PRINCIPLES OF QUANTUM MECHANICS (cont'd)

COMMUTATORS

Order counts when applying multiple operators!

e.g. $\hat{A} = \hat{p}\hat{x}$ = ? and can we write $\hat{p}\hat{x} = \hat{x}\hat{p}$?

$\Rightarrow$ operate on function to obtain .

$$\hat{A}f(x) = g(x) \quad \Rightarrow \quad (\hat{p}\hat{x})f(x) = \left(-i\hbar\frac{d}{dx}\right)(x)f(x)$$

$$= -i\hbar x\frac{d}{dx}f(x) - i\hbar f(x) = \left(-i\hbar x\frac{d}{dx} - i\hbar\right)f(x)$$

$$\therefore \quad \hat{A} = \left(-i\hbar x\frac{d}{dx} - i\hbar\right) = (\hat{p}\hat{x})$$

Now try $\hat{B} = \hat{x}\hat{p}$

$$\hat{B}f(x) = (x)\left(-i\hbar\frac{d}{dx}\right)f(x) = \left(-i\hbar x\frac{d}{dx}\right)f(x)$$

$$\therefore \quad \hat{B} = -i\hbar x\frac{d}{dx} = \hat{x}\hat{p}$$

$$\therefore \quad \hat{x}\hat{p} \neq \hat{p}\hat{x}$$

Define commutator

For two operators $\hat{A}$ and $\hat{B}$,

$$\left[\hat{A},\hat{B}\right] = \hat{A}\hat{B} - \hat{B}\hat{A} = \hat{C}$$

← need not be zero!

e.g.

$$\left[\hat{x},\hat{p}\right] = \hat{x}\hat{p} - \hat{p}\hat{x} = i\hbar \neq 0!$$

Important general statements about commutators:

1) For operators that <u>commute</u>

$$\left[\hat{A},\hat{B}\right]=0$$

- it is possible to find a set of wavefunctions that are eigenfunctions of both operators simultaneously.

e.g. can find wavefunctions ψ_n such that

$$\hat{A}\psi_n = a_n\psi_n \quad \text{and} \quad \hat{B}\psi_n = b_n\psi_n$$

- This means that we <u>can</u> know the exact values of both observables *A* and *B* simultaneously (no uncertainty limitation).

2) For operators that do <u>not</u> commute

$$\left[\hat{A},\hat{B}\right]\neq 0$$

- it is <u>not</u> possible to find a set of wavefunctions that are simultaneous eigenfunctions of both operators.

- This means that we <u>cannot</u> know the exact values of both observables *A* and *B* simultaneously $\Rightarrow$ *uncertainty*!

e.g. $\left[\hat{x},\hat{p}\right]=i\hbar\neq 0 \quad \Rightarrow \quad \Delta x\Delta p\geq\frac{\hbar}{2}$