6.581/20.482

LECTURE 20: DECONVOLUTION

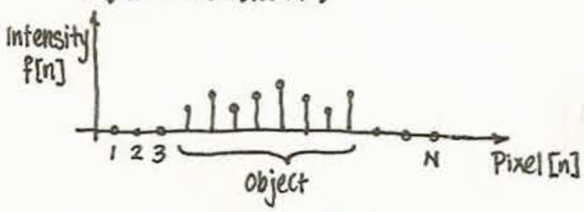
THURSDAY 27 APRIL 2006

DECONVOLUTION

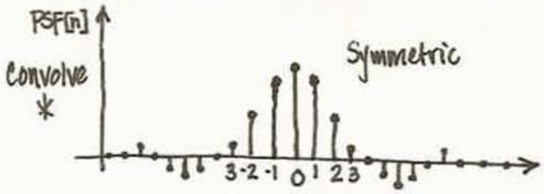
1.) Point spread function representation
2.) Straightforward deconvolution
3.) Handling noise (SVD)

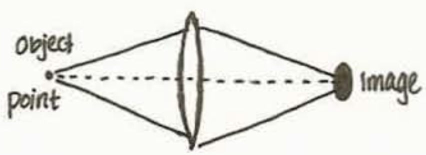
1-D

Object (black & white)



Point Spread Function (PSF)





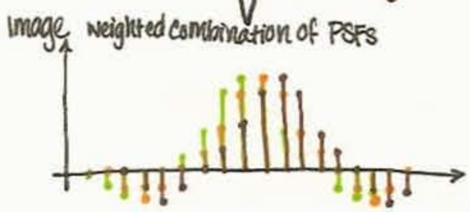
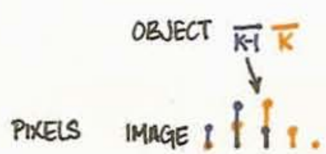


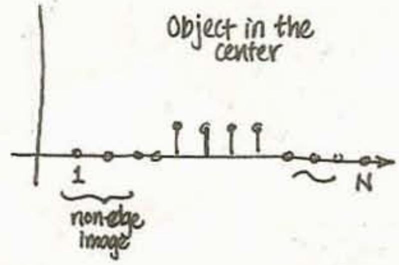
Image $\qquad\qquad$ $k-1 \quad k \quad k+1$ $\qquad$ $\underbrace{\text{assume periodicity}}$ $\qquad$ Object

$$\begin{bmatrix} \hat{G}[1] \\ \vdots \\ \hat{G}[k] \\ \vdots \\ \hat{G}[N] \end{bmatrix} = k \begin{bmatrix} PSF[0] & PSF[-1] & & & PSF[2] & PSF[1] \\ & & & & & \\ \cdots PSF[2] & PSF[1] & PSF[0] & PSF[-1] & PSF[-2] \cdots & \\ & & & & & \end{bmatrix} \begin{bmatrix} \hat{F}[1] \\ \vdots \\ \hat{F}[N] \end{bmatrix}$$

$\begin{bmatrix} \vdots \\ f[N-1] \\ f[N] \end{bmatrix}$ pretend object repeats periodically



K-1's contribution at K is the +1 contribution from K-1 $\Rightarrow$ PSF[1]



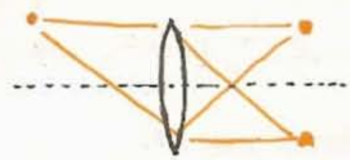
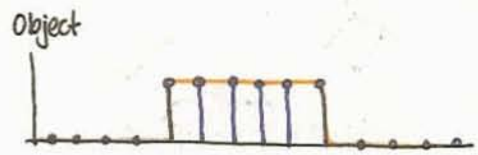


Image $\qquad$ Object

$$\vec{G} = \underbrace{H}_{\text{Matrix}} \vec{F}^{\circ}$$



$$\vec{G}^{\circ} = H\vec{F}^{\circ} \qquad \vec{F} = H^{-1}\vec{G}^{\circ}$$

Image Pixels ($\vec{G}^{\circ}$), Object Pixels ($\vec{F}^{\circ}$), reconstructed image ($\vec{F}$)

When the smallest bit of noise is added to the image & then reconstructed, the resulting reconstructed image is garbage
$\Rightarrow$ very fragile

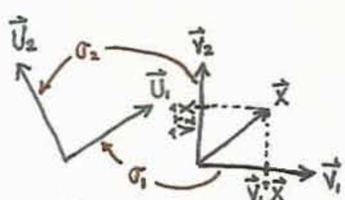
NOISE

$$\vec{G} = \vec{G}^{\circ} + \underbrace{\vec{W}}_{\text{vector of noise}} = H\vec{F}^{\circ} + \vec{W}$$

$$\vec{F}^{R} = H^{-1}(H\vec{F}^{\circ} + \vec{W}) = \vec{F}^{\circ} + \underbrace{H^{-1}\vec{W}}_{\text{How large?}}$$

$$H = U \Sigma V^{T}$$

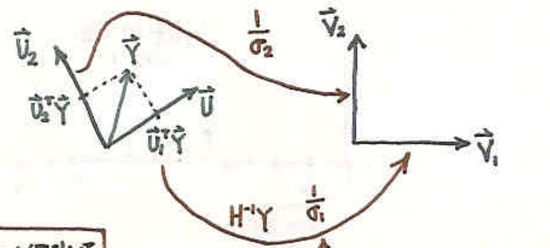
U: output coordinate system; V^T: input coordinate system; $\Sigma = \begin{bmatrix} \sigma_1 & & & \\ & \sigma_2 & & \\ & & \ddots & \\ & & & \sigma_n \end{bmatrix}$



$$HX = \sum_{i=1}^{N} \vec{U}_i \sigma_i \vec{V}_i^T X$$

σ_i values are small

$H = U\Sigma V^T$



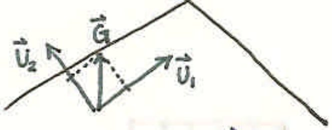
$\boxed{H^{-1} = V\Sigma^{-1}U^T}$

$\frac{1}{\sigma_i}$ values are huge

$$\vec{G}^\circ = H\vec{F}_o = \sum_i \vec{u}_i \sigma_i \vec{v}_i^T \vec{F}^\circ = \sigma_1 \vec{v}_1^T \vec{F}^\circ \begin{bmatrix} \vdots \\ \vec{u}_1 \\ \vdots \end{bmatrix} + \sigma_2 \vec{v}_2^T \vec{F}^\circ \begin{bmatrix} \vdots \\ \vec{u}_2 \\ \vdots \end{bmatrix} + \ldots + \sigma_N \vec{v}_N^T \vec{F}^\circ \begin{bmatrix} \vdots \\ \vec{u}_N \\ \vdots \end{bmatrix}$$

~~Perfect Reconstruction (PR)~~

$$\cancel{\vec{F}^{PR} = \vec{v}_1^T \vec{F}^\circ [\vec{u}_1] + \vec{v}_2^T \vec{F}^\circ [\vec{u}_2] + \ldots}$$



$$\vec{G} = \vec{G}^\circ + \vec{W} = \overbrace{\sum_i \vec{u}_i \sigma_i \vec{v}_i^T \vec{F}^\circ}^{H\vec{F}^\circ} + \vec{W}$$

$$\vec{F}^R = \underbrace{\sum_i \vec{v}_i \frac{1}{\sigma_i} \vec{u}_i^T \vec{G}}_{H^{-1}\vec{G}}$$

$$\vec{F}^R = \sum_i \vec{v}_i \frac{1}{\sigma_i} \left(\sigma_i \vec{v}_i^T \vec{F}^\circ + u_i^T \vec{W} \right)$$

$$= \sum_i \left[\overbrace{\underbrace{\vec{v}_i \vec{v}_i^T \vec{F}^\circ}_{\text{perfect reconstruction}}}^{\text{"SIGNAL"}} + \overbrace{\vec{v}_i \frac{1}{\sigma_i} (u_i^T \vec{W})}^{\text{"NOISE"}} \right]$$

$$\vec{F}^R = \left(\vec{v}_1 \vec{v}_1^T \vec{F}^\circ + \vec{v}_1 \frac{1}{\sigma_1} \vec{u}_1^T \vec{W} \right) + \vec{v}_2 \left(\vec{v}_2^T \vec{F}^\circ + \frac{1}{\sigma_2} \vec{u}_2^T \vec{W} \right) + \ldots + \vec{v}_N \left(\vec{v}_N^T \vec{F}^\circ + \frac{1}{\sigma_N} \vec{u}_N^T \vec{W} \right)$$

→ Remove terms with small σ