10.34 – Fall 2006

Homework #6

Due Date: Wednesday, Oct. 18th, 2006 – 9 AM

Run each problem using the explicit solver ode45 and a stiff implicit solver (ode23s). Compare the time-to-solution, and check whether ode45 gives accurate answers.

1) For a slightly modified chemostat system of Quiz 1, consider what happens at start-up for a couple of different cases. *(useful Matlab code is included on the web site)*

$$Cell\ Multiplication = \frac{k_1 N_{cells}\left[Nutrients\right]}{\left(1+c_1\left[Nutrients\right]\right)\left(1+d\left[P\right]\right)}$$

$$\frac{dN_{cells}}{dt} = Cell\ Multiplication - Rate\, at\, which\, cells\ flow\, out\, of\ reactor$$

$$Nutrient\, Cons.\, Rate = k_2 N_{cells}\left(\left[Nutrients\right] - 1\times10^{-6}M\right) + c_2\left(Cell\, Multiplication\right)$$

$$P\ production\, rate = \frac{k_3 N_{cells}\exp\left(-d\left[P\right]\right)}{\left(1+c_1\left[Nutrients\right]\right)}\cdot\left(\left[Nutrients\right]-0.01\right)^2$$

$k_1 = 1\times10^{-3}\ M^{-1}s^{-1}$ $\qquad c_1 = 0.1\quad M^{-1}$

$k_2 = 1\times10^{-9}\ moles\ cell^{-1}\, M^{-1}\, s^{-1}$ $\qquad c_2 = 1\times10^{-7}\quad moles\ cell^{-1}$

$k_3 = 1\times10^{-8}\ moles\ cell^{-1}\, s^{-1}\, M^{-2}$ $\qquad d = 0.01\quad M^{-1}$

First, you run a 150 liter CSTR seeded with cells to steady state (the flow rate is 2.3 L/min with $[Nut]_{in}$ = 0.2 M). Then, you fill an empty 230 liter CSTR with 229 liters of nutrient solution (0.2 M) and then stop the nutrient flow. The flow rate is constant at 2.3 L/min for all parts of the problem. At time = 0, you pour in a one liter aliquot of the output of the steady-state 150 L reactor into the 230 L chemostat. Then at time $t = t_{nutrient}$ you turn on the nutrient flow to the 230 L reactor. Run the cases with $t_{nutrient}$ = *0 hr*, *2.0 hr*, and *8.0 hr*. Note that for $t_{nutrient} > 0$, the reactor will operate in a batch mode until the flow is started.

a. Plot (log-log) the number of cells vs. time in the 230 L CSTR for each case (x-axis range from 0.01 hr → 25 hr).
b. How long does it take the system to come within 1% of the long-time steady-state concentrations with each different start-up procedure?
c. Also, discretize the 0 → 25 hr time frame in 100-second bins, determine the number of integrator steps taken within each 100-second period, and plot the results on a semilogx plot (x-axis range from 0.01 hr → 25 hr).
d. Give the total number of integrator steps taken for each case, and compare the CPU time required for the implicit/explicit solvers.

2) When designing filters and other optical components for high-power lamps and lasers, one must be concerned about the possibility that the optical component will be overheated or damaged by thermal shock when the power is turned on. Consider systems described by the differential equations presented in an earlier homework, at steady-state:

$$\frac{dI}{dy} = -(a + cT^2)I$$

$$k\frac{d^2T}{dy^2} - \frac{dI}{dy} = 0$$

$I_{(x=0)} = I_0 \qquad T_{(x=0)} = T_{(x=L)} = T_a$

a) Convert these steady-state equations into standard 1st-order ODE form.

b) Solve the equations using the ODE-IVP solvers, assuming that the filter is 1 cm wide and the intensity of laser beam is 0.3 MW/m^2. The ambient temperature is 300 K and the thermal conductivity of the filter is 0.6 W/m/K. For the following values of a and c

   1. a = 10 c = 1e-7
   2. a = 1000 c = 1e-7
   3. a = 10 c = 3e-4
   4. a = 10000 c = 1e-7

   plot the graphs of temperature with the filter width.

   Since one of the boundary conditions is given at x=L, not x=0, use the “shooting” method to vary your initial guess for the variable whose value at x=0 is not known. (You might want to use bisection to vary this initial guess).

c) For coefficients in part 4 of the previous section, what is the distance at which the temperature in the filter is maximum. How many values of x are used by the ODE solver is the region of filter before the maximum temperature is reached. In HW 3, we had solved essentially the same problem by formulating it as a BVP, if you wanted to use that method would 500 grid points be sufficient for this problem?