Combinatorics

When working with particles on a lattice (whether it be 1, 2, or 3 dimensional), use the equations below to determine Ω, the number of possible configurations the system can take.

For M = # of sites
N = # of particles in system

Case #1:
Particles are : non-interacting (more than one can occupy same site)
distiguishable

$$\Omega = M^N$$

Case #2:
Particles are : exclusionary (only one can occupy a given site)
distiguishable

$$\Omega = \frac{M!}{(M-N)!}$$

Case #3:
Particles are : exclusionary (only one can occupy a given site)
indistiguishable

$$\Omega = \frac{M!}{N!(M-N)!}$$

Case #4:
Particles are : non-interacting (more than one can occupy same site)
indistiguishable

$$\Omega = \frac{(M+N-1)!}{N!(M-1)!}$$

These equations can be used to determine the entropy for the microcanonical ensemble. For this ensemble, since each microstate has the same energy, the entropy is determined solely by the number of possible configurations of the system:

$$\underline{S} = \underline{S}_{\text{config}} = k \ln \Omega$$