Table 2.7 Complete Description of Electrodynamics

Name	Differential form
(1) Gauss' law	$\nabla \cdot \epsilon \boldsymbol{E} = \rho_e$
(2) Faraday's law	$\nabla \times \boldsymbol{E} = -\dfrac{\partial \mu \boldsymbol{H}}{\partial t}$
(3) Ampère's law	$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \dfrac{\partial \epsilon \boldsymbol{E}}{\partial t}$
(4) Magnetic flux	$\nabla \cdot \mu \boldsymbol{H} = 0$
(5) Charge conservation	$\nabla \cdot \boldsymbol{J} = -\dfrac{\partial \rho_e}{\partial t}$
(6) Lorentz force law	$\boldsymbol{F} = \rho_e [\boldsymbol{E} + \boldsymbol{v} \times \mu \boldsymbol{H}]$
(7) Newton's law (single charged particle)	$m\,(\partial \boldsymbol{v}/\partial t) = \underbrace{q(\boldsymbol{E} + \boldsymbol{v} \times \mu \boldsymbol{H})}_{\boldsymbol{f}^{\text{elec}}} + \boldsymbol{f}^{\text{other}}$

Constitutive Laws for Linear, Isotropic Media

$\boldsymbol{D} = \varepsilon \boldsymbol{E} = \varepsilon_o \boldsymbol{E} + \boldsymbol{P}$

$\boldsymbol{B} = \mu \boldsymbol{H}$

$\boldsymbol{J} = \sigma \boldsymbol{E}$

Source: Grodzinsky, Alan. Field, Forces and Flows in Biological Systems. Garland Science, 2011. [Preview with Google Books]

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either H = 0, or
- ∂/∂t ≈ small
- low enough freq
- $\lambda >> L^{\text{char}}$

Constitutive Law

$$\boldsymbol{J} = \sigma \boldsymbol{E} + \ldots$$

Source: Grodzinsky, Alan. Field, Forces and Flows in Biological Systems. Garland Science, 2011. [Preview with Google Books]

From a painting at the Deutsches Museum, Munich.

GEORG SIMON OHM

1789-1854

Mathematician and experimentalist

Current Flow in conductors

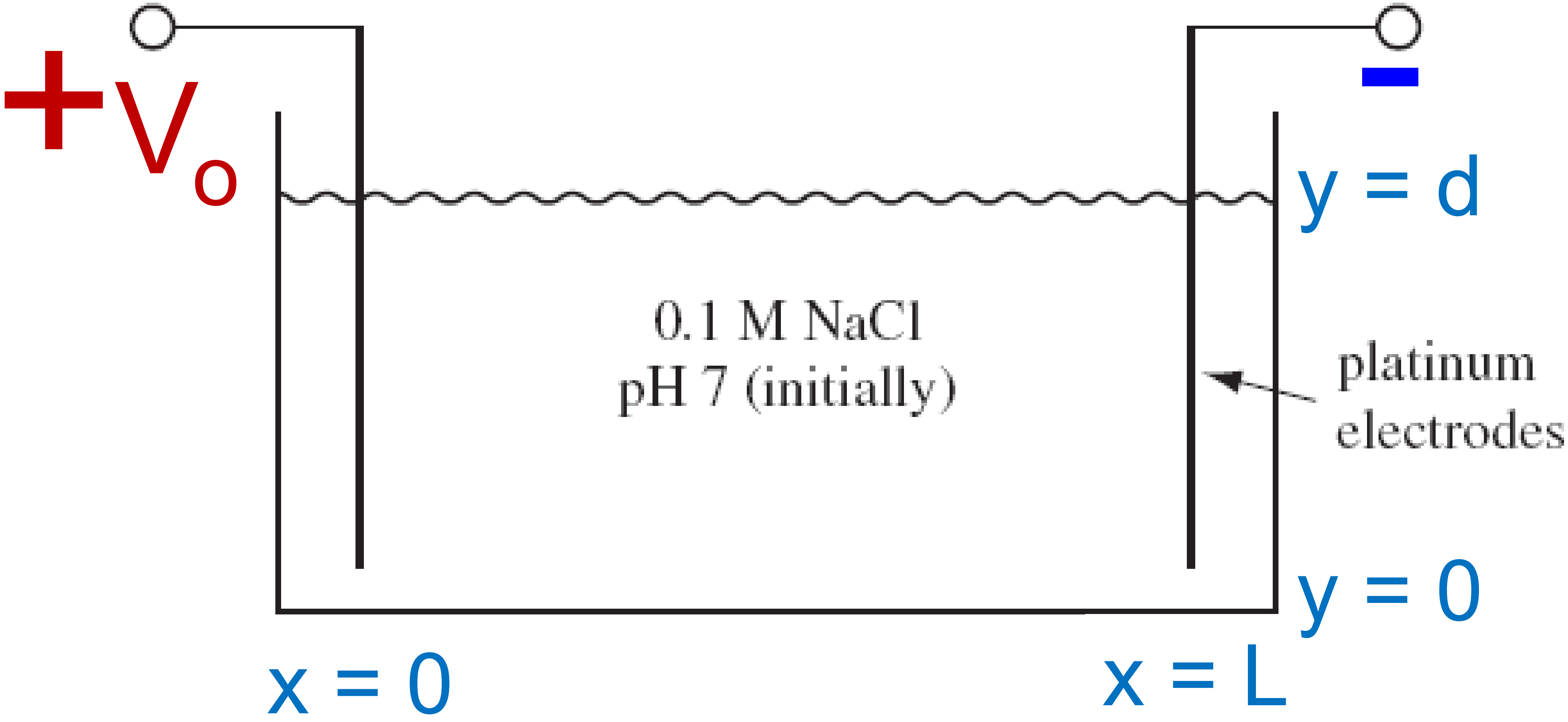
Ohm's Law:

$$\boxed{\mathbf{J} = \sigma \mathbf{E}}$$

↕

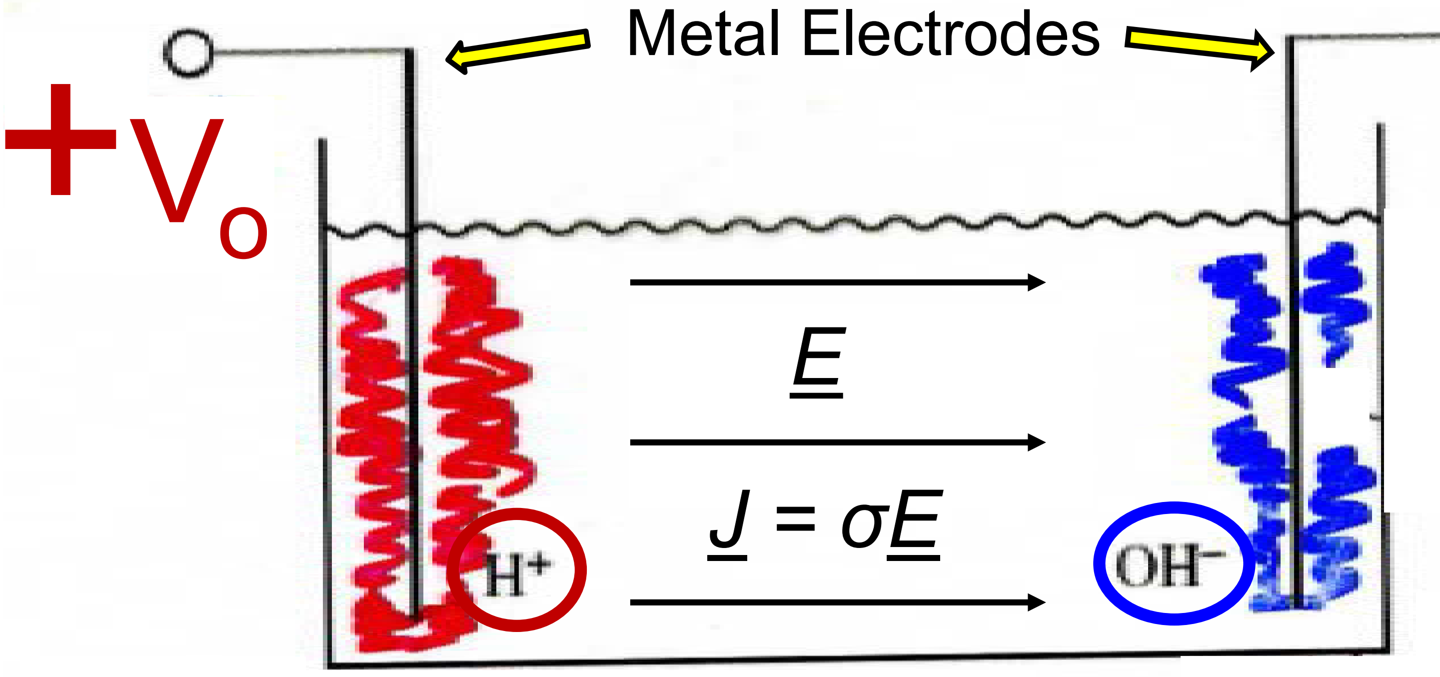
(empirical)

Electro-Statics:



But: Electrolysis Reactions at Electrodes

$$H_2O \longrightarrow 2H^+ + \overbrace{\tfrac{1}{2}O_2 \uparrow}^{*\ \text{Gas bubbles}} + 2e^- \qquad 2e^- + 2H_2O \longrightarrow \overbrace{H_2 \uparrow}^{*} + 2OH^-$$



DEMO

Really: $\mathbf{J} = \sigma\mathbf{E}$ **+ *diffusion + convection***

ElectroStatics: $\nabla\bullet\underline{J} = -(\partial\rho_e/\partial t) \equiv 0$

$\nabla\bullet J = 0 = \nabla\bullet\sigma E = \sigma[\nabla\bullet(-\nabla\Phi)] = 0 \rightarrow \nabla^2\Phi = 0$ Laplace

$(J = \sigma E)$

$$\frac{\partial^2\Phi}{\partial x^2} = 0$$

$$\Phi = V_o\,(1 - x/L)$$

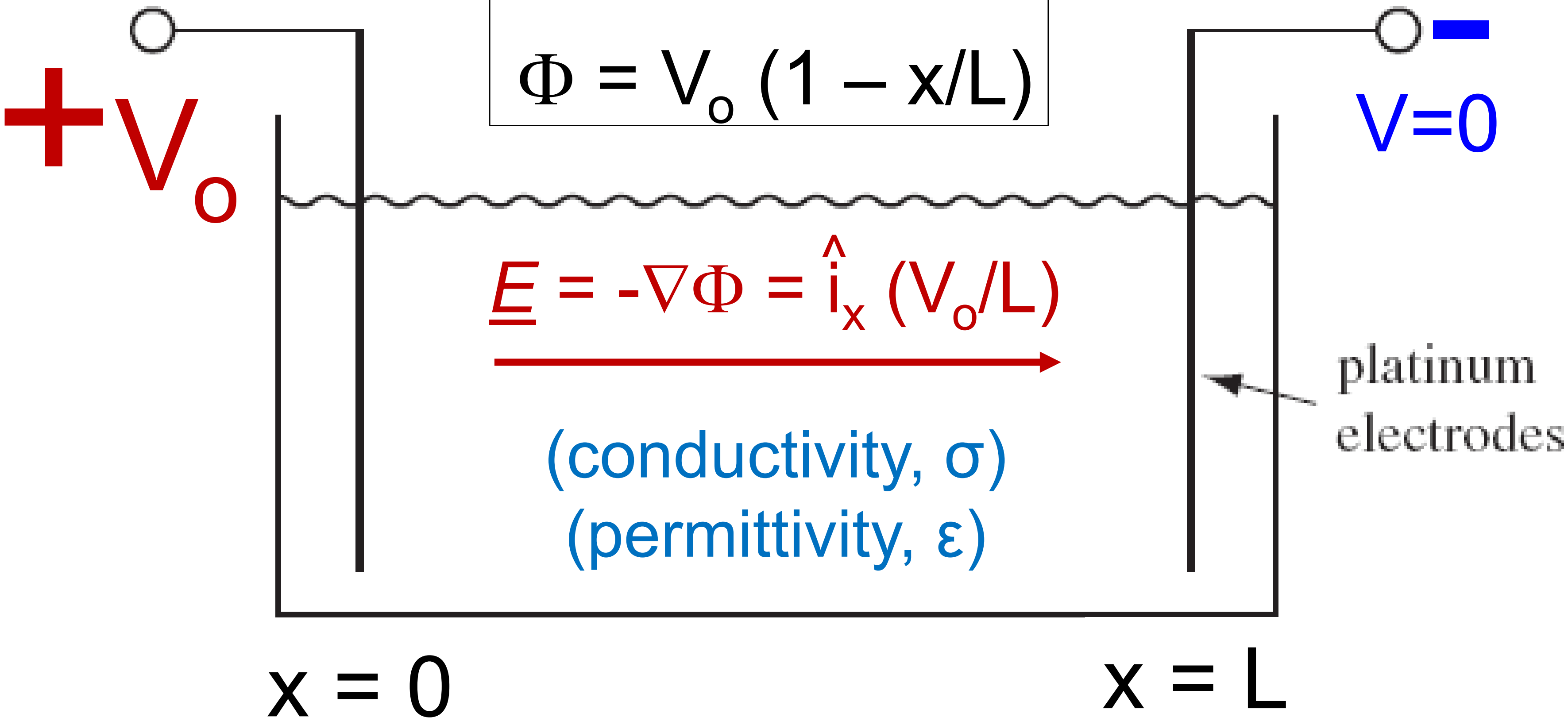


Table B.7
page 297

Solutions of Laplace's Eq.

$$\nabla^2\Phi = 0$$

in 2 - dimen's

Rectangular Coordinates
(independent of z)

e^{kx} and e^{-kx} may be replaced by sinh kx and cosh kx.

$$\Phi = e^{kx}(A_1 \sin ky + A_2 \cos ky) + e^{-kx}(B_1 \sin ky + B_2 \cos ky)$$

$$\boxed{\Phi = Axy + Bx + Cy + D;} \qquad (k = 0)$$

Cylindrical Coordinates
(independent of z)

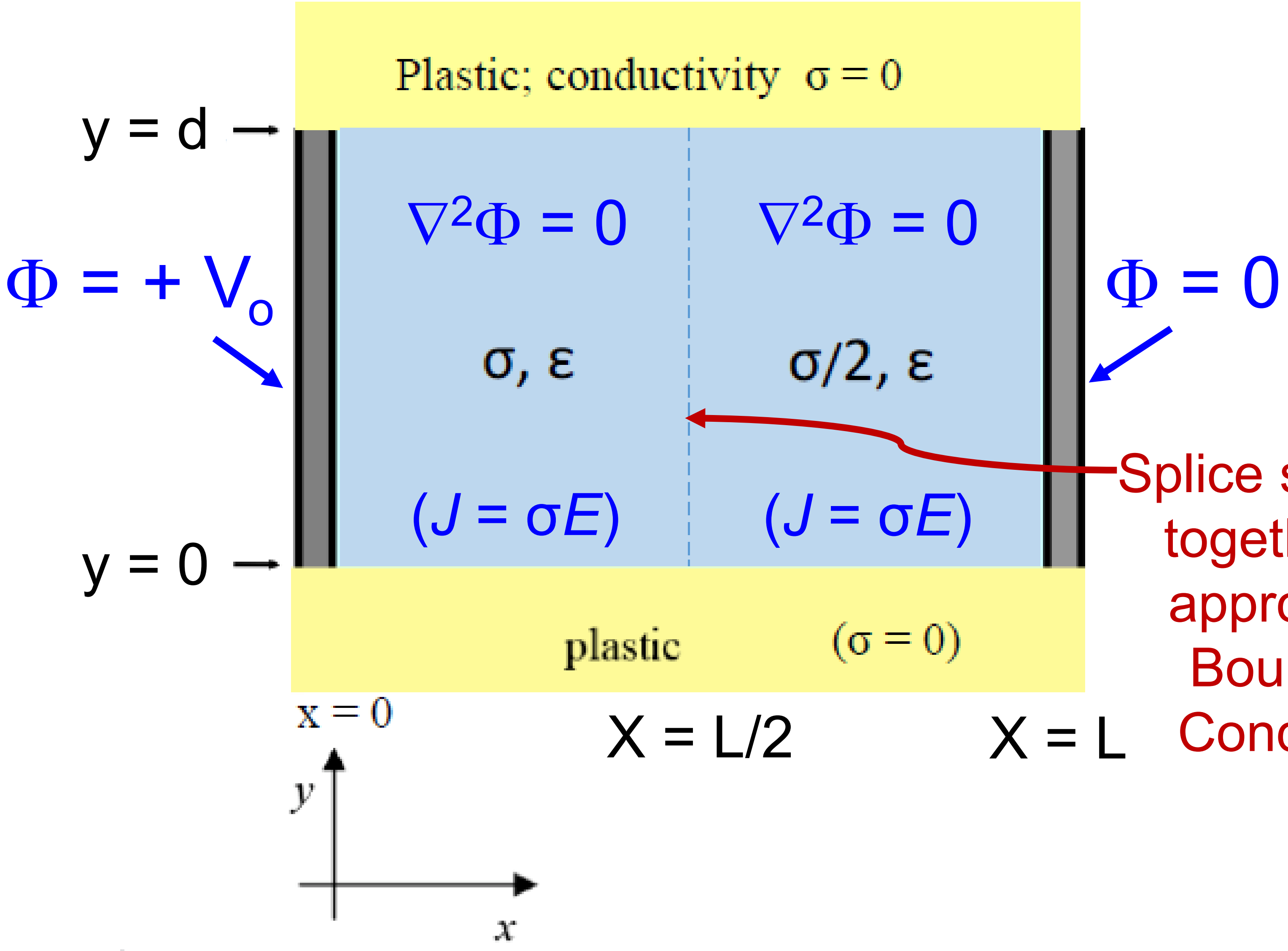
$$\Phi = r^n(A_1 \sin n\phi + A_2 \cos n\phi) + r^{-n}(B_1 \sin n\phi + B_2 \cos n\phi)$$

$$\Phi = (A_1\phi + A_2)\ln\frac{R}{r} + B_1\phi + B_2; \qquad (n = 0)$$

Spherical Coordinates
(independent of ϕ):

$$\Phi = A\,r\cos\theta + \frac{B}{r^2}\cos\theta + \frac{C}{r} + D$$

PSet 4, P3: Gradient Gel Electrophoresis



Splice solutions together via appropriate Boundary Conditions

EQS subset of Maxwell's Eqns

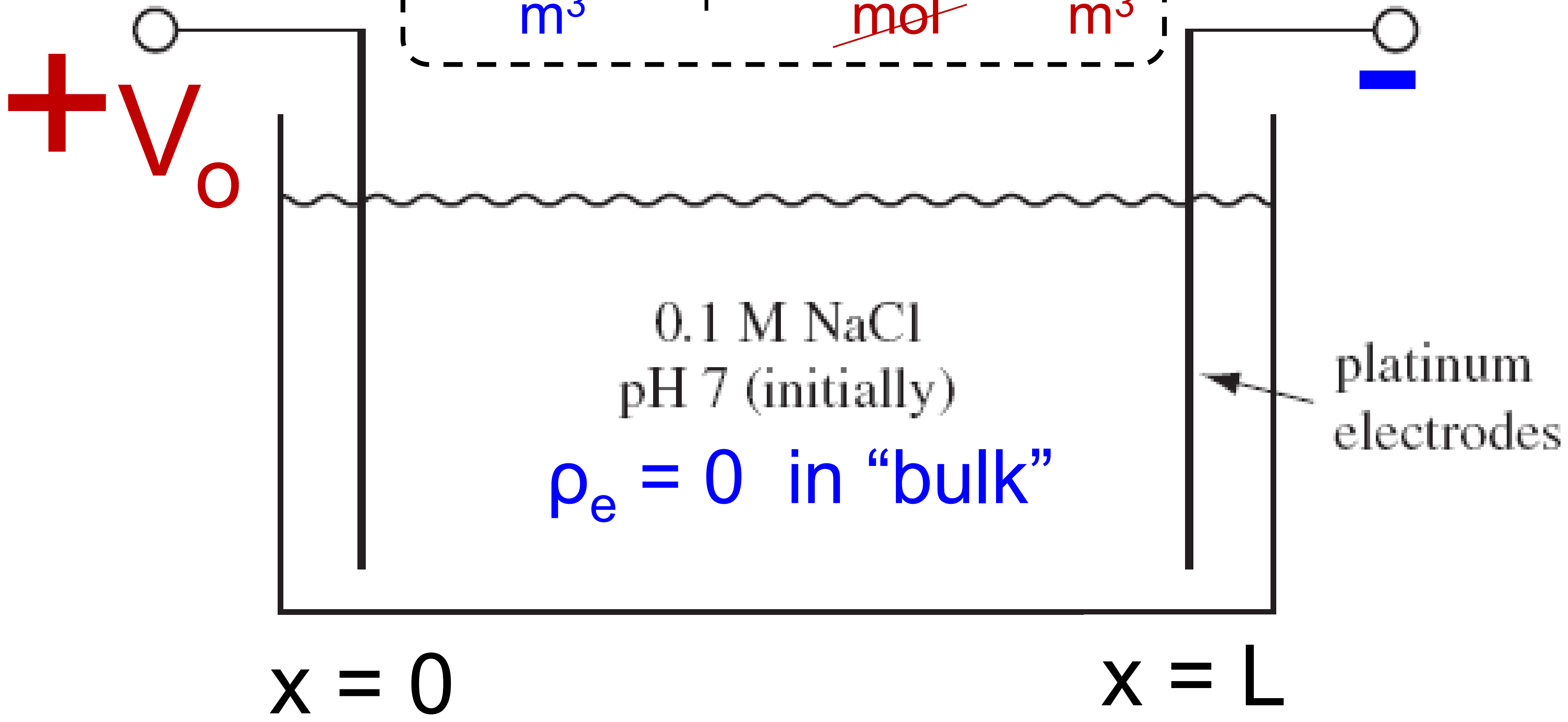
LAW	Boundary Cond.
(1) $\nabla\cdot\epsilon\underline{E}(\underline{r},t)=\rho_e(\underline{r},t)$ **Gauss**	(1') $\boxed{\underline{n}\cdot(\epsilon_1\underline{E}_1-\epsilon_2\underline{E}_2)=\sigma_s(t)}$
(2) $\nabla\times\underline{E}=0$ $\Rightarrow \boldsymbol{E}=-\nabla\Phi$ **Faraday**	(2') $\underline{n}\times(\underline{E}_1-\underline{E}_2)=0$ $\Leftrightarrow \boxed{\Phi_1=\Phi_2}$ * (E_{tan} continuous)
(3) $\nabla\cdot\underline{J}(\underline{r},t)=-\frac{\partial}{\partial t}\rho_e(\underline{r},t)$ **Cons. of Charge**	(3') $\underline{n}\cdot(\underline{J}_1-\underline{J}_2)=-\frac{\partial\sigma_s(t)}{\partial t}$ * $\boxed{\underline{n}\cdot(\sigma_1\underline{E}_1-\sigma_2\underline{E}_2)=-\cancel{\frac{\partial\sigma_s}{\partial t}}}$ 0 static

(4) $J=\sigma E$ (+ other) (constitutive law)

Electro-Statics:

$$\nabla \bullet \varepsilon \underline{\underline{E}} = \rho_e = 0 \rightarrow \nabla^2 \Phi = 0 \quad \text{Laplace}$$

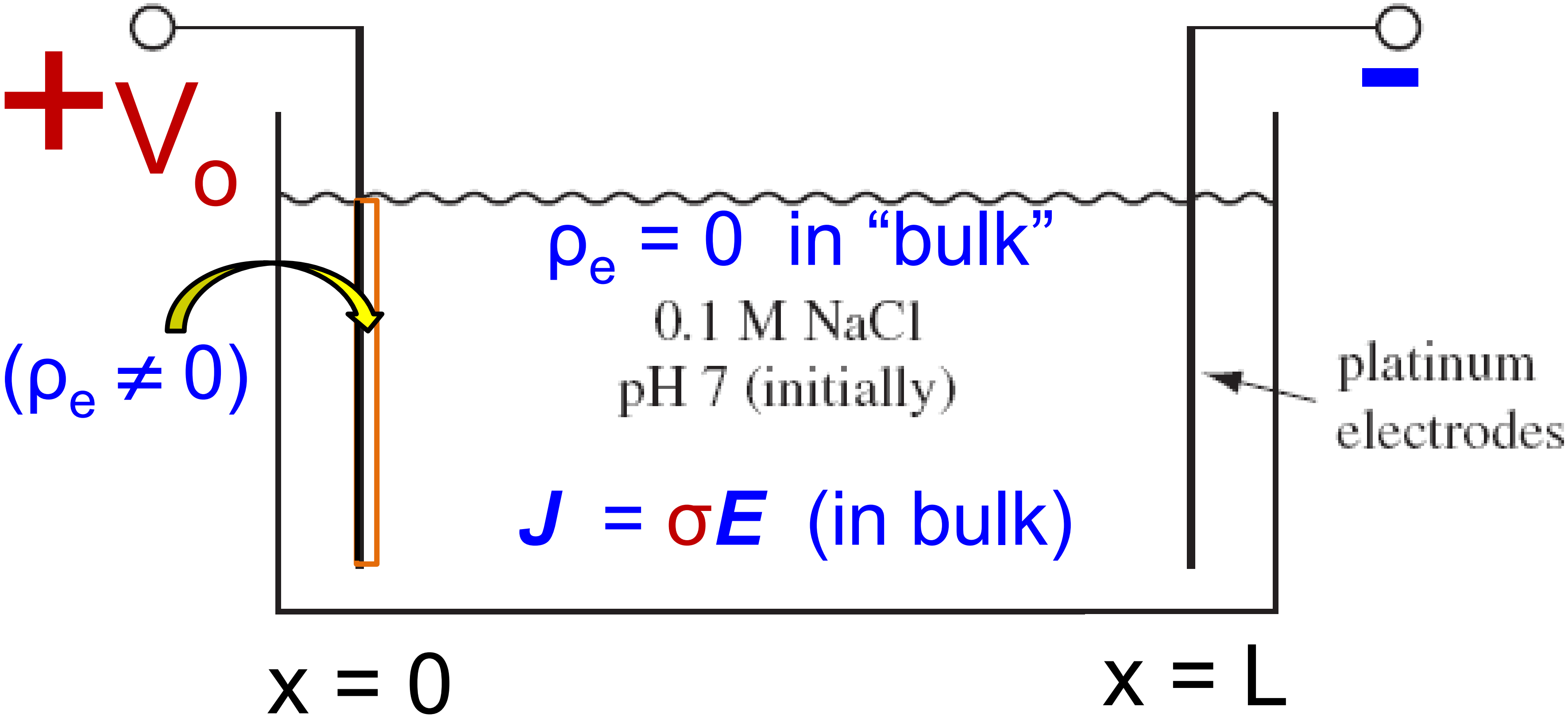
$$\rho_e \left(\frac{\text{coul}}{\text{m}^3}\right) = \sum_i z_i F \left(\frac{\text{coul}}{\cancel{\text{mol}}}\right) c_i \left(\frac{\cancel{\text{mol}}}{\text{m}^3}\right)$$



$\rho_e = 0$ in "bulk"

But is ρ_e zero everywhere?

$$\nabla \cdot \varepsilon \underline{E} = \rho_e = 0 \rightarrow \nabla^2 \Phi = 0 \quad \text{Laplace}$$

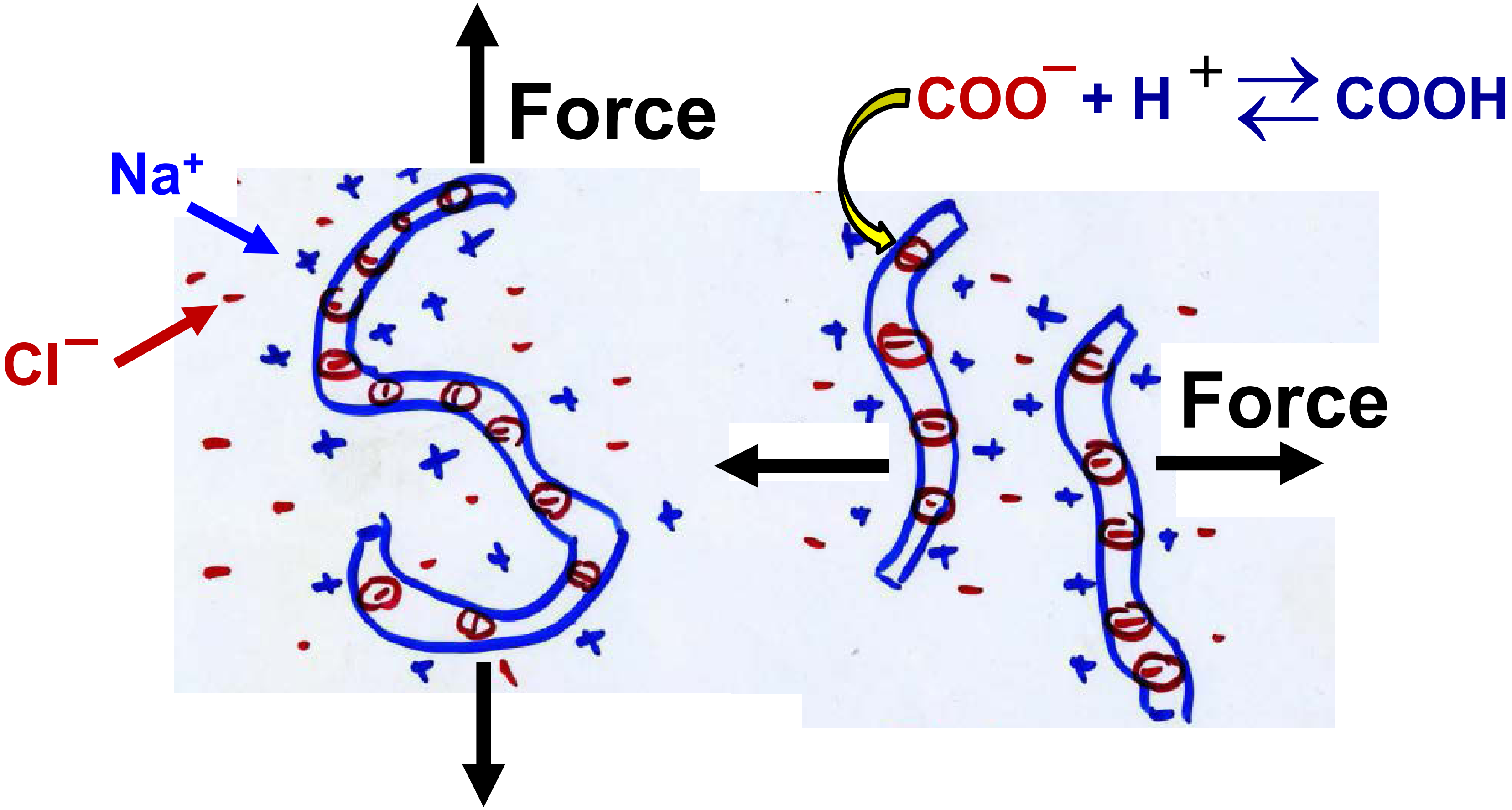


($\rho_e \neq 0$)

$\rho_e = 0$ in “bulk”

$\boldsymbol{J} = \sigma \boldsymbol{E}$ (in bulk)

Poisson-Boltzmann: Molecular Electrostatic Interactions



Intra-molecular Electrostatic Interactions

Inter-molecular Electrostatic Interactions

PSet 4, P2

In Figure 2.24, we picture an idealized metal electrode/electrolyte interface where the metal is known to have a net surface charge σ_d at $x = 0$. This leads to a net space charge of mobile ions in the adjacent electrolyte phase. We wish to find the *equilibrium* potential and space charge distribution in the electrolyte.

(**a**) For the one-dimensional model of Figure 2.24, write Poisson's equation for the electrolyte region $x \geq 0$.

Further, assume that the distribution of all mobile ions can be adequately represented by Boltzmann statistics, so that the probability of finding a given ion of species i and valence z_i at position x can be written as $\exp[-z_iF\Phi(x)/RT]$, and therefore

Poisson's Eqn

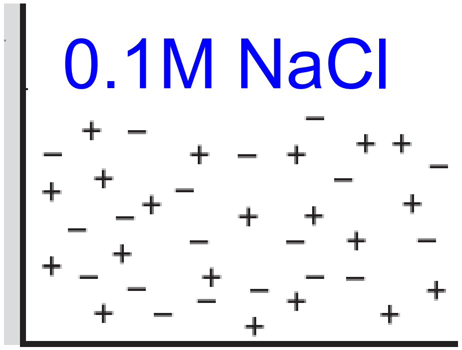
$$\boxed{c_i = c_{i0}e^{-z_iF\Phi(x)/RT}} \qquad (2.127)$$

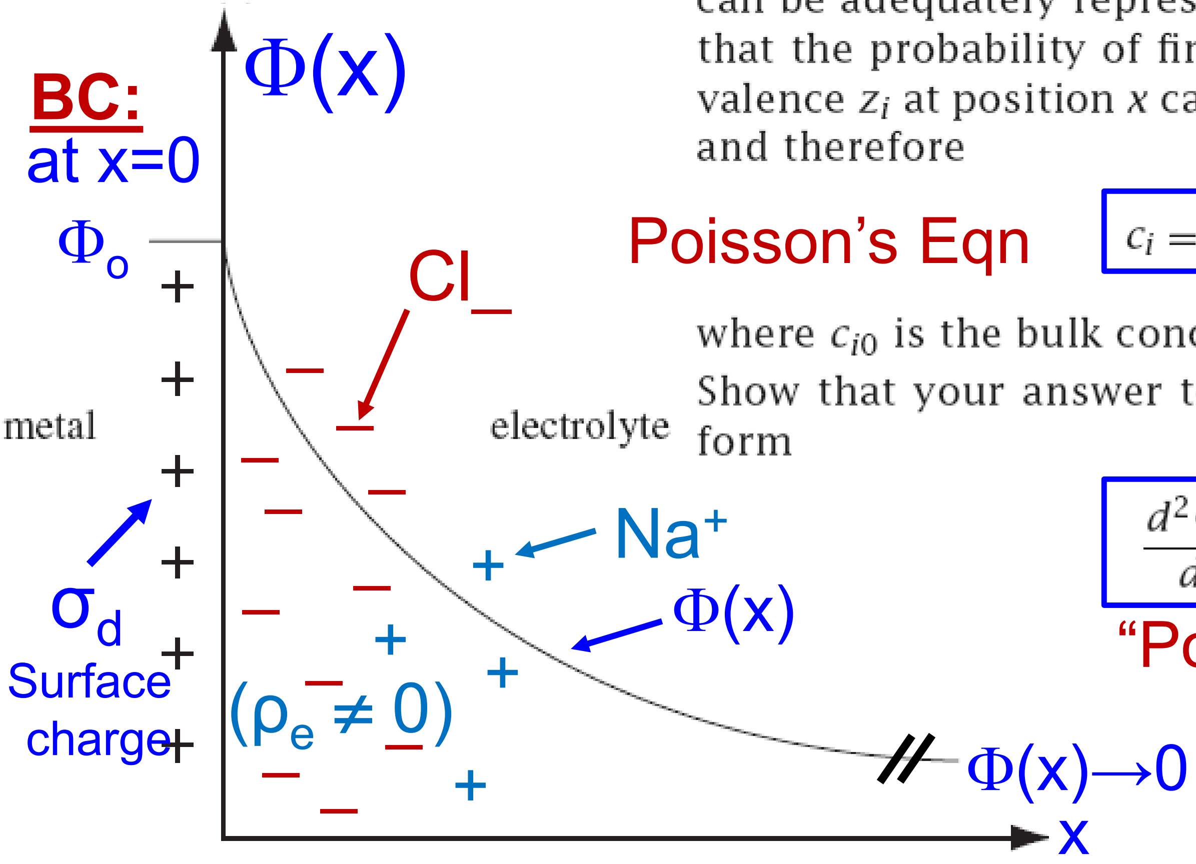
where c_{i0} is the bulk concentration of the ith species.

Show that your answer to part (a) reduces to the limiting form

$$\boxed{\frac{d^2\Phi(x)}{dx^2} = \kappa^2\Phi(x)} \qquad (2.128)$$

"Poisson-Boltzmann Eqn" (linearized)





....Find Φ(x)

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Constitutive Law

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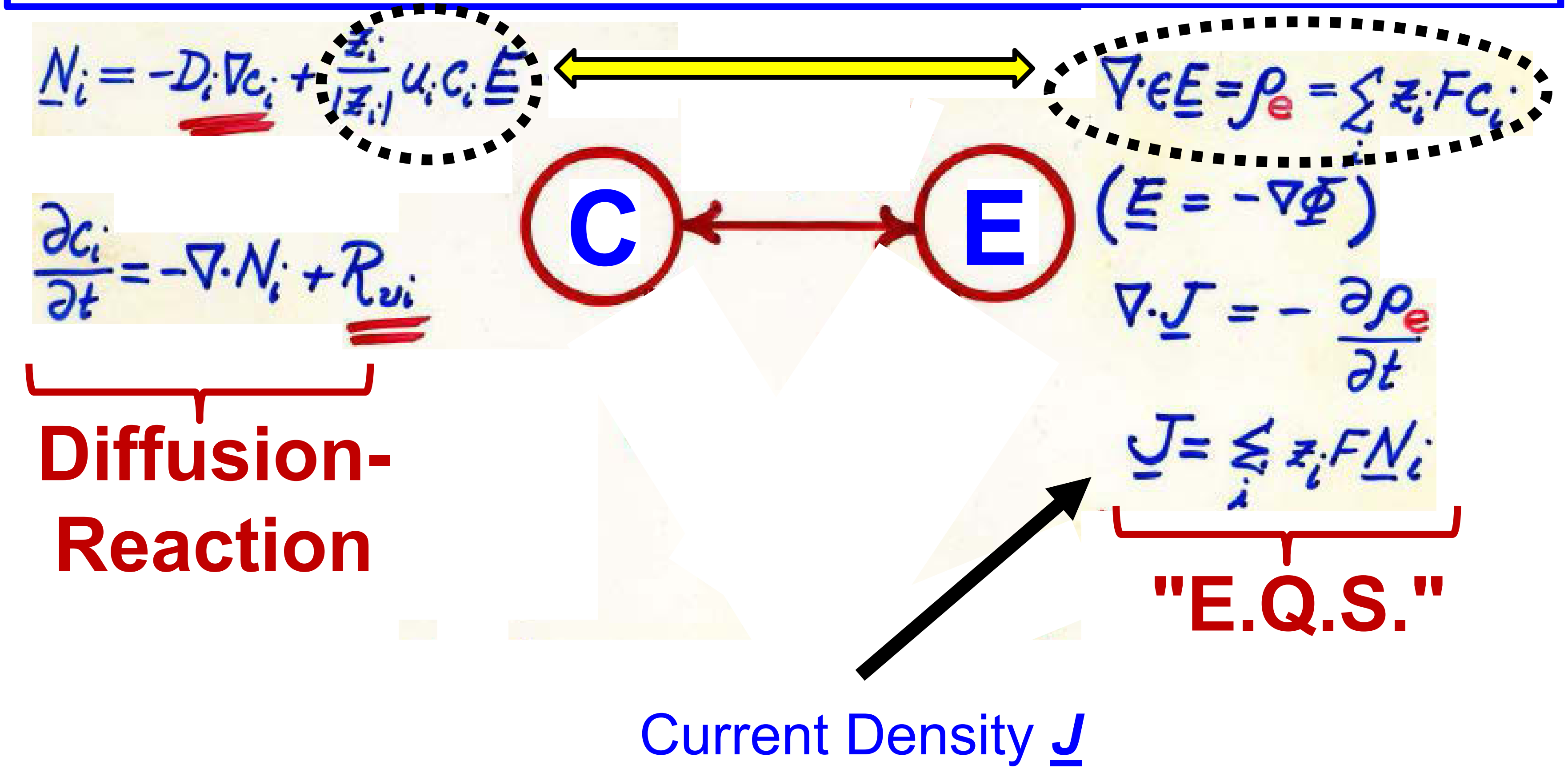
FFF: Complete Description of Coupled Transport and Biomolecular Interactions



$$\underline{N}_i = -D_i \nabla c_i + \frac{z_i}{|z_i|} u_i c_i \underline{E}$$

$$\frac{\partial c_i}{\partial t} = -\nabla \cdot \underline{N}_i + R_{vi}$$

$$\nabla \cdot \epsilon \underline{E} = \rho_e = \sum_i z_i F c_i$$

$$(\underline{E} = -\nabla \Phi)$$

$$\nabla \cdot \underline{J} = -\frac{\partial \rho_e}{\partial t}$$

$$\underline{J} = \sum_i z_i F \underline{N}_i$$

Diffusion-Reaction

"E.Q.S."

Current Density ***J***

20.430J / 2.795J / 6.561J / 10.539J Fields, Forces, and Flows in Biological Systems
Fall 2015