent. Moreover, the distribution of V_i is the same for all years i. Therefore, the variables V_1, ..., V_N may be considered independent and identically distributed (iid), with some common cumulative distribution $F_V(v)$.

Empirical data indicate that, at most locations, the distribution $F_V(v)$ is either Extreme Type I or Extreme Type II. The latter has the form

$$F_V(v) = e^{-(v/u)^{-k}} \qquad (1)$$

where u and k are positive parameters. In Boston, plausible values of u and k are u = 49.4 mph and k = 6.5. Therefore, for Boston

$$F_V(v) = e^{-(v/49.4)^{-6.5}} \qquad (2)$$

where V is in mph.

Consider now the problem of choosing the design wind speed v* for a building in Boston, such that the probability of non-exceeding v* during the design life of the building equals a target reliability value. The probability that any given v* is not exceeded in N years (the reliability of the building) is $Rel(v^*,N) = P\left[\max_{i=1,N} V_i \le v^*\right]$ and may be calculated as a function of v* and N as:

$$\begin{aligned} Rel(v^*,N) &= P\left[(V_1 \le v^*) \cap (V_2 \le v^*) \cap \ldots \cap (V_N \le v^*)\right] \\ &= \left\{F_V(v^*)\right\}^N \\ &= e^{-N(v^*/49.9)^{-6.5}} \end{aligned} \qquad (3)$$

Plots of Rel(v*,N) against v* are shown in Figure 1 for N = 10 and N = 50.

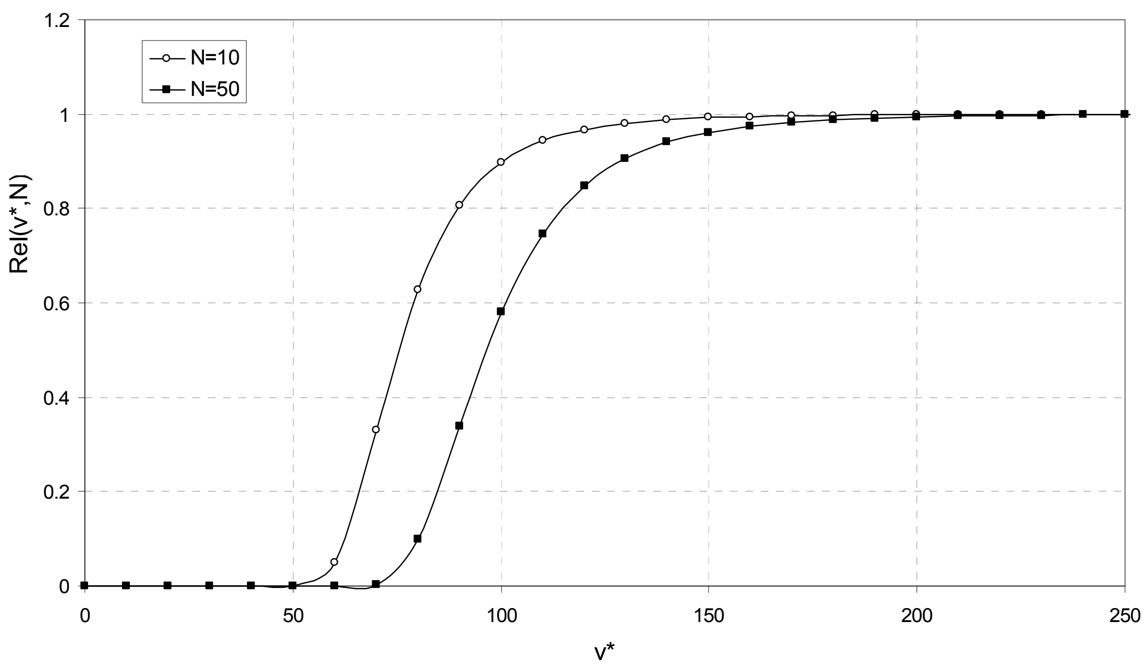

Figure 1: Reliability of a building in Boston against wind as a function of design wind speed v*, for exposure periods of 10 and 50 years.

Notice that:

1. For any given N, the reliability approaches 1 quite slowly, meaning that one must design for very high wind speeds in order to attain high safety levels. This is due to the fact that the distribution in Eq. 2 has a long upper tail. Had one chosen an Extreme Type I distribution for V, one would have found less extreme design demands;

2. Viewing the probability in Eq. 3 as the reliability of the building against wind loads is conservative, because exceeding v* does not necessarily imply serious damage to the building. Typically, the actual strength of a building is much greater that the nominal strength, due to built-in conservatism in design codes, nominal material properties, and good construction practice. Therefore, the actual reliability may be much higher that the probability in Eq. 3;

3. Interestingly, if the yearly maximum winds V_i are iid with the Extreme Type II distribution in Eq. 1, then the maximum wind in N years, $V_{max,N} = \max_{i=1,N} V_i$ has itself an Extreme Type II distribution, with the same parameter k but different parameter u_N. In fact,

$$\begin{aligned} F_{V_{max,N}}(v) &= e^{-N(v/u)^{-k}} \\ &= e^{-(v/u_N)^{-k}} \end{aligned} \qquad (4)$$

where $u_N = uN^{-1/k}$.

Problem 10.1

To better understand various aspects of the problem,

(i) plot the cumulative distribution function in Eq. 1 and its derivative (the probability density function of V) for different values of u and k, to see how these parameters affect the distribution of yearly wind speed (you might use the four combinations u = 40 or 60 mph and k = 4 or 7);

(ii) make plots analogous to those in Figure 1 for the case when yearly maximum winds have Extreme Type I distribution, with the same mean value m and variance σ^2 *as implied by Eq. 2, i.e. with m =* $u\Gamma\left(1-\frac{1}{k}\right)$ *= 55.1 mph and* σ^2 *=* $u^2\left[\Gamma\left(1-\frac{2}{k}\right)-\Gamma^2\left(1-\frac{1}{k}\right)\right]$ *= 157.5 (mph)*^{2} *where* Γ *is the gamma function [on these expressions for the mean and variance of the Extreme Type II distribution, see for example Benjamin and Cornell (1970), p. 279]. The cumulative distribution of the Extreme Type I distribution is given by*

$$F_V(v) = 1 - e^{-e^{\alpha(v-u')}} \qquad (5)$$

where α *and u' are parameters. The values of* α *and u' for the above mean value and variance are* $\alpha = \frac{\pi}{\sqrt{6}}\frac{1}{\sigma} = 0.102$ *and* $u' = m - \frac{0.577}{\alpha} = 43.93$ *mph. Comment on the results, in comparison with Figure 1.*

Problem 10.2

The following data on maximum yearly wind speed were recorded at Great Falls, Montana, 10 m above ground, during the period 1944-1977 (from Simiu and Scanlan, 1996).

year	***max speed (mph)***	***year***	***max speed (mph)***
1944	*57*	*1961*	*60*
1945	*65*	*1962*	*66*
1946	*62*	*1963*	*55*
1947	*58*	*1964*	*51*
1948	*64*	*1965*	*60*
1949	*65*	*1966*	*55*
1950	*59*	*1967*	*60*
1951	*65*	*1968*	*51*
1952	*59*	*1969*	*51*
1953	*60*	*1970*	*62*
1954	*64*	*1971*	*51*
1955	*65*	*1972*	*54*
1956	*73*	*1973*	*52*
1957	*60*	*1974*	*59*
1958	*67*	*1975*	*56*
1959	*50*	*1976*	*52*
1960	*74*	*1977*	*49*

(i) Calculate the sample mean and sample variance of this dataset.

(ii) Find the parameters of the Extreme Type I distribution for which the theoretical mean and variance match the sample mean and sample variance (use formulas given above as part of Problem 9.1).

(iii) Produce reliability plots analogous to those in Figure 1.

Benjamin, J. R. and C. A. Cornell, *Probability, Statistics and Decision for Civil Engineers*, McGraw-Hill, 1970.

Simiu, E. and R. Scanlan, *Wind Effects on Structures*, 3rd ed., Wiley, 1996.