Problem Set 7 (due Thurs Apr 29)
Simulation of synaptic inputs

April 23, 2004

1. Simulate an integrate-and-fire neuron with a whole mess of synaptic inputs, as follows. (This model is based on that in S. Song, K.D. Miller, and L.F. Abbott, “Competitive Hebbian learning through spike-timing-dependent synaptic plasticity”, *Nat. Neurosci.*, **3**(9):919-26, Sep. 2000. You won’t have to deal with synaptic plasticity in your version, for which you can be grateful.)

   The time evolution of the membrane potential V is given by

   $$\tau_m \frac{dV}{dt} = V_{rest} - V + g_{ex}(t)(E_{ex} - V) + g_{in}(t)(E_{in} - V)$$

   When V reaches V_{thresh}, the neuron fires and the potential returns to V_{reset}. There are N_{ex} excitatory neurons and N_{in} inhibitory neurons providing input to the cell, with synaptic conductances g_{ex} and g_{in}. These conductances change according to the firing of their respective input neurons: when input neuron i fires, $g_{ex}(t)$ is incremented by $\bar{g}_{ex}$ or $g_{in}(t)$ is incremented by $\bar{g}_{in}$, depending on whether the input neuron was excitatory or inhibitory. Between incoming spikes, both synaptic conductances decay exponentially:

   $$\tau_{ex} \frac{dg_{ex}}{dt} = -g_{ex}; \qquad \tau_{in} \frac{dg_{in}}{dt} = -g_{in}$$

   Have all the input neurons fire according to a Poisson process, with rate λ_{ex} or λ_{in}.

   Got all that? Here are your parameter values: $\tau_m = 20$ ms, $V_{rest} = -70$ mV, $E_{ex} = 0$ mV, $E_{in} = -70$ mV, $V_{thresh} = -54$ mV, $V_{reset} = -60$ mV, $\tau_{ex} = \tau_{in} = 5$ ms, $N_{ex} = 1000, N_{in} = 200$.

   By adjusting the values of the constants $\bar{g}_{ex}, \bar{g}_{in}, \lambda_{ex}, \lambda_{in}$, you can make the output neuron fire regularly, or very irregularly. Turn in plots of V as a function of time demonstrating both kinds of firing, along with the values you used for these parameters. Try $\bar{g}_{ex} = 0.015, \bar{g}_{in} = 0.05, \lambda_{ex} = \lambda_{in} = 10$ Hz to start with.

2. In the regular firing case, estimate the firing rate analytically by computing the average synaptic conductances, and deriving a result similar to problem 4b of problem set 5. Check your result against your simulation.