MIT Department of Brain and Cognitive Sciences
9.641J, Spring 2005 - Introduction to Neural Networks
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Antisymmetric networks

Antisymmetry

- Idealization of interaction between excitatory and inhibitory neuron

Olfactory bulb

- Dendrodendritic connections between and granule (inhibitory) cells
- 80% are reciprocal, side-by-side pairs

Linear antisymmetric network

$$\dot{x} = Ax, \quad A^T = -A$$

- Superposition of oscillatory components
- $x^T A^n x$ is conserved for even *n*

Eigenvalues of a real antisymmetric matrix are either zero or pure imaginary.

Simple harmonic oscillator

- antisymmetric after rescaling

$$\dot{q} = \frac{p}{m}$$

$$\dot{p} = -kq$$

$$\dot{x}_1 = -\omega x_2$$

$$\dot{x}_2 = \omega x_1$$

Antisymmetric network

$$\dot{x} = f(b + Ax), \quad A = -A^T$$

- bias can be removed by a shift, if A is nonsingular
- decay term is omitted because it's symmetric

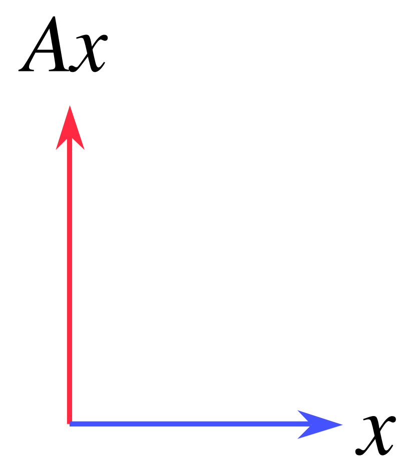
Conservation law

$$H = 1^T F(Ax) = \sum_i F((Ax)_i)$$

- *H* is a constant of motion

Ax is perpendicular to *x*

$$x^T A x = \sum_{ij} x_i A_{ij} x_j = 0$$



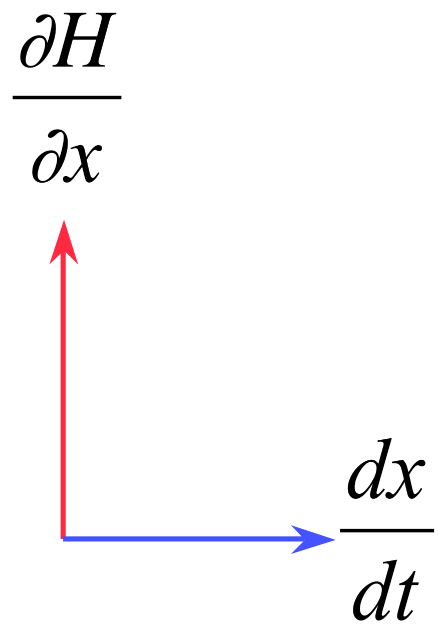
Velocity is perpendicular to the gradient of *H*

$$H = 1^T F(Ax)$$

$$\frac{\partial H}{\partial x} = A^T f(Ax)$$

$$= A^T \dot{x}$$

$$\dot{H} = \dot{x}^T A^T \dot{x} = 0$$



Cowan (1972)

Action principle

- Stationary for trajectories that satisfy $x(0)=x(\mathrm{T})$

$$S=\int_0^T dt\left[\frac{1}{2}\dot{x}^T A\dot{x}-1^T F\left(Ax\right)\right]$$

Effect of decay term

- Introduction of dissipation

$$\dot{x} + x = f(Ax), \quad A = -A^T$$

$$L(x) = 1^T F(Ax) + 1^T \overline{F}(x)$$

Lyapunov function

$$-\frac{\partial L}{\partial x} = A^T f(Ax) + f^{-1}(x)$$

$$= -A\dot{x} - Ax + f^{-1}(x)$$

$$= -A\dot{x} - f^{-1}(\dot{x} + x) + f^{-1}(x)$$

$$\dot{L} = \dot{x}^T \frac{\partial L}{\partial x}$$

$$= -\dot{x}^T A\dot{x} - \dot{x}^T\left[f^{-1}(\dot{x} + x) - f^{-1}(x)\right]$$

$$= -\dot{x}^T\left[f^{-1}(\dot{x} + x) - f^{-1}(x)\right] \leq 0$$