10.52
Mechanics of Fluids
Spring 2006
Problem Set 4

Problem 1

Consider a sinusoidal traveling wave of small amplitude ($Ak \ll 1$) such that the free surface is given by:

$$\eta = A \cos (kx - \omega t)$$

The corresponding velocities are given by:

$$u = A\omega e^{ky} \cos (kx - \omega t)$$

and

$$v = A\omega e^{ky} \sin (kx - \omega t)$$

(y is the vertical coordinate and is negative in the direction of g; x is the horizontal coordinate)

a) Sketch the streamlines at $t = o$ that pass through: $x = o, y = o$; $x = 0, y = \frac{-2\pi}{k}$

b) Sketch the particle line at $t = \frac{2\pi}{\omega}$ for the particle which was at $x = o, y = o$ at $t = o$

c) Sketch the particle line at $t = \frac{2\pi}{\omega}$ for the particle which was at $x = o, y = o$ at $t = \pi/\omega$

d) Sketch the streak line at $t = \frac{2\pi}{\omega}$ formed by all particles emitted from $x = o$, $y = -A$ since $t = o$

Note: This problem is hard if you fail to take advantage of the fact that $Ak \ll 1$, but it is not hard if you do take advantage of this fact.

Problem 2 (Problem by Ain Sonin and Ascher Shapiro. Used with permission.)

For an ideal rectilinear vortex, the velocity profile is given by $v_r = v_z = 0$; $v_\theta = K/r$. A qualitatively similar result is associated with an ideal smoke ring.

a) Why does a smoke ring propel itself? Why does the velocity of translation diminish with time?

b) As shown in the figure on page 3, given two successive smoke rings, the trailing ring will tend to overtake and pass through the leading ring. Why?

Problem 2 - Figure

Image removed due to copyright reasons.

Problem 3 (Problem by Ain Sonin and Ascher Shapiro. Used with permission.)

In two-dimensional planar flow, the potential function of a point source located at the origin is given (in polar co-ordinates) by:

$$\phi = \frac{Q}{2\pi} \ln r$$

Q is referred to as the strength of the source.
Similarly, a sink has potential:

$$\phi = -\frac{Q}{2\pi} \ln r$$

If a source of strength Q is located at $r = b/2$, $\theta = 0$, and a sink of strength Q at $r = -b/2$, $\theta = \pi$; what is the potential function? What is the stream function? Sketch both. Also sketch for the case in which $b \to 0$, but Qb remains constant.

Problem 4

The stream function of a certain planar flow is, in polar co-ordinates:

$$\Psi = A\, r^{2/3} \sin(2/3\, \theta)$$

Consider only the region $0 \le \theta \le \frac{3\pi}{4}$

a) Sketch several streamlines.

b) Find the x and y velocity components at (x, y) = (0, 0); (-1, 1); (∞, 0).

c) Find the volumetric flow rate across any line joining (0, 0) and (-1, 1).

d) If the pressure at (-1, 1) is p*, what is the pressure at (0, 0)?

Problem 5

Consider a planar flow for which the stream function, as expressed in polar coordinates, is:

$$\Psi = A\, r^{\frac{2\pi}{\theta_0}} \sin\left(\frac{2\pi\theta}{\theta_0}\right)$$

where A and θ_0 are constants.

a) Sketch several streamlines for the region $\theta_0 < \theta < \frac{\pi}{2}$.

b) Give an expression for v_r along the line $\theta = \theta_0$.

c) For a fixed value of r, sketch (vs. a coordinate perpendicular to the line $\theta = \theta_0$) the velocity component in the direction parallel to the line $\theta = \theta_0$.

d) For the same fixed value of r, repeat Part (c) for the velocity component in the direction normal to the line $\theta=\theta_0$.