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PROFESSOR: OK. So misconceptions. So misconceptions, alternative conceptions, understanding the student view of the world, why is that important? Well one of the best ways to introduce the importance of the idea is a story. Now I'm not 100% sure the story is true, but it's so good that it ought to be true even if it isn't. I think it is actually true.

So this was two, let's say physics, TAs, physics grad students, who were in a philosophy class. And the teacher in the philosophy class said well-- they were talking about things being relative, and morals being relatives, and relativity in philosophy-- as an example of relativity, and things being relative depending on the situation, if you drop a pencil on the Earth, it will fall to the ground. But if you drop the pencil on the moon, it won't fall to the ground. It will just float.

And so most of the people in the class just nodded, basically thinking, oh that's a good example. But the two physicists in the class were a bit shocked and said well wait a minute, wait a minute. That can't be true. Surely, something is wrong here. So they raised their hand. They said, well wait a minute. If you drop a pencil on the moon, surely it's going to fall to the ground, to the ground of the moon.

And the other people in the class said, no, no, that's not right. That's not right. So they thought, OK, well we have some way of convincing them that the pencil's not going to float. And they said, OK well you all saw the pictures of the astronauts walking on the moon, right? And they said, oh yeah. Well, when the astronauts walk, did they just float away, or did they stay on the moon? Oh, they stayed on the moon. Well, how'd that happen? They thought, oh we have them now.

And what was the response? The response was, oh, well that was because the astronauts were wearing heavy boots. So here was a chance to do a bit of

educational research. So they thought, oh my god. Maybe is this just this class? Let's actually do a survey. So they took out the university phone directory and just randomly dialed people. And they called them up and said, OK if you drop a pencil on the moon, will it float away or will fall to the ground? And most of the people said, it's going to float away.

So then they said, oh yeah, but what about the astronauts? And they gave them the whole thing about the astronauts again. And about half of the people said, oh yeah, that's because the astronauts were wearing heavy boots. So again, they were shocked and flabbergasted. And I think that reaction is a very healthy reaction. But what it shows is that the students-- so in this case they weren't officially their students-- but students-- they could easily be your students-- can have a very different view of the world, of the physical world, than you have. And if that's true, that means that as you're teaching them, what you're teaching is actually being interpreted in this completely different way.

I think Goethe said that-- he said that mathematicians, or maybe it was Frenchmen. It was either Frenchmen or mathematicians. Maybe it was mathematicians. They have their own language, and everything you tell them, they first translate into their own language, and only then do they understand it. So that's the same thing. The students are translating the things you're saying into their own language. But to understand what they're translating into, you have to know what their language is.

So in other words, you have to understand not just the misconceptions, but in general the conceptions that students have, so that you can actually plan your teaching. And here is a diagram to illustrate that. So this is a model of teaching to see where all the pieces fit in.

So here, this is the system, which is how students think. This is the output, which is what students can do after they go through our teaching. And this is the input, which is what we do, what kind of classroom structure we set up, what assignments we do. And by this, I mean how students think individually and collectively, how students work together, as well. So generally speaking the way-- so this is the system. And

call this the result. And this is the teaching. Our problem is trying to figure out the teaching.

And how do you figure out the teaching? Well you start with figuring out what result you want. What do you want students to be able to do? What is your goal for the class? And then you have to run it back through the system. Right so you figure out the result, and you run it through the inverse of s . And then there's your teaching. That's the teaching equation let's call it.

Now those of you from engineering fields know that basically the only thing you can do anything with is linear systems. And unfortunately anything that's even slightly non linear basically is a bag of tricks, and there's nothing you can do. And you're just hosed. Well unfortunately this system is incredibly non-linear. So it's probably kind of hard to work out s inverse, even if you can work out r . But it's hard, but it doesn't mean you can't start. And a necessary condition for even doing anything in this direction is you have to know what is in s , how do students think.

Another necessary condition is you need to work out r . And that's actually something-- what is the result you want? So that we're going to talk about in the session on course design. That's something that's often forgotten in teaching, and it's fundamentally important. That if you don't plan your goals, if you don't plan where you want to go, your chance of getting there is pretty low.

So assume you've done that. The next thing you want to do is you want to understand how students think, so that you can plan your teaching to reach them, in other words, so you have some chance of working out s inverse. Now you can't actually write down some equation for s inverse, but you can invert it mentally in your head. And a necessary condition for doing that is to understand s . So you want to understand where students are coming from, so that you can work out t .

So that's the signals and systems model of teaching. Now, an example again to show the importance of doing that and what happens if you don't actually understand where the student's are is yet another example from freshman physics. I'll leave this here. So this is one of the examples from the readings. So not

everyone may have read that reading. It's the reading by Reif. And it's this problem.

So this is a pendulum at five points in its arc. So these are the endpoints. And the question people were asked was to draw the acceleration vector at these five locations. So what's the acceleration right here, here, here, here, and here? So I'm not so interested in what the actual answer is as so much as how people did on this. So this is actually really shocking. So of students in intro physics who've had acceleration-- so they'd done acceleration-- zero out of 124 students could do that problem.

So that's 0%. So of grad TAs, it was 15%. So these are grad TAs in physics who TA that course. And PhD students-- so was PhD students on their qualifying exam. It went up to 22%.

OK so now this is a fundamentally important idea, acceleration. But you'd expect PhD students on their qualifying exam to be at 100%. And you'd like the TAs in the course to be at 100%. Because there's no chance of them teaching the students an idea that they themselves don't understand. As Polya said, there's no method of teaching yet invented that will allow the students to understand what their teacher does not. And that's true. There's no amount of sort of trickery with style and things you can do to convey an understanding that you don't have. So this is a very shocking number to me. And--

AUDIENCE: Who said that?

PROFESSOR: Pardon?

AUDIENCE: Who said that?

PROFESSOR: George Polya. So Polya was a great mathematics teacher from Hungary and then eventually at Stanford. And he's written-- I'll put references to his books on the website. But they are some of the best books on math teaching ever written. And so he said that in one of his books. And it's I think very true.

So what this example shows is that, a, the teachers didn't understand it. But that

really fundamental ideas can be completely skipped over for years and years and years. Yes, [INAUDIBLE].

AUDIENCE: It's a special example of some vague, conceptually difficult problem. I'm just wondering-- so I read the article. Did they focus on how to figure the answer out? Because a lot of physics majors don't.

PROFESSOR: OK, so the question is, how difficult is this problem? It's an interesting problem, because it's difficult if you try to do it by just blasting out the equations. So they actually asked a bunch of faculty members the same question. And one of them just got completely tangled in knots, physics faculty, trying to write out the differential equation and figure out what the acceleration is by brute force. So yeah, it is hard if you do it that way. But if you actually understand-- I'll show you-- what the acceleration means, then it's no problem.

So for example, here, the thing isn't moving. So it has v equals zero. But, what velocity is it going to have just a bit later. Well it's going to be going that way. So that means acceleration is that way.

AUDIENCE: But were they given enough time to do this, to do this thinking problem?

PROFESSOR: Yeah, so they were given quite a while. I think it was-- when the study on the faculty members, I think it was a big long think aloud protocol. So they were given like half an hour. In fact, the time is what hurt them. Because they all thought, oh I have so much time. I'm going to just blast it out with equations. And if you were only given one minute, you might have actually had a chance. Because you realized, oh I have to think about this intuitively.

So here, same thing by symmetry. And then this one is hard. And this one is hard. But this one right at the center is much easier. So this one catches a lot of people. But here it's moving in a circle, and it's at its maximum speed. So here it's moving at its maximum speed. So it's not going to be going faster that way or that way. So its speed is actually at a max, which means there's no change in speed. So you're not going to get any acceleration in that direction. But because it's moving in a circle,

somebody's yanking it upwards. So it has to be accelerating upwards to move in a circle, because it was going this way. And then later, it's going that way. So it has to be accelerating upwards. So there's an acceleration vector upwards, like that. So this is a bit exaggerated. It's generally not that big.

And then the question is, with these guys, well it's just interpolation. It's somewhere between that and that. So it's going to be something like that and something like that, depending on where exactly they are. But this is the one that tripped up a lot of people. And in fact, a lot of people got tripped up because they memorize the following thing, which is an object in simple harmonic motion has no acceleration at the equilibrium point. And that's true, except this isn't a simple harmonic motion. It's almost. But it's a pendulum. It's not a pure spring. So it has circular motion. So if you reason about it from the motion, it's actually very quick. But if you try to blast it out with equations, you're basically hosed.

So I wanted to check also, how deep are these misconceptions? So I actually did a survey of students in Cambridge in the physics major. And so one of the questions I asked them was the following-- so I made a survey of nine questions. And I wanted to see how they reasoned intuitively on these nine questions. And to force intuitive reasoning, I only gave a few minutes to get around the problem. So they knew that basically writing out a bunch of equations wasn't going to help, because you had no time to actually write them all out.

So the question was, which is going to go faster down the plane? So these are two identical planes, inclined planes with angle α . And there is either going to be a disk like a coin rolling down the plane, or a ring, like somebody's wedding ring rolling down the plane. And the question is, which goes faster, or which has a bigger acceleration. So this we'll call the hoop, and this is the disk. And they're going down the same angle.

AUDIENCE: The mass is the same.

PROFESSOR: I didn't say that. The question is, is the mass the same? So I'm going to give you that choice. So the question is, which goes faster in acceleration or velocity? So the

choices were the hoop-- So to answer the question about mass, one of the choices is that it depends on mass. Depends upon radius or depends on alpha. So it could depend on the mass, the radius, or on alpha. It could be the same. The disk could be faster. Or the hoop could be faster. OK, so just for fun-- and I realize this is not a physics class-- but it's fun to actually try to guess the answer to this, even if you're not a physicist.

So choose A, B, C, D, E, or F in cooperation with one or two neighbors. And then I'll talk about my analysis of this problem. OK, take another 30 seconds and collect a vote. Again, it doesn't really matter whether you get it right or not. I just want you to think about it, so you realize it is a subtle question.

OK, so who votes for the hoop being faster? Who votes for the disk being faster? Who votes that they're the same? That it depends on the mass, you'd like to know the mass? Fair enough. That you'd like to know the radius? Maybe 12. That you'd like to know the inclined plane angle? Nobody like that one, OK. And now who's sure of their answer. OK, so that's a much lower number.

So now I can show you what numbers came up when I asked the students this. So this was 11%. 35%. 13%. This one is 11% as well. Radius was 26%. And 3%. So kind of similar, kind of similar to your percentages. And again, only about a third of the people were sure of their answer.

OK, so why is this interesting? Well first of all, all the students had the technical knowledge to do the problem. These were all physics majors in various years, one through four, so freshman through senior. But the freshman had seen the material to actually do this and calculate it if they need to. So now this problem illustrates well first of all that even an intuitive question like this can actually trip people up, even if they've done all the calculations. So the question is, how are they reasoning?

Now in Cambridge I had the advantage of also doing tutorials, because that's how half the teaching goes. So in tutorials I actually I set this problem for my students, and I got a chance to talk to them about it. So I got to figure out, how are they reasoning about it? So I asked them, well what do you think is going to happen?

And then please explain why, more than you can get just from multiple choice. And the answers were very revealing.

What they said was, many of them said, well, just as heavier objects fall faster, so should heavier objects roll faster. So that was basically to justify they wanted to know either the radius or the mass, because that would affect the mass, and that would also affect the mass. So, let me put that up again, because that's a really interesting statement.

So actually what I call this, this is like the American theory of the British accent. Some of you know my explanation of that. So the American theory of the British accent is that if you go to an English person and you step on their feet in the middle of the night, they'll actually cry out in an American accent. Which is to say that the American, sort of folk theory of the British accent is that the British accent is a fake that people just put on, and in moments of stress they forget to do the put on act. OK, so now why do I bring that up? Well because it's very similar.

So if you ask the students-- suppose you didn't tell them this question about inclined planes. You just say, do heavier objects fall faster? You don't know about air resistance or anything. They say, oh no definitely not. Galileo showed that. But what this shows is that in a new situation, in other words where they don't have their automatic reactions, all of a sudden the folk theory comes out again. They really do think heavier objects fall faster.

Right, so how can you reconcile that with if you ask them, they'll say no they don't? Well that's because they had so much experience with every time they said that, someone said, whack. Heavier objects fall at the same speed. Galileo showed that, Leaning Tower of Pisa. So it becomes this linguistic string which hasn't actually been incorporated into their way of thinking about how the world works.

They think force is related to velocity, heavier objects fall faster. And to get that to come out, you have to, for example, equivalent to stepping on them in the middle of a night, surprising them. So you surprise them with a completely new context, which is the rolling. Rolling, they haven't thought much about rolling. So all of a sudden,

they're now thinking about rolling. And they're not thinking about all the policing rules that they were told about what happens to heavier objects. And so now their true way of looking at the world comes out, which is that heavier objects fall faster.

So again, if you don't realize that that's what they're doing, your teaching is going to be completely pointless. And what's going to happen is the following, which is that here are two models. The students will develop two models of the world in their mind. One let's call it the school model. And another is their intuitive model.

And let's say they start out a bit different from each other. And now you do a whole bunch more teaching, and you don't take account of the fact that this belongs in their intuitive model. And you teach a bunch of stuff about rolling down the plane. You do it with a bunch of equations. By the way, I should tell you what the right answer is. The right answer is this one.

It actually doesn't matter how heavy they are, because it's a gravitation problem. But suppose you just calculated it out. Well what have you done? You've overlaid on top of this, you've put in-- so this is their intuitive model. You've done a whole bunch of calculations which contradict it. And the two models diverge farther and farther from each other. So now next time they have to solve a problem, they have to decide whether I'm going to use the school model or the intuitive model.

And they learn, OK, well the intuitive model doesn't work very well. Let me use the school model. So the school model basically keeps diverging. And the intuitive model and the school model never meet again, which means that they can't use their intuition for solving any of the problems you give them, or that they're going to solve later. So that is an educational tragedy.

So the divergence, and in my view, that is the fundamental reason why most teaching produces no results a year later. Because what you've done is you've developed the school model, the symbolic crunching, whatever it may be. But you haven't actually yanked the intuitive model and the school model together. You need to actually work on bringing them together. So now the natural question is, how do you do that? So let me give you another example of a similar misconception

and a way of yanking the two together. Yes, question.

AUDIENCE: Would you mind explaining why the disk rolls faster?

PROFESSOR: Oh, yeah, why is it the disk? Yeah, sure. So first of all, why doesn't it depend on let's say the mass? So that's one thing to clear up right away. And one way to reason about that, maybe the best way in a physics class is to do it by dimensions. But another way is the following thought experiment. Which is, suppose I have two disks-- let's see, I might be poor today. I only have one disk in my pocket. Well here's two disks. The chalk is also a disk. So now, suppose I have one piece of chalk, and I want to roll it down the incline plane.

And now I ask about a second piece of chalk. Let's say they're identical pieces of chalk. They're both going to go at the same speed. OK, so now suppose I stick a bit of glue on this chalk and that chalk, and I stick them together. They're going to go, again, at the same speed, as if the glue weren't there. Because the glue isn't being stressed. They're just moving identically. So what I've shown by that experiment is that a heavy piece of chalk moves just the same as a light piece of chalk. So it's quite plausible from that reasoning that mass doesn't matter.

Why doesn't radius matter? Well you could do a couple of scaling arguments to show that the extra torque you get from being bigger is exactly canceled out by the extra mass because you're bigger. So the radius doesn't matter. Mass and radius don't matter.

So then it's just a question of shape, which is a dimensionalist thing. And what it is intuitively is how concentrated is the mass near the center versus near the edge. So suppose I actually made an extreme version of the hoop. And actually I did this for the students in Cambridge. We manufactured-- and you could actually imagine what it is.

So imagine this were hollow inside. That would be your ring or your hoop. Well can you make anything that would move even slower? Because I'm planning the hoop moves slower than the disk. Yes, what you do is you put giant ears on the thing. So

it rolls like those dumbbell things you do for weightlifting. So it rolls on this radius, but it has these giant ears. Well it's just going to move so slowly, because it's just being yanked by this really weak force. And it has to accelerate this giant mass. No chance.

So it moves really slow. In fact, that thing it can take 20 seconds to go down a meter long inclined plane. So that's an extreme version. So it rolls on this, and this is these big ears. So that's really slow. That's medium slow. And that's faster. And if you put all the mass right at the center, it would be even faster, because the rolling wouldn't suck up any of the energy. So that's the intuitive explanation.

OK so now an example of a question showing a kind of similar misconception, a related misconception. But I'm going to show it to you to illustrate what you can do about this. So what can you do about it? So the question is the following.

So that's a steel ball, which I have in my hand, and I drop on a steel table, say from a meter up high. Forget about air resistance. It bounces off the steel table. And the question is I'm interested in the forces on it. So what are the forces. So here's my table. So while it's falling, I'm interested in the forces on it at three different points in its flight. First while it's falling. I've let it go, and it hasn't yet hit. At the instant that it's stationary during its bounce. And while it's rising.

So when you ask the students this-- and I've done this several times-- mostly they know it's just gravity while it's falling, mg . So if you actually use this question, don't ask-- this is chronologically next, but don't ask that next. Ask this one, when it's rising. And what you'll find-- what do you think many students say, not all but many? What are the forces on it while it's rising? Yeah, [? Adrian. ?]

AUDIENCE: So of them will say the force goes up.

PROFESSOR: Right, some will say the force goes up. They'll say mg up. I see people laughing, and that's a good reaction, because that shows you're learning something about how students think. That's kind of amazing. Why would they possibly say that? Well let's talk about this one, and you'll see that it's actually a similar mode of reasoning

to this. So what you'll find is even students who get this correct-- so this isn't right. It's mg down. But there's a deep seated misconception that if something's moving upwards, it has to have a force pushing it upwards. And then the force somehow gets used up, and then it stops.

OK, well what about here? So now they'll agree that there's weight pushing down. So that's the mg . And then the question is, what's the other force? And they call it by different names. In America, it's called the normal force, which is a terrible name. And in England it's called the reaction force, which is a terrible name for another reason. Which is that it implies that it's the result, somehow a reaction. So it's a terrible name. But let's ask the students, how big is this force?

OK, so it's upwards. Everyone agrees that it's upwards. But how big, r or n , depending on which country you're in? So take a minute or two and for yourself figure out how big the reaction force or the normal force is, in comparison with the weight. So this is a small steel ball dropped from say a meter and bouncing off a steel table. So two questions, first figure out what you think. And second, figure out what you think students are going to say. OK so you're doing both, trying to get into the student mind.

Let's do a quick vote. So what's the ratio between that normal force and the weight? So who votes for equal? First we'll do what you think, and then we'll do what students think. So who thinks they're equal? Two to one? Greater than 10, but not quite 1,000? Greater than 1,000 to 1?

OK now what do you think students say? How many say one to one? How many say two to one? How many say greater than 10? Greater than 1,000? And how many are not sure what the students are going to say?

So actually when you ask the students it depends on the population. But generally, what I find is it's 90% for one to one, And about 10% for two to one. And no one says anything else. OK so now why would students say one to one? What's the reasoning going on in their mind?

Has no velocity. So that is exactly the same reasoning as this, with mg upwards. But what's interesting is even the students who know it's mg downwards still say, well the velocity is zero. So v equals zero. Therefore, it's in equilibrium. Therefore, there's no forces acting on it. Therefore, the reaction force, or the normal force, and the weight have to cancel. N equals mg because it's stopped. So that what I call that is I call that the f equals mv theory of physics. Now maybe like in the linguistic books, I'll put a star by that saying don't say that. That's not how you translate that sentence into real life.

But why f equals mv ? Why is it so deep seeded? Well we have a gazillion hours of experience with it. Suppose I move a object, and I push it. Stop pushing it, and it stops moving. If I have a light object, I just push it a little less and it moves. And I stop pushing, and it stops moving. So f is related to m , and f is related to v . This seems to explain most of our world. It's hard to get around that.

So the question is, how do you actually try to force the students to see that that's actually completely junk what they said, and that it can't be equal? And the way I want to do it is I want to do it in a way that recombines their intuitive model and their school model. I want to meld them together.

So the way I do that is the following, which is I get a rock. So let's call this a rock. And I ask one of the students, could I please borrow your hand? And so somebody gives me their hand. And I say, OK, I'm going to put the rock in your hand. Now, what's the weight of the rock? mg . OK, great. So what's the force of the rock on your hand? mg . Great. Now I hold the rock about the same place I'm going to drop the steel ball, and I say, OK on the count of three, I'm going to drop the rock on your hand. One, two, and then I'm ready. What are they going to do? Like that. And soon as they try to do that, I grab their hand and say, wait a minute. Let's reason about this with physics.

Why are you moving your hand? You just told me that when the thing bounces, it's still mg . The reaction force and the normal force is still mg . And you just showed me that mg on your hand didn't hurt your hand. You could hold the rock on your hand

no problem. So why are you moving your hand?

So then we'll have a discussion, and many of them will then change to two to one. They'll say, oh, well it's actually two to one. And I say OK, two to one, well I have an answer for that. Two rocks. So I have two rocks, and I put them on their hand. So now compared to the first rock, mg, 2mg. So now I take the first rock, and I'm going to drop it on your hand. And one, two, three. And they try to move. And I say wait, wait. You just told me 2mg was fine. This felt fine to you, right? And they say, I know it's going to hurt like hell.

So now what they're doing is they now have some kind of contradiction that's evident between the school model and the intuitive model. And now is your opportunity to introduce and explain, yeah you're right. Your intuitive model is actually correct in this case. And your school model reasoning is incorrect. So actually what we're going to do is try to move the school model towards a piece of your intuitive model that is correct. We're trying to get away from the $f = mv$ part of the intuitive model and emphasize the part that does know. And the part that says, I don't want to have that rock in my hand is correct.

So then they're very ready to do a school model calculation of how big the force is. And it turns out to be something like 100,000 or 10,000 times the weight. So it's huge because the impact time is so short. And then to make that plausible to them, we can actually calculate the impact time or estimate it. But then you can actually say, look you already know this. You intuitively understand that. Because if you-- again I'll stand on the table. Suppose you're going to jump off an object like this, what do you do when you land? What does everyone do? Bend your knees. Right, so you do that. And what does that do? That increases the contact time.

And why are you increasing the contact time? Because that decreases the acceleration. The acceleration is a change in velocity divided by the contact time. So you can't affect the change in velocity very much. That's either v or $2v$ or something like that. But the contact time you can change. So with the steel ball, contact time is actually very short. Because the speed of sound is so fast. So the impact force is

really high. So yes, your intuitive model was correct. And now we're yanking the school model towards it. So you actually bring the two together. And you fight the divergence. So you want to look for examples like that whenever you can.

And to summarize the lesson-- because it was pointed out that I should summarize more often-- the summary of the entire theme about misconceptions is that you need to understand that so that you can teach properly. Because there's no way to deduce this without doing that, which we're going to talk about later. But even knowing that, if you don't know this, you can't run the system backwards and figure out what goes here. And so then, how do you actually learn about this? Well you've done one way. You've done one way already.

Let me write down two or three ways for you. So the question is, how to learn about s , the students' system. Well one way is to read. So in many fields, there's lots of research about what particular misconceptions students have. And I've given you some of those readings on the website. But there's a whole vast body of literature, and many fields have that. So you can look at that.

Another way, and this is-- so this is shut your mouth. Now why is that? Well there's no way to really understand what students are thinking unless you ask them. And if you don't let them talk, you're not going to actually hear what they're thinking. You'll actually put words into their mouth. Because you won't imagine that they think f equals mv . You never would imagine that. So you have to shut your mouth and listen. So office hours is a chance for that.

And related to that is the feedback sheet. If students get in the habit of asking you questions about what was confusing, you start to build a model of the students' system, what worked and what didn't work and what was confusing for them. So all of these ways are ways to build models of s , which is absolutely necessary for deducing t . Any questions?

OK so if you could do two things. One is to spend one minute filling out the feedback sheet. And the other is everyone who has a homework equation treatment, please raise your hand. OK that's great. So now just find one person near you to swap with.

Question? Yeah, OK. So that's great. So just find one person who has one near you to swap with and give each other comments. And then just write your name over the next week and bring it in next time with you.

So basically this is a way-- and if there's no one near you, or they've already swapped, just make a group of three. It's no big deal. Does anyone not have someone to swap with? OK great. And so introduce yourself to the other person. It doesn't matter if you're in different fields. It often can be an advantage, because you'll get a fresh set of eyes on it. And I'll give you some directions by email about what to look for and put it on the website. But generally just give each other suggestions and work together on what you thought was a good and what wasn't good. And just write your name on the person you commented on. And then you'll bring them both in. You'll bring the commented sheet from the other person in the next time. Or you'll just give it back to them and bring it. This is not a big police exercise. I just want you to meet somebody else and have practice discussing these things and learn from each other. So questions about that?

So if you could finish filling out the sheets and bring the sheets either here or here, I'll pack up. And meanwhile we'll have any questions that people want to talk to me about individually outside, because I think there's another class that 's going to come streaming in with like 150 people. So remember, learn about the student system. It's probably the most important thing to do to make your teaching stand out and have long lasting, good effects.

NARRATOR: Answers from lecture four to questions generated in lecture three.

PROFESSOR: So first, questions from before. One of the comments-- well, let's see, two of them were that could we have less time for questions, but also they're very helpful. So there's always a tension there. So what I'm going to try to do is shorten some of the total answers to the questions and then maybe put some answers to the questions towards the end of the lecture as well. I do like to try to answer most of the questions. And the reason being is that it provides also pacing.

And you'll find that too. If you find yourself generating four times as many questions

as you have time to answer the next time, it probably means you did way too much stuff the previous time. So it gives you feedback signal on how much material to discuss in the class time. And so that's one reason I'm reluctant to just put all the answers on the web. Because that's sort of like teaching with slides where you can just flip through all the equations really fast, and it seems like you got through a lot of equations. But actually no one really understood the equations. So it's sort of a symptomatic treatment for a more fundamental problem which may be that I'm doing too many things and not allowing enough time for questions in the class itself.

A general question, which is, how do you evaluate your own teaching? When you feel good or bad about how class just went? And what metrics do I use popularity, covering what I set out to? How do I know?

So one major way is I use these. It's not really that I say, oh did people hate it or like it? I mean, I want to know if people hated it or liked it. And I just I don't just add up the likes and the hates and say OK, 10 likes minus 3 hates equals 7. That's great. You're batting 80%. But I try to get a sense from reading the comments of what people were confused about, what was interesting to people, and sort of integrate that up and imagine the whole class as one giant student mind. And I want to know how I connect the material to the collective student mind. So I evaluate it that way, sort of intuitively from the comments that people make on the sheet. Which is why I hand out the sheet every time, and I think the sheet is so valuable.

Now another way, covering the materials I set out to. Well I'm probably sort of on one extreme of that, which I don't really care how much material I covered. But let me explain why I'm on that extreme. So some people only care about how much material is covered. And some people are more on my end. So I'll give you the reason. And most people I would say on this edge. So I don't need to really justify that, because you've heard lots of reasons for that. But it's the other side, why do I not care?

Well I don't care so much just because I think class time and a lecture course is such a small part of a student's life that if you haven't connected to the students, if

you haven't made it click for them and got them interested, made them want to continue studying, then you've lost most of the effect of the teaching. The effect of the teaching isn't really going to be so much in the class. Because what you can do is you can kindle interest, kindle a way of looking at the world. But it's really up to the students to carry that on afterwards.

And if you've actually bored them, or they've decided that this whole material is useless, they're not going to do that. And then it's all just going to go away anyway. So actually what I'm much more interested in is how much I've kindled those interests. And that's what I try to judge using the sheets. And I think that's actually what produces the long term change in ways of thinking.

Another comment was that the rock experiment rocked, which I thought was a good pun. If you're not willing to write out detailed derivations on the board, is it more important to have more detail published lecture notes? And yeah I think that's right. In fact, that's generally necessary. But it doesn't have to be your own published notes. It could just be a book you refer people to. So we've sort of gone backwards from Gutenberg. In the days before Gutenberg, everyone read out their notes. That was what monasteries did. You read out the bible. And at the end of a year or two of doing that, you had 50 bibles, because everyone took notes on what you read. That was the old photocopier, back in say the year 1300.

And then when Gutenberg came along the universities then did the same thing. That's what universities were. They basically grew out of the monasteries. Then Gutenberg came along and said, oh actually books are cheap now, well much cheaper than before. Books used to cost roughly say \$5,000 a book in today's dollars. But then with Gutenberg and automatic printing, they could be much cheaper.

So you can-- and we've developed printing technology much farther. So actually for a while until word processing came along and typesetting on your computer, people would actually just refer people to books and say, OK you're a student. You are expected to read these books. But now that people can make their own notes,

people have sort of forgot how to tell people to use books. And instead people, the lecturers, just tend to write their own notes out. And the students actually seem for some crazy reason to basically except notes specifically for that topic and don't want to read books.

So it gets really strange. You get in these bizarre cycles where you make notes. And as long as you have notes that seem like they're key to the lecture, that's great. Students will read them. And then you assemble all the notes for the very same lectures into a book, and now suppose the book gets published. The students will say, well now we want notes again. You can't say, well there's the book which was the notes. Because it's not specifically targeted.

So part of the cure for that is reading memos. Or any way of teaching students to do reading is to shift back to taking advantage of Gutenberg. And then once we take advantage of Gutenberg, we can worry about taking advantage of all the web technology and word processing technology that we have. But yeah, it is important, to answer the question directly, to have the details somewhere for the students. And the best place generally is in printed material, whether in a book or in something you write.

Because writing it on the board is very noisy. By which I mean that it's very easy to mis-copy. It's easy for you to make mistakes. It's easier for the student to make mistakes when they're taking notes. It's best to have it all in print, and also it's best to put the high level stuff in lecture, and have all the details with the students in their own comfortable space when they have time to go back and forth.

OK, so then there were several questions about how do you meld the intuitive model and the school model? Are their general principles? So I talked about how for example with the rock example that the school model is that when the rock isn't moving, the force is zero because it's in equilibrium. So the net force is zero. So the reaction force equals the weight. So that's the school model.

The intuitive model, somewhere in there is an intuitive model that you don't want the rock to fall on your hand. So what's a general ideas behind melding those two

models? So that demonstration I showed of borrowing someone's hand and dropping a rock on it, well not quite dropping the rock on it, but threatening to, is one way. But what are the general principles behind that? So the general principles are that you want to connect to where all the mental hardware is.

So generally symbolic processing it's a sort of surface phenomenon in the brain. It's linguistic. It's only about 100,000 years old, whereas perceptual reasoning, visual reasoning, visceral reasoning, those are hundreds of millions of years old. Organisms have been seeing things for hundreds of millions of years. So there's a huge difference in the amount of hardware that we have.

So symbolic hardware say has evolved for 10 to the five years. And perceptual, rough estimates 10 to the five versus 10 to eight years. So there's about 1,000 times more evolutionary help for these things. So if you want to meld-- and this is basically where the intuitive model, the internal model lives. So if you want to change it, you have to connect to where the model lives. You have to do things that are perceptual. So visual arguments, visceral arguments, auditory demonstrations, things they create conflict.

Somehow some kind of conflict is also another general principle, where you expose some kind of obvious contradiction. And that's so beneficial, because that's self teaching. As long as there's a contradiction, the students know that the problem isn't done. So in the rock example, there's a contradiction between, I don't want the rock dropping on my hand on the one hand-- that's the perceptual view-- with the symbolic one, which is net force is zero so the reaction force equals mg . So there's a contradiction. As long as you can't get the same answer by two different ways, you know you still have more to learn.

So it's self-teaching. You don't have to wait for the teacher to say, oh wait a minute. That's not the right answer. You know on your own. So know on your own. You internally feel that there's something wrong. So you're tapping in, I would say contradictions and puzzles. They're somewhat symbolic, but they're also kind of visceral. You feel them like oh, my god that can't be right. So if you can tap into that

as well that's another way of trying to meld the models. It's interesting. I think it's a fundamentally important point about teaching these two models. And I've never seen it discussed anywhere. So you're the sort of first people to hear it officially as far as I can tell. So any questions actually about that model? Because I think it is so important. Yes.

AUDIENCE: I'm just curious, what if you're in a field that doesn't lend itself to perceptual-- like if you're in a field that tends to be more symbolic. And physics is really nice because it has this physical aspect to it. So I'm just curious if you've thought about other--

PROFESSOR: So the question was, what happens if you're in a field that doesn't lend itself to perceptual reasoning so well?

AUDIENCE: I guess I wouldn't [INAUDIBLE]

PROFESSOR: Yeah there's visual in everything. And I think it's hard to imagine a field that doesn't have it. Can you give an example?

AUDIENCE: Well, some computer science. And if you spend a lot of time with computers, then you start to develop it. But you could think, well I know that it's not possible that it could just randomly decide to give a wrong answer. There has to be a reason. How the program develops, you can develop intuition about where the bugs might be. But when you're just starting out, that's not something that we necessarily have built in to us in an intuitive way.

PROFESSOR: That's true.

AUDIENCE: [INAUDIBLE]

PROFESSOR: Yeah so computers and programming and computer science was offered as an example. Yeah I agree with you. Some fields are harder. And what makes computer science harder is that computers are symbol processing devices. So it pushes them over here.

But even there I think you can do stuff. Like for example, what you could do? Well there are some things like for example, loop invariants, and you can actually turn

those into perceptions. So once you have get a way of looking at programs. You say, oh that can't be right because it has a fencepost error in it or it keeps keeps decrementing the counter, but there's nothing to stop it from having a floor. So you start to see problems like that. And you have tools like different kinds of magnifying glasses to look at programs. So you could actually shift it that way. And really good programmers actually do look at programs perceptually.

AUDIENCE: See, that's encouragement to build up [INAUDIBLE]

PROFESSOR: Yeah, so you want to encourage that. And it's true-- pardon?

AUDIENCE: It's not built in?

PROFESSOR: I would say that the field itself pushes things towards symbolic. Just because it is a symbol manipulation device. So that's what makes computers so much better at what they do compared to humans. That's why we use computers. Because we're so bad at symbol manipulation, and computers are so good at it. So our ways of solving problems are going to be very different than computers. But the problem that we have when we're using computers, we have to solve with human hardware.

So when the computer isn't doing what we want, we have to try to find some way of understanding what's going on. And it can't be just purely symbolic, because we're bad at that. So you can't-- I mean in desperation sometimes you'll trace through every step of a program and see what went wrong. But generally that just overflows you with tons of data. It's much better to actually try to take a step back, break the program into parts, see big chunks, and make sure the big chunks work.

And experts do that, not sort of automatically because they've been trained as experts. So in any field the experts see differently than the novices, because their perception is different. So yeah, when you're teaching computer science to students, you want to teach them ways that are perceptual. You want to teach them visceral feelings. So one way to do that for example is running time of algorithms. Different sorting algorithms take different times. So suppose you have-- for all the non CS people, a huge area of study in CS is sorting. Suppose you have a giant list

of numbers or list of words you want to alphabetize. Text indexing uses it. All kinds of stuff using sorting.

So how fast can you sort? Well the naive sort of simplest thing to program is called bubble sort. And if you have n numbers, it takes something times n squared, so flip operations. So time is of order n squared. But then there's merge sort, which is a divide and conquer algorithm, which takes $n \log n$. Now you could just say that. But it's also very helpful for students to have a feel, because generally students don't have an understanding of functions is what I found. They don't have a good feel for what functions do and mean. So what's the difference between n squared and $n \log n$? Well one way is to actually program a sort algorithm and then try it.

Try both algorithms and make your list bigger and bigger. When do you get fed up waiting for the damn thing to finish? So that's a visceral feeling. And actually that will tell you that there's a huge difference between $n \log n$ and n squared. Because this guy, basically when n is around thousand-- well, actually now computers are much faster, let's let's say 10,000-- you're pretty much going to be fed up with that. Because that's 100 million sort operations. It's going to take five or 10 seconds, maybe even bit longer if it's written in a higher level language. So maybe you're fed up around n equals 10,000.

Here 10,000 times $\log n$. Well, the log of anything is never bigger than 10, as a rough rule of thumb The reason being that if the log of the thing were bigger than 10, then n itself will be so big your computer couldn't even fit it. So you're not even dealing with numbers whose logs are bigger than 10. It depends on the base, but roughly speaking in base 10 logarithm, it's never going to be bigger than 10. So this is say at most 10. And so if you have a 10,000 only times a 10 here. Whereas here you have 10,000 times 10,000. Here you can go up easily to a million before you get fed up.

So the fed up is a connection to a visceral reasoning. And actually it then makes the difference between these really, really apparent. So there's ways you can do it even there. Question?

AUDIENCE:

I think that in the science fields that, if you really understood what you're teaching, then you should be able to basically put it into a simplification. Because then it's basically summarizing everything, and it's just so much better and closer. I mean, if you write for example a term paper or a proposal or whatever, you can write a one page text basically, or you can make a figure or a nice sketch. It's the same information.

PROFESSOR:

Right so the comment was that if you really understand something, you can make a really compact pictorial representation. That's generally true. And that's why it takes really long time to make those. So when you're making talks, for example, when we talk about this later in the term, when you're making talks and slides, a really good model for making slides is-- here's a slide. This is the four to three ratio sort of. So here is a sentence. So this is some assertion, some message you want to get across. And here is the evidence. The body of the slide is the evidence. And that's a picture. The best is to have some kind of visual evidence.

But what you find when you do this-- and it is, I think, the best way to make slides-- it takes a long time to really find the best picture. And even to find a good picture because it's just so tempting to write a bunch of words out. But actually if you look for that picture and you really try to capture the main idea in a picture, you will be much more successful at conveying the messages that you want to convey. So in general, try to talk to the perceptual system.

Now there was a question related to that which is that maybe I'm too harsh or too negative about the symbolic ways of doing things. And the particular example was so if you remember last time, I gave this example. So this is a pendulum, and this is the extreme of the pendulum motion. And the question was what is the acceleration vector at these various points in motion. And I said, well the way to do it is to reason intuitively about it, to figure out what it is here and here, understand what acceleration is, and do this one by interpolation.

And the comment was well, even if you can't do that, you can still use the Lagrangian. If I know the Lagrangian for the system, which for the non-physicist is

sort of the elephant gun. You can solve pretty much anything with the Lagrangian. So you can solve it with the Lagrangian and work out the forces everywhere. And that's true. But I still maintain that it's better to have-- I'm not saying it's bad to have a Lagrangian analysis. The more ways of doing things is better.

But there's a fundamentally important reason why it's important to have the intuitive way of doing it. And that is a search argument. So if you remember from before, we talked about chunking. The experts have big chunks, and the novices have small chunks. Well we looked at the effect of that of perception, which is that one of the proxies for it was the chess masters could remember whole positions, and the novices couldn't. Because their chunks were too small, and if you remember only seven chunks, which is typical, you can't remember a whole position. Whereas the chess master's chunks are like three, four, five pieces. And they can easily fit a whole position in seven chunks.

Well there's another, perhaps even more fundamental consequence of the different chunk size, and that is the exponential explosion in search space. So suppose you're here. This is your state now, and you're looking for a solution to the problem. So suppose the solution is here. But you don't know that yet. You're looking for the solution. Now if your chunk size is really big-- well let's say suppose your chunk size is really big-- you'll try a few possible chunks. OK, do any of those get me closer? Well this one looks a bit likely. Let me try expanding that one, maybe that one. Try this one. And then eventually you get your way over here.

So here suppose there's four possible approaches you could try, and because the chunks are big, there's only three levels. So this gives you four to the three items to search. OK, now suppose you make your chunks really, really small. Suppose your chunks are only half the size. So you now at every half branch you have to search for possibilities. So this is the large chunk.

So here we had one, two, three levels. Here we're going to have six levels, in other words, four to the six possibilities to search. So the difference doesn't show up in simple problems. Because you may have only one level to go through versus two

levels to go through, and either method works. But when you're trying to solve interesting new problems, you want your chunks to be as large as possible so your search space collapses to something manageable. So you don't want to have to reason with the Lagrangian as the default mode. It's fine for that particular problem. But if that problem is embedded in a bigger problem, you want to just know what the answer's going to be roughly. So you can continue making progress and decide should I go this route or this route.

So that's why I'm not negative on the standard ways. But I think the standard ways are actually genuinely bad for problem solving in new domains. And you want to augment your perception so that you guide your search in the right direction with big chunks. So that's fundamentally important.

So I think that was sort of the main themes in the questions, and I'll look over them and see which ones to answer probably at the beginning of next time. I'll try to leave more time for questions today, so we don't generate so many questions next time to overflow it. OK, so before I continue, any questions about these general principles of thinking about your teaching? Yes?

AUDIENCE: So the approach is trying to focus more on the perceptual way to understand thing, but on the other hand, the symbolic manipulations are useful. So how do you make sure that the students will themselves engage in the symbolic. How do you encourage them to do the symbolic reasoning themselves without scaring them?

PROFESSOR: So how do you encourage the students to do the symbolic reasoning? Because it is important. And I agree it is important. Just pure perception isn't quite enough. I think most courses I would say, and most teaching, is of 99% symbolic and 1% perceptual. And it depends on the field, but a rough estimate is I think it should be say something like 20% symbolic and maybe 80% perceptual initially. And then as you go to more and more advanced levels, if you've done it mostly perceptual earlier on, and you've developed the intuition, then you can increase the amount of symbolic.

So I think the way you do it is you build it into the structure of the curriculum. So one

particular course might just have one mix. And the more advanced students who got the perceptual material earlier are now ready for the symbolic. So there's actually a graph that I show for this. So that's the fraction of symbolic. And that's the fraction of perceptual material as you sort of go on in the major. So I think this is the ideal structure for a major. Because this is going beyond course design to whole design of a major in a curriculum.

So early on, say for the freshmen, it should be mostly perceptual. And there's many reasons for that. First of all their symbolic capacities-- we're all bad at symbolic capacities to start with. As freshmen, they're worse than the seniors. They haven't had time for example get sophisticated understanding of differential equations. Their algebra's rusty. It's not really practiced much. So don't rely on those modes of learning. Rely on the perceptual modes.

And the perceptual modes give you the big chunks. And so then once you have the big chunks, well then sure they're ready for the symbolic more and more. Because the symbolic fits into the big chunks and helps guide the search. It helps actually do the search. I have to calculate this. Then I calculate this. Well each calculation you do you need the symbolic. So by the time they're graduate students, hopefully the perceptual stuff they've had it up here. It's really solid. And now they're ready for partial differential equations and things like that. But earlier on, you don't want to avoid partial differential equations. You want to solve them in intuitive ways.

And an example of that is early on, when you're doing fluid mechanics, you would solve the cones using dimensional analysis, as we did before. You wouldn't solve the Navier-Stokes equations. And later on, for example, in a graduate class on computational fluid mechanics, you'd actually try to simulate that and figure out what the drag coefficient is numerically. So that's the mix between perception and symbolic that I think is a very good way to construct a whole curriculum. Does that answer your question? OK.