ELECTRON SPIN

Experimental evidence for electron spin

Compton Scattering (1921): AH Compton suggested that "the electron is probably the ultimate magnetic particle."

Stern-Gerlach Experiment (1922): Passed a beam of silver atoms ($4d^{10}5s^1$) through an inhomogeneous magnetic field and observed that they split into two beams of space quantized components.

Uhlenbeck and Goudsmit (1925) showed that these were two angular momentum states – the electron has intrinsic angular momentum – "SPIN" angular momentum

Pauli Exclusion Principle (1925): no more than 2 electrons per orbital, or, no two electrons with all the same quantum numbers. Additional quantum number, now called m_s, was postulated.

Postulate 6: *All electronic wavefunctions must be antisymmetric under the exchange of any two electrons.*

Theoretical Justification

Dirac (1928) developed relativistic quantum theory & derived electron spin angular momentum

Orbital Angular Momentum

L = orbital angular momentum

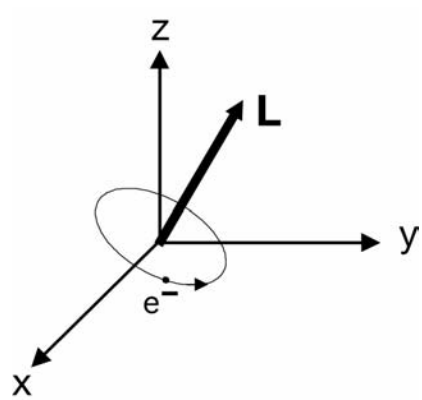
$|L| = \hbar\sqrt{l(l+1)}$

l = orbital angular momentum quantum number

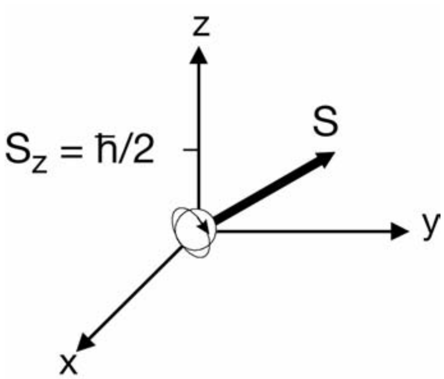
$l \le n-1$

$L_z = m\hbar$

$m = 0, \pm1, \pm2, \ldots, \pm l$



Spin Angular Momentum



$S \equiv$ *spin* angular momentum
$|S| = \hbar\sqrt{s(s+1)} = \hbar\sqrt{3}/2$
s = *spin* angular momentum quantum number
$s = 1/2$
$S_z = m_s\hbar$
$m_s = \pm 1/2$

Define spin angular momentum operators analogous to orbital angular momentum operators

$$L^2 Y_l^m(\theta,\phi) = l(l+1)\hbar^2 Y_l^m(\theta,\phi) \qquad l = 0,1,2,...n \text{ for H atom}$$
$$L_z Y_l^m(\theta,\phi) = m\hbar Y_l^m(\theta,\phi) \qquad \text{m} = 0,\pm1,\pm2,...\pm n \text{ for H atom}$$

$$\hat{S}^2\alpha = s(s+1)\hbar^2\alpha \qquad \hat{S}^2\beta = s(s+1)\hbar^2\beta \qquad s = \frac{1}{2} \text{ always}$$
$$\hat{S}_z\alpha = m_s\hbar\alpha \quad m_s^\alpha = \frac{1}{2} \qquad \hat{S}_z\beta = m_s\hbar\beta \qquad m_s^\beta = -\frac{1}{2}$$

Spin eigenfunctions α and β are not functions of <u>spatial coordinates</u> so the equations are somewhat simpler!

$$\alpha \equiv \text{"spin up"} \qquad \beta \equiv \text{"spin down"}$$

Spin eigenfunctions are orthonormal:

$$\int \alpha^*\alpha d\sigma = \int \beta^*\beta d\sigma = 1 \qquad \sigma \equiv \text{ spin variable}$$
$$\int \alpha^*\beta d\sigma = \int \beta^*\alpha d\sigma = 0$$

Spin variable has no classical analog. Nevertheless, the angular momentum of the electron spin leads to a magnetic moment, similar to orbital angular momentum.

Electron orbital magnetic moment

$$\boldsymbol{\mu}_L = -\frac{e}{2m_e}\mathbf{L}$$

$$|\boldsymbol{\mu}_L| = -\frac{e\hbar}{2m_e}\sqrt{l(l+1)} \equiv -\beta_0\sqrt{l(l+1)}$$

$$\mu_{L_z} = -\frac{e}{2m_e}L_z = -\frac{e\hbar}{2m_e}m = -\beta_0 m$$

Electron spin magnetic moment

$$\boldsymbol{\mu}_s = -\frac{e}{2m_e}g\mathbf{S}$$

$$|\boldsymbol{\mu}_s| = -\frac{e\hbar}{2m_e}g\sqrt{s(s+1)} = -\beta_0 g\sqrt{s(s+1)}$$

$$\mu_{S_z} = -\frac{e}{2m_e}gS_z = -\frac{e\hbar}{2m_e}gm_s = -\beta_0 g m_s \approx \pm\beta_0$$

$$g \equiv \text{"electronic g factor"} = 2.002322$$

Total electronic wavefunction has both SPATIAL and SPIN parts.
Each part is normalized so the total wavefunction is normalized

$$\Psi(r,\theta,\phi,\sigma) = \psi(r,\theta,\phi)\alpha(\sigma) \quad \text{or} \quad \psi(r,\theta,\phi)\beta(\sigma)$$

e.g. for H atom the ground state total wavefunctions (in atomic units) are

$$\Psi_{100\frac{1}{2}} = \left(\frac{Z^3}{\pi}\right)^{1/2} e^{-Zr}\alpha \qquad \Psi_{100-\frac{1}{2}} = \left(\frac{Z^3}{\pi}\right)^{1/2} e^{-Zr}\beta$$

which are orthogonal and normalized. Note the quantum numbers are now --

$nlmm_s$