Kinetics of Elementary Reactions: Radioactive Decay

See lecture 31 for an introduction to kinetics and lecture 32 for the kinetics of radioactive decay.

Radioactive Decay. The decay of a nucleus is **independent** of the number of surrounding nuclei that have decayed. We can apply first order integrated rate laws:

$[A] = [A]_0\, e^{-k\,t}$ and $t_{1/2} = \dfrac{0.6931}{k}$

However, instead of concentration, the first order integrated rate law is expressed in terms of N (number of nuclei):

$N = N_o e^{-k\,t}$ $k \equiv$ decay constant

$t \equiv$ time

$N_0 \equiv$ number of nuclei originally present

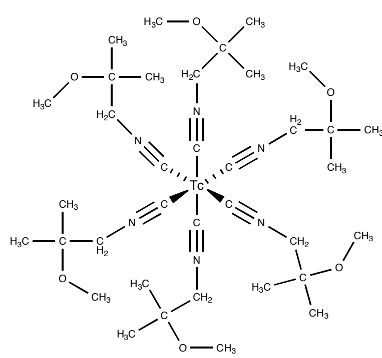
Nuclear kinetics – monitor rate of occurrence of decay events with a Geiger counter (radiation detector). Decay rate is also called Activity (A)

$N = N_o e^{-k\,t}$ can be expressed as $A = A_o e^{-k\,t}$ $A \equiv$ Activity $A_o \equiv$ initial Activity

Example from pg 3 of Lecture 32 notes: Medical Applications of Radioactive Decay.

Technetium(Tc)-99 is the most widely used radioactive nuclide in medicine. It is used for diagnostic organ imaging and bone scans, with over 7 million uses annually in the US.

One of the patent holders for the technetium-based imaging agent cardiolite^{TM} is MIT Professor of Chemistry Alan Davison.



Cardiolite^{TM} is a coordination complex, and Prof. Davison figured out which ligands to use (CN^-) to obtain the desired properties of solubility and stability to be applied to medical imaging. Cardiolite has saved many lives in diagnosing coronary artery disease.

In a Cardiolite stress test, the molecule is administered by IV and travels through the blood into the heart. Since the drug cannot access areas of the heart with insufficient blood supply, a subsequent scan reveals any blocked arteries.

Recitation or homework example:
Calculate the total activity (in disintegrations per second) caused by the decay of 0.5 microgram of ^{99m}Tc (an excited nuclear state of ^{99}Tc), which has a half-life of 6.0 hours.

To calculate the activity of a sample of 1.0 mg of ^{99m}Tc, we can use the following equation: A=kN. We need to first determine the decay constant, k and the number of nuclei.
To calculate the number of nuclei:

$$\#\ \text{of nuclei} = \left(0.5 \text{ x } 10^{-6}\text{g}\right)\left(\frac{\text{mol}}{99.00 \text{ g}}\right)\left(\frac{6.022 \text{ x } 10^{23} \text{ atoms}}{\text{mol}}\right)$$

$$= 3.0414 \text{ x } 10^{15}$$

To calculate the decay constant:

$$t_{\frac{1}{2}} = 6.0 \text{ hrs}\left(\frac{60 \text{ min}}{\text{hr}}\right)\left(\frac{60 \text{ sec}}{\text{min}}\right)$$

$$= 2.16 \text{ x } 10^{4} \text{ s}$$

$$k = \frac{\ln 2}{t_{\frac{1}{2}}}$$

$$= \frac{0.6931}{2.16 \text{ x } 10^{4} \text{ s}}$$

$$= 3.2088 \text{ x } 10^{-5} \text{ s}^{-1}$$

We can now substitute those values into the equation A=kN;

$$A = kN$$

$$= \left(3.2088 \text{ x } 10^{-5} \text{ s}^{-1}\right)\left(3.0414 \text{ x } 10^{15} \text{ nuclei}\right)$$

$$= 9.759 \text{ x } 10^{10}$$

$$= 1 \text{ x } 10^{11} \text{ disintegrations per second}$$

A sample of 0.5 μg of ^{99m}Tc has the activity of 1 x 10^{11} disintegrations s^{-1}