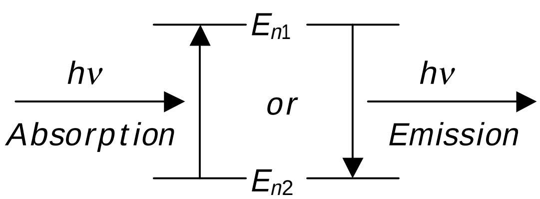
The ATOM of NIELS BOHR

Niels Bohr, a Danish physicist who established the Copenhagen school.

(a) Assumptions underlying the Bohr atom

(1) Atoms can exist in stable "states" without radiating. The states have discrete energies E_n, n = 1, 2, 3,..., where n = 1 is the lowest energy state (the most negative, relative to the dissociated atom at zero energy), n = 2 is the next lowest energy state, etc. The number "n" is an integer, a *quantum number*, that labels the state.

(2) Transitions between states can be made with the absorption or emission of a photon of frequency ν where $\nu = \dfrac{\Delta E}{h}$.



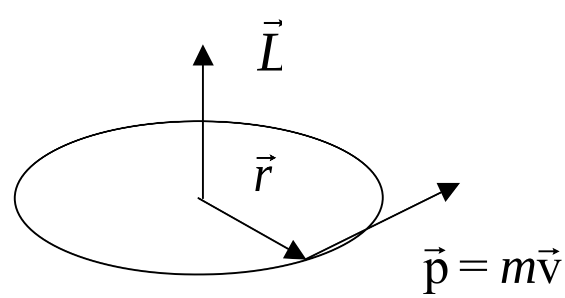
These two assumptions "explain" the discrete spectrum of atomic vapor emission. Each line in the spectrum corresponds to a transition between two particular levels. *This is the birth of modern spectroscopy*.

(3) Angular momentum is quantized: $\ell = n\hbar$ where $\hbar = \dfrac{h}{2\pi}$

Angular momentum

$$\vec{L} = \vec{r} \times \vec{p} \qquad \ell = \left|\vec{L}\right|$$

For circular motion:
$\vec{L}$ is constant if $\vec{r}$ and $\left|\vec{p}\right|$ are constant



$\boxed{l = mrv}$ is a constant of the motion

Other useful properties

$$\underbrace{v}_{\substack{\text{velocity}\\ \text{(m/s)}}} = \underbrace{(2\pi r)}_{\substack{\text{circumference}\\ \text{(m/cycle)}}} \cdot \underbrace{\nu_{rot}}_{\substack{\text{frequency}\\ \text{(cycles/s)}}} = r\ \underbrace{\omega_{rot}}_{\substack{\text{angular}\\ \text{frequency}\\ \text{(rad/s)}}}$$

$$\Rightarrow \quad \ell = mvr = mr^2\omega_{rot}$$

Recall the moment of inertia $I = \sum_i m_i r_i^2$

$\therefore$ For our system $I = mr^2$

$\Rightarrow \quad \ell = I\omega_{rot}$

Note: Linear motion vs. Circular motion

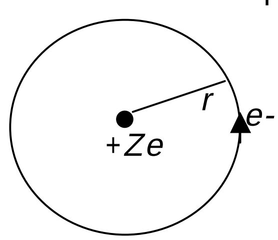
mass	m	$\leftrightarrow$	I	moment of inertia
velocity	v	$\leftrightarrow$	ω_{rot}	angular velocity
momentum	$p = mv$	$\leftrightarrow$	$\ell = I\omega$	angular momentum

Kinetic energy is often written in terms of momentum:

$$\text{K.E.} = \frac{1}{2}mv^2 = \frac{p^2}{2m} \qquad \text{K.E.} = \frac{1}{2}\frac{m^2r^2v^2}{mr^2} = \frac{\ell^2}{2I}$$

Introduce Bohr's quantization into the Rutherford's planetary model.

For a 1-electron atom with a nucleus of charge $+Ze$



$$r = \frac{Ze^2}{4\pi\varepsilon_0 mv^2} \quad \Rightarrow \quad r = \frac{n^2}{Z}\left(4\pi\varepsilon_0\right)\frac{\hbar^2}{me^2}$$ The radius is quantized!!

$$\left(4\pi\varepsilon_0\right)\frac{\hbar^2}{me^2} \equiv a_0$$ the *Bohr radius*

For H atom with $n = 1$, $r = a_0 = 5.29 \times 10^{-11}$ m = 0.529 Å (1 Å = 10^{-10} m)

Take Rutherford's energy and put in r,

$$E = -\frac{1}{2}\frac{Ze^2}{4\pi\varepsilon_0 r} \quad \Rightarrow \quad \boxed{E_n = -\frac{1}{n^2}\frac{Z^2 me^4}{8\varepsilon_0^2 h^2}}$$

Energies are quantized!!!

For H atom, emission spectrum

$$\bar{\nu}\left(\text{cm}^{-1}\right) = \frac{E_{n2}}{hc} - \frac{E_{n1}}{hc} = \frac{me^4}{8\varepsilon_0^2 h^3 c}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Rydberg formula ! with $R = \dfrac{me^4}{8\varepsilon_0^2 h^3 c} = 109{,}737 \text{ cm}^{-1}$

Measured value is 109,678 cm^{-1} (Slight difference due to model that gives nucleus no motion at all, i.e. infinite mass.)