ELECTRICAL SUBSYSTEM

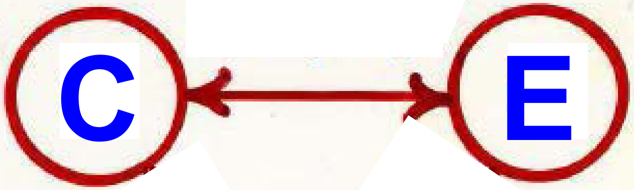
Lect	Date	A. ELECTRICAL SUBSYSTEM: Fundamentals & Applications
8	Oct 7	E-fields and transport; Maxwell’s equations for electric & magnetic fields
9	Oct 13	Define electrical potential; conservation of charge; Electro-quasistatics
10	Oct 14	Laplacian solutions; examples with electrodes; Electric field boundary conditions; Ohmic transport; Charge Relaxation; Electrical migration vs. chemical diffusive fluxes
	Oct 19	Fundamentals and applications of EQS: MEMs; cell electroporation; EKG
		B. ELECTRICAL SUBSYSTEM: Transport, binding, molecular interactions
11	Oct 21	Electrochemical coupling; Electrical double layers; Poisson–Boltzmann Equation
12	Oct 26	Donnan equilibrium in tissues, gels, polyelectrolyte networks
13	Oct 28	Charge group ionization & electro-diffusion-reaction in molecular networks
14	Nov 2	Transport of charged proteins into charged tissues with Donnan BCs

FFF: Complete Description of Coupled Transport and Biomolecular Interactions

$$\underline{N}_i = -D_i \nabla c_i + \frac{z_i}{|z_i|} u_i c_i \underline{E}$$

$$\frac{\partial c_i}{\partial t} = -\nabla \cdot \underline{N}_i + R_{vi}$$

Diffusion-Reaction



$$\nabla \cdot \epsilon \underline{E} = \rho_e = \sum_i z_i F c_i$$

$$(\underline{E} = -\nabla \Phi)$$

$$\nabla \cdot \underline{J} = -\frac{\partial \rho_e}{\partial t}$$

$$\underline{J} = \sum_i z_i F \underline{N}_i$$

"E.Q.S."

Start with Maxwell's Equations

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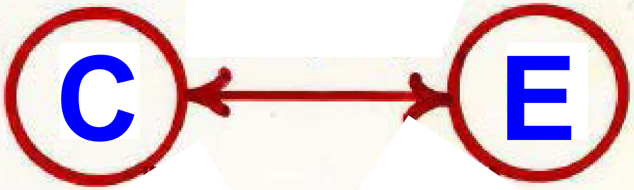

$$\frac{\partial c_i}{\partial t} = -\nabla \cdot \underline{N}_i + R_{vi}$$

Diffusion-Reaction

$$(\underline{E} = -\nabla \Phi)$$

$$\nabla \cdot \underline{J} = -\frac{\partial \rho_e}{\partial t}$$

$$\underline{J} = \sum_i z_i F \underline{N}_i$$

"E.Q.S."

Current density $$\underbrace{F \overset{(10^5)}{\left(\frac{\text{coul}}{\text{mol}}\right)}}_{\text{Faraday's constant}} N_i \left(\frac{\text{mol}}{(\text{m}^2 \bullet \text{s})}\right) = J \left(\frac{\text{A}}{\text{m}^2}\right)$$

(Last Time):

(Table 2.7, p. 63)

Name	Integral form
(1) Gauss' law	$\oint_S \epsilon \boldsymbol{E} \cdot d\boldsymbol{a} = \int_V \rho_e$
(3) Ampère's law	$\oint_C \boldsymbol{H} \cdot d\boldsymbol{s} = \int_S \boldsymbol{J} \cdot d\boldsymbol{a}$
(2) Faraday's law	$\oint_C \boldsymbol{E} \cdot d\boldsymbol{s} = -\frac{d}{dt} \int_S \mu \boldsymbol{H} \cdot d\boldsymbol{a}$
(4) Magnetic flux	$\oint_S \mu \boldsymbol{H} \cdot d\boldsymbol{a} = 0$

charges

currents

Magnetic Induction

No magnetic “monopoles” …only dipoles

(Last Time):

(Table 2.7, p. 63)

	Name	$\boxed{\text{Integral form}}$
(1)	Gauss' law	$\oint_S \epsilon \boldsymbol{E} \cdot d\boldsymbol{a} = \int_V \rho_e$
(3)	Ampère's law	$\oint_C \boldsymbol{H} \cdot d\boldsymbol{s} = \int_S \boldsymbol{J} \cdot d\boldsymbol{a} + \boxed{\frac{d}{dt} \int_S \epsilon \boldsymbol{E} \cdot d\boldsymbol{a}}$
(2)	Faraday's law	$\oint_C \boldsymbol{E} \cdot d\boldsymbol{s} = \boxed{-\frac{d}{dt} \int_S \mu \boldsymbol{H} \cdot d\boldsymbol{a}}$
(4)	Magnetic flux	$\oint_S \mu \boldsymbol{H} \cdot d\boldsymbol{a} = 0$

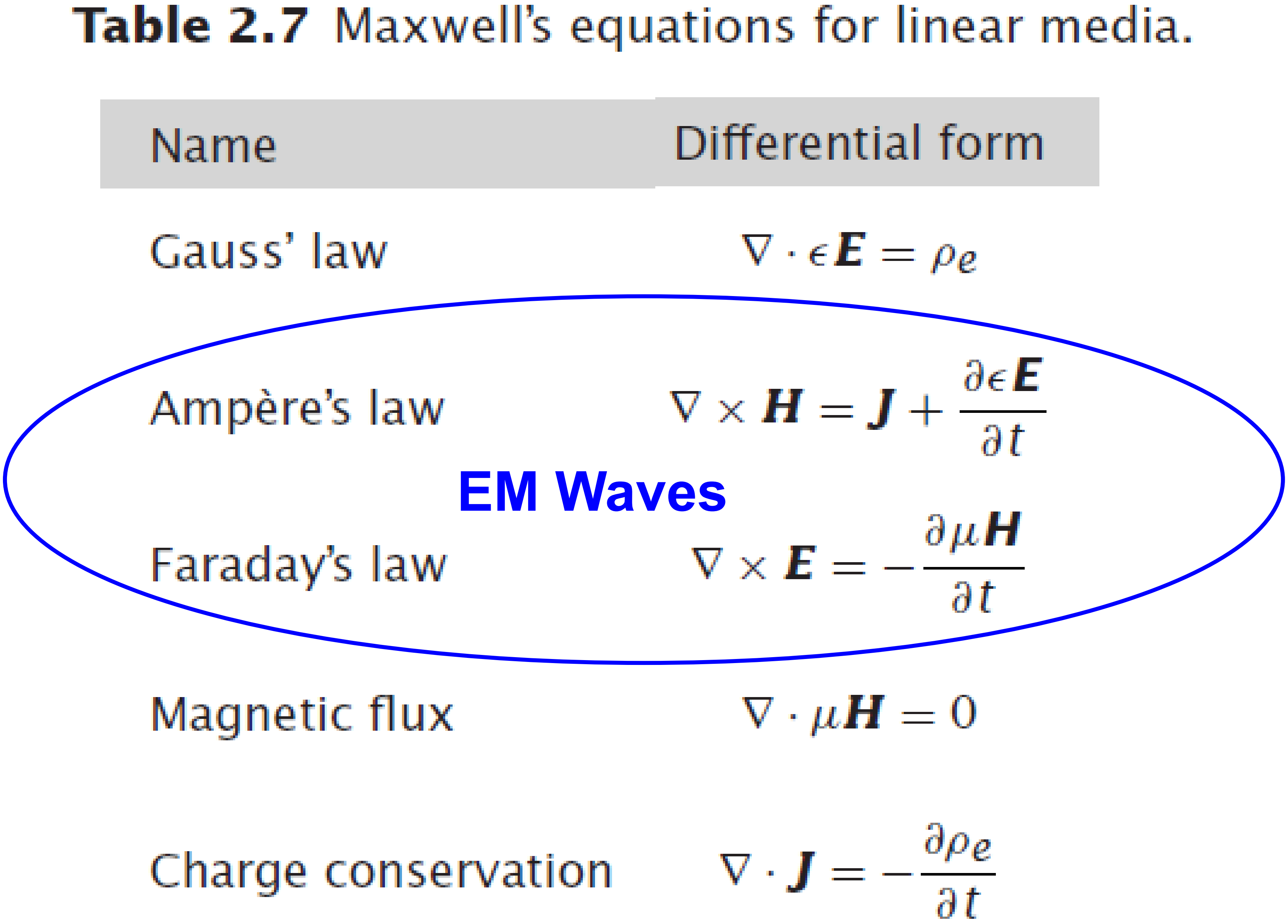

Table 2.7 Maxwell's equations for linear media.

Name	Differential form
Gauss' law	$\nabla \cdot \epsilon \boldsymbol{E} = \rho_e$
Ampère's law	$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \epsilon \boldsymbol{E}}{\partial t}$
Faraday's law	$\nabla \times \boldsymbol{E} = -\frac{\partial \mu \boldsymbol{H}}{\partial t}$
Magnetic flux	$\nabla \cdot \mu \boldsymbol{H} = 0$
Charge conservation	$\nabla \cdot \boldsymbol{J} = -\frac{\partial \rho_e}{\partial t}$

EM Waves

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Source: Grodzinsky, Alan. Field, Forces and Flows in Biological Systems. Garland Science, 2011. [Preview with Google Books]

Table 2.7 Maxwell's equations for linear media.

Page 44, 46

EM Waves

Ampère's law $$\nabla \times \boldsymbol{H} = \cancel{\boldsymbol{J}} + \frac{\partial \epsilon \boldsymbol{E}}{\partial t}$$

Faraday's law $$\nabla \left[\nabla \times \boldsymbol{E} = -\frac{\partial \mu \boldsymbol{H}}{\partial t}\right]$$

$$\nabla \times (\nabla \times \boldsymbol{E}) = -\mu\epsilon \frac{\partial^2 \boldsymbol{E}}{\partial t^2}$$

Speed of light $$\boxed{c = 1/\sqrt{\mu\epsilon} = f\lambda}$$

$$\nabla^2 \boldsymbol{E} = \mu\epsilon \frac{\partial^2 \boldsymbol{E}}{\partial t^2}$$

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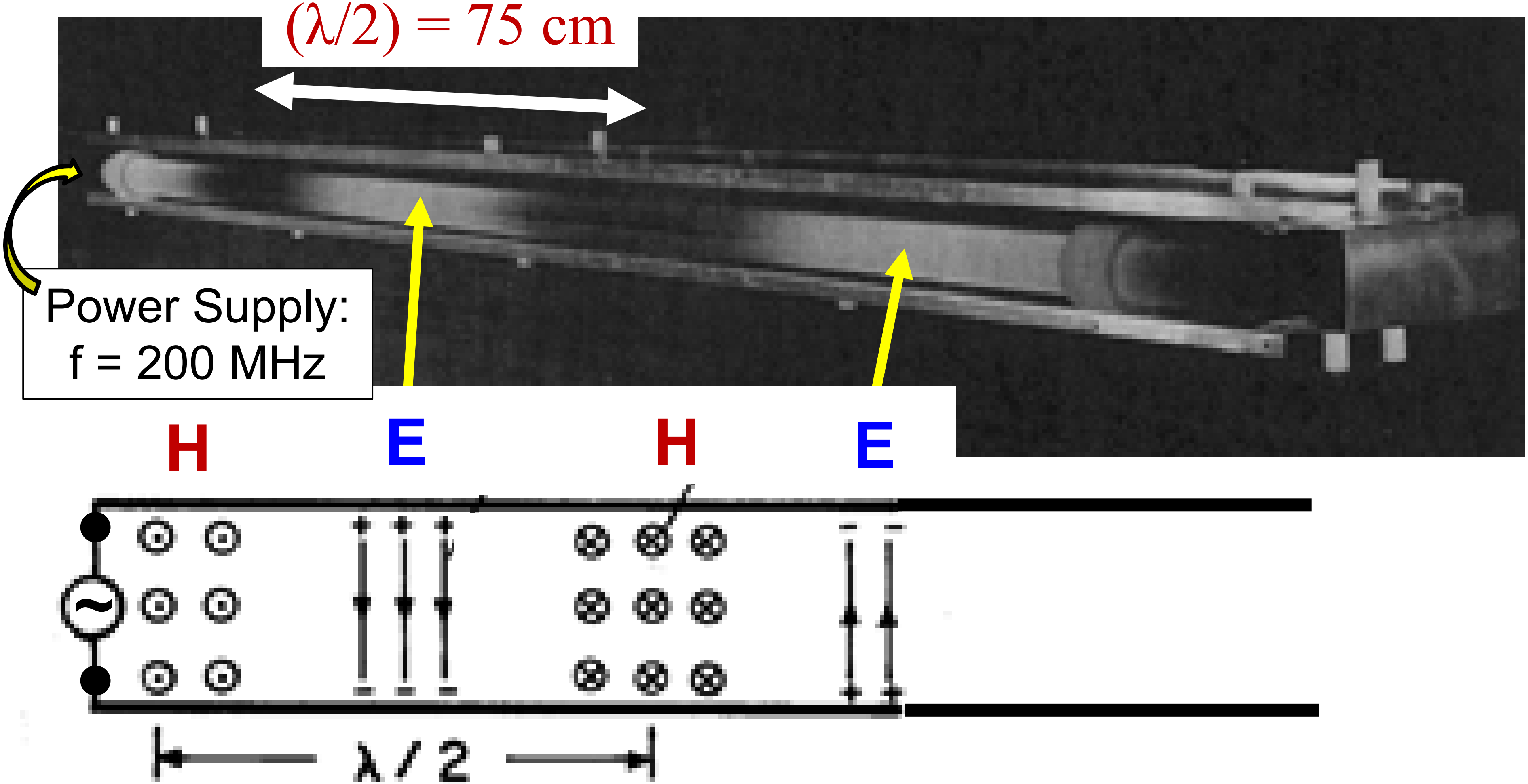
Demo from Last Lecture:
Electromagnetic "Standing Wave"



$(\lambda/2) = 75$ cm

Power Supply:
f = 200 MHz

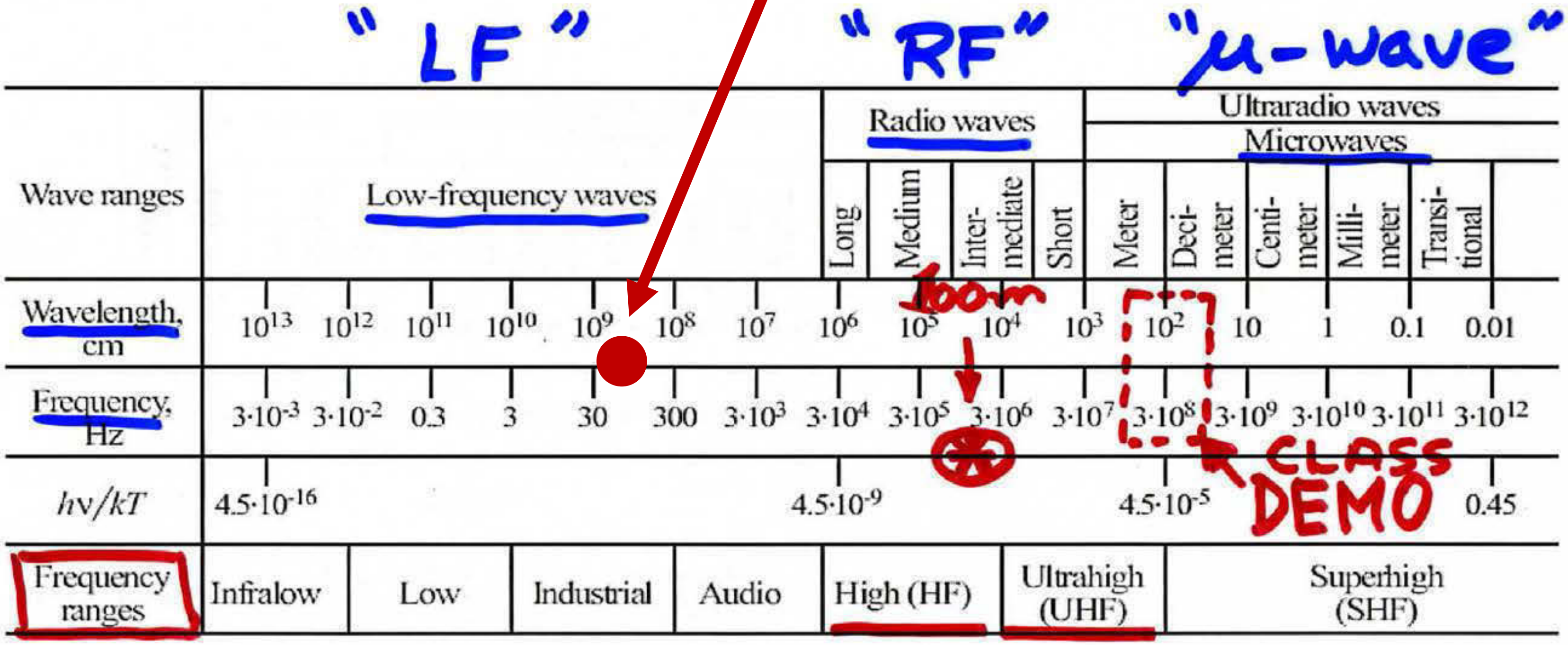
$f = 200$ MHz; $\lambda = [c/f] =$ $\underline{\lambda = 1.5 \text{ m}}$

Electromagnetic Spectrum

Text Table 2.6

Region of electromagnetic spectrum from infralow to superhigh frequencies in which $h\nu < kT$

60 Hz ↔ 3,100 mile wavelength



"LF" "RF" "μ-wave"

Wave ranges	Low-frequency waves							Radio waves				Ultraradio waves Microwaves				
								Long	Medium	Inter-mediate	Short	Meter	Deci-meter	Centi-meter	Milli-meter	Transi-tional
Wavelength, cm	10^{13}	10^{12}	10^{11}	10^{10}	10^{9}	10^{8}	10^{7}	10^{6}	10^{5}	10^{4}	10^{3}	10^{2}	10	1	0.1	0.01
Frequency, Hz	$3 \cdot 10^{-3}$	$3 \cdot 10^{-2}$	0.3	3	30	300	$3 \cdot 10^{3}$	$3 \cdot 10^{4}$	$3 \cdot 10^{5}$	$3 \cdot 10^{6}$	$3 \cdot 10^{7}$	$3 \cdot 10^{8}$	$3 \cdot 10^{9}$	$3 \cdot 10^{10}$	$3 \cdot 10^{11}$	$3 \cdot 10^{12}$
$h\nu/kT$	$4.5 \cdot 10^{-16}$							$4.5 \cdot 10^{-9}$				$4.5 \cdot 10^{-5}$				0.45
Frequency ranges	Infralow		Low		Industrial	Audio		High (HF)		Ultrahigh (UHF)		Superhigh (SHF)				

100m

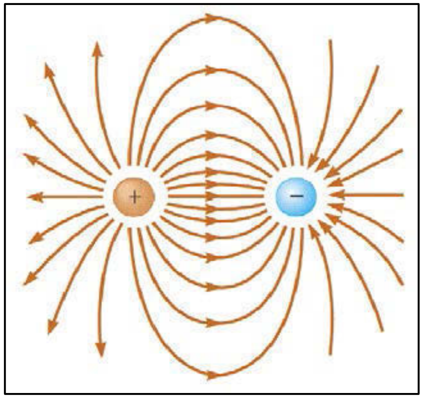
CLASS DEMO

Chemical Subsystem: Conservation of Mass

$$\underbrace{\frac{d}{dt}\int_V c_i\,dV}_{\text{rate of increase of } c_i \text{ inside volume}} = \overset{\downarrow}{-}\underbrace{\oint_S \mathbf{N}_i\cdot\mathbf{n}\,da}_{\text{outward Flux of solute}} + \underbrace{\int_V R_i\,dV}_{\text{reaction: generation of species}}$$

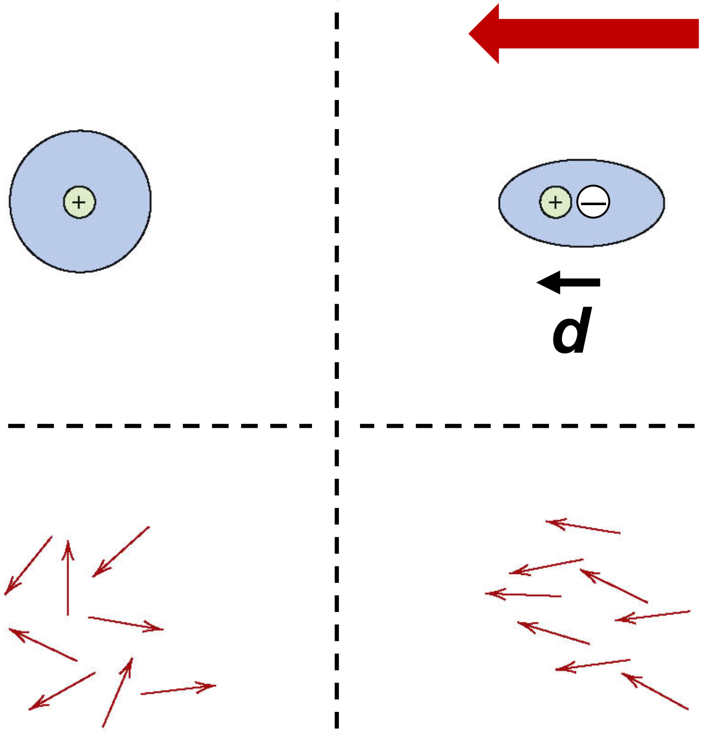
Figure 1.1 removed due to copyright restrictions. Source: Grodzinsky, Alan. Field, Forces and Flows in Biological Systems. Garland Science, 2011. [Preview with Google Books]

"Polarization"



No ***E***-field

Applied ***E***

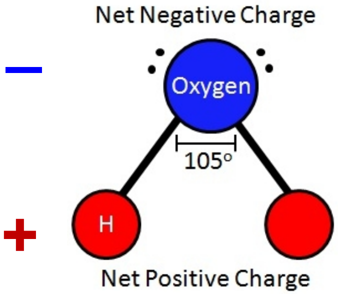


Induced polarization

(atomic/molecular)

Orientation polarization

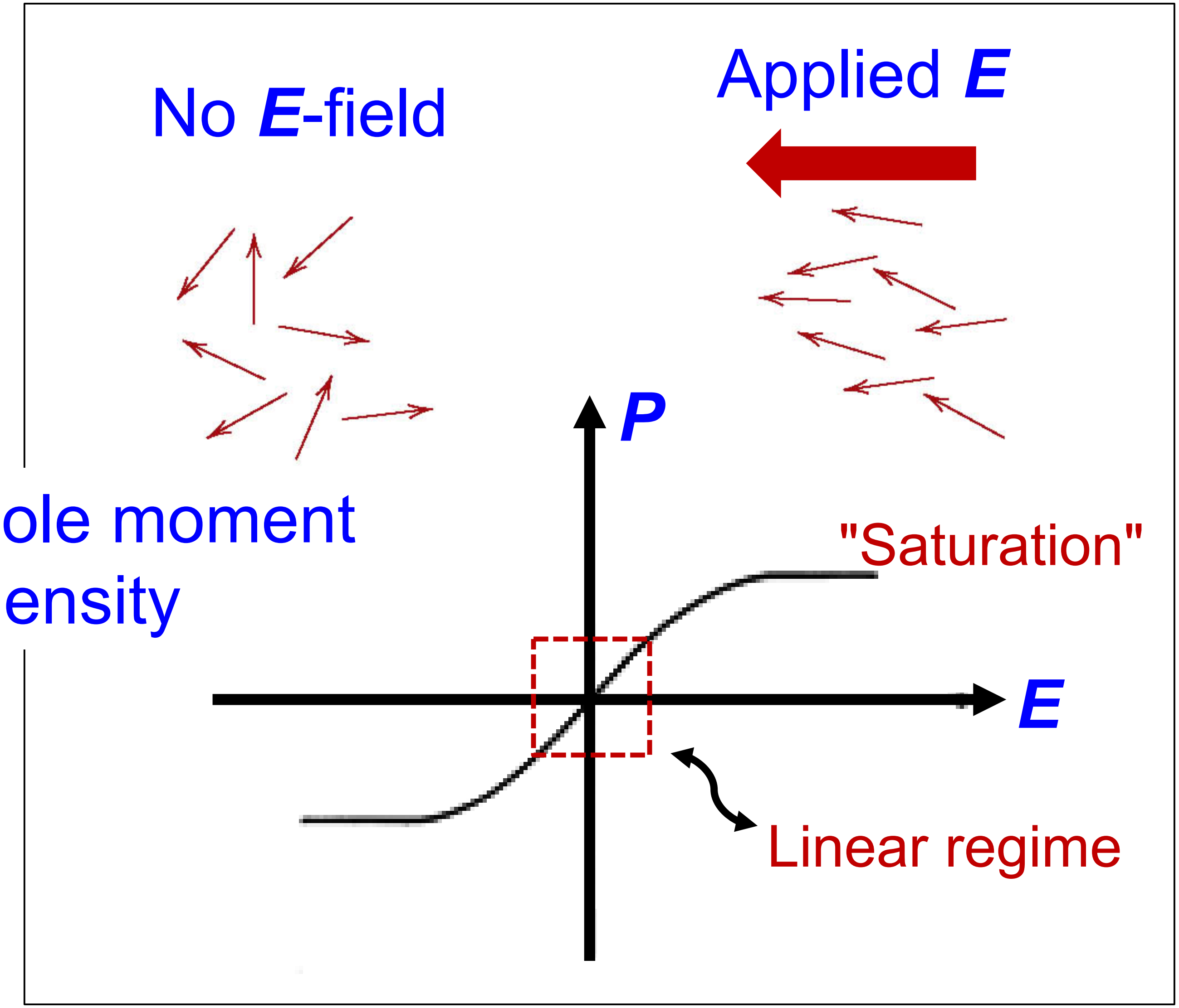
(orientation of H_2O dipoles)



Courtesy of flikr

Homogeneous, isotropic, nonlinear Polarization:

(Example: E-induced orientation of water dipoles)



P **=** dipole moment density

Homogeneous, isotropic, nonlinear Polarization:

(Example: E-induced orientation of water dipoles)

P **=** dipole moment density

In general, ***P*** **can be:**

- Non-linear
- Anisotropic (e.g., a tensor in a crystal)
- A function of frequency
  (dipoles acting like harmonic
  oscillators in a sinusoidal E field)

From a painting at the Deutsches Museum, Munich.

GEORG SIMON OHM

1789-1854

Mathematician and experimentalist

Current Flow in conductors

Ohm's Law:

(empirical)

$$\boxed{\mathbf{J} = \sigma \mathbf{E}}$$

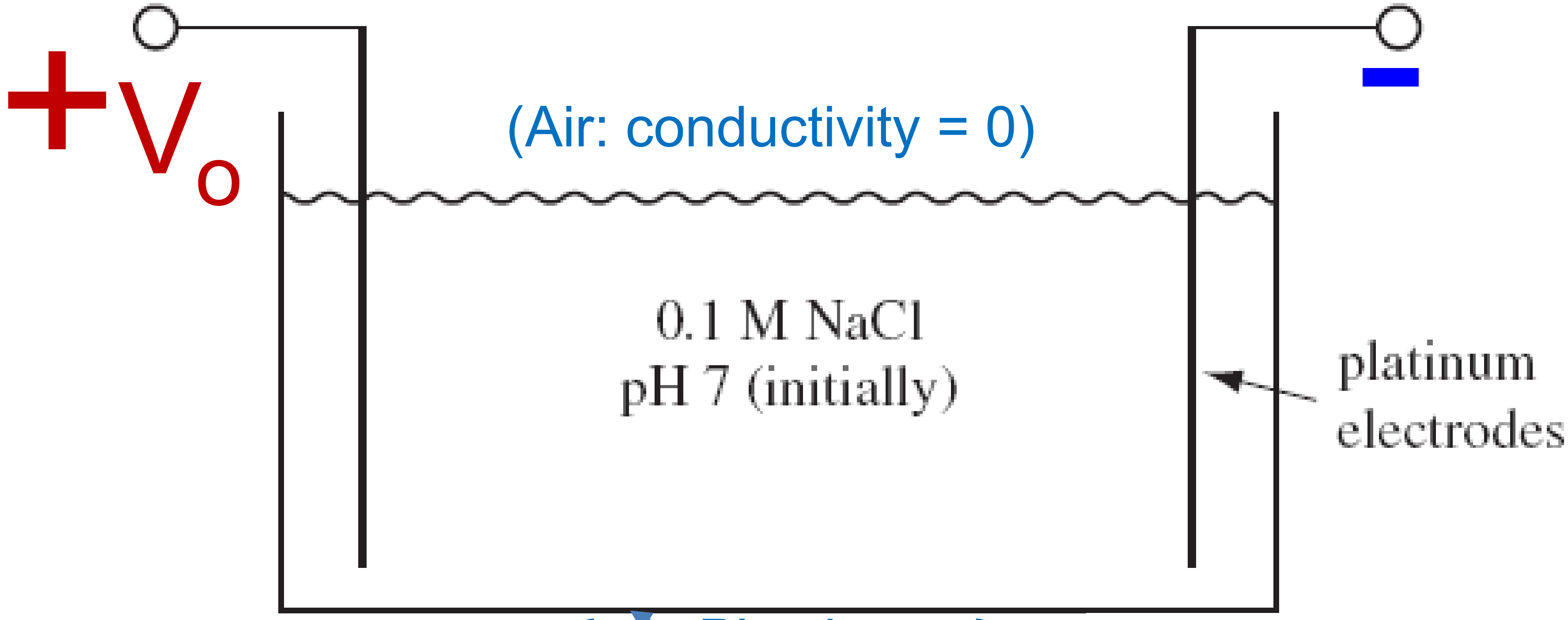
Always?

$$\begin{tikzcd}\mathbf{J} \arrow[d, dashed] & = & \sigma\mathbf{E} \arrow[d, dashed] \\ (iR & = & V)\end{tikzcd}$$

(circuits)

Electro-Statics:

$$\nabla^2\Phi \Rightarrow \left[\partial^2\phi/\partial x^2 \ = 0\right]$$ “Laplace’s Eqn”

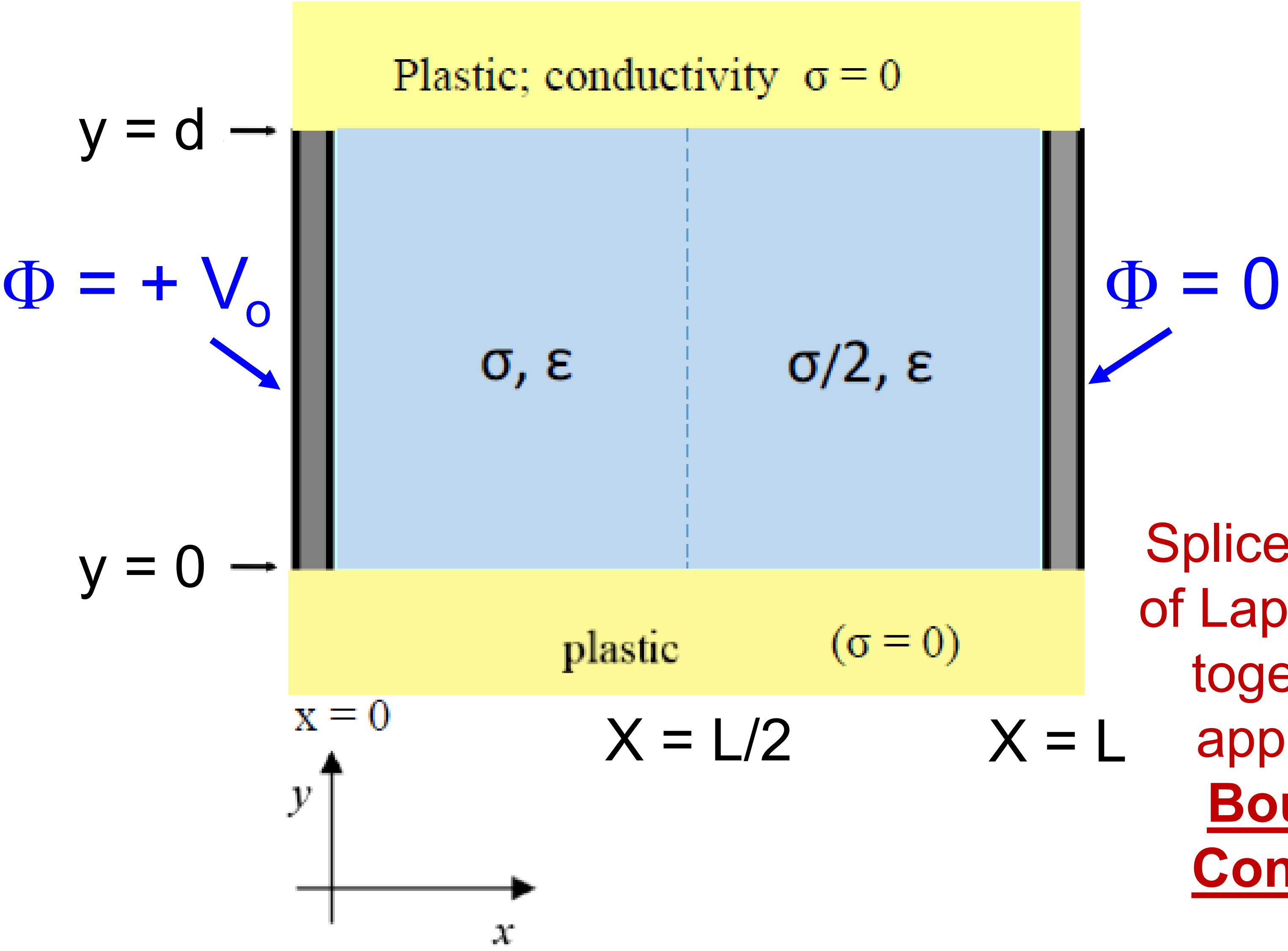


x = 0

Plastic:
conductivity = 0

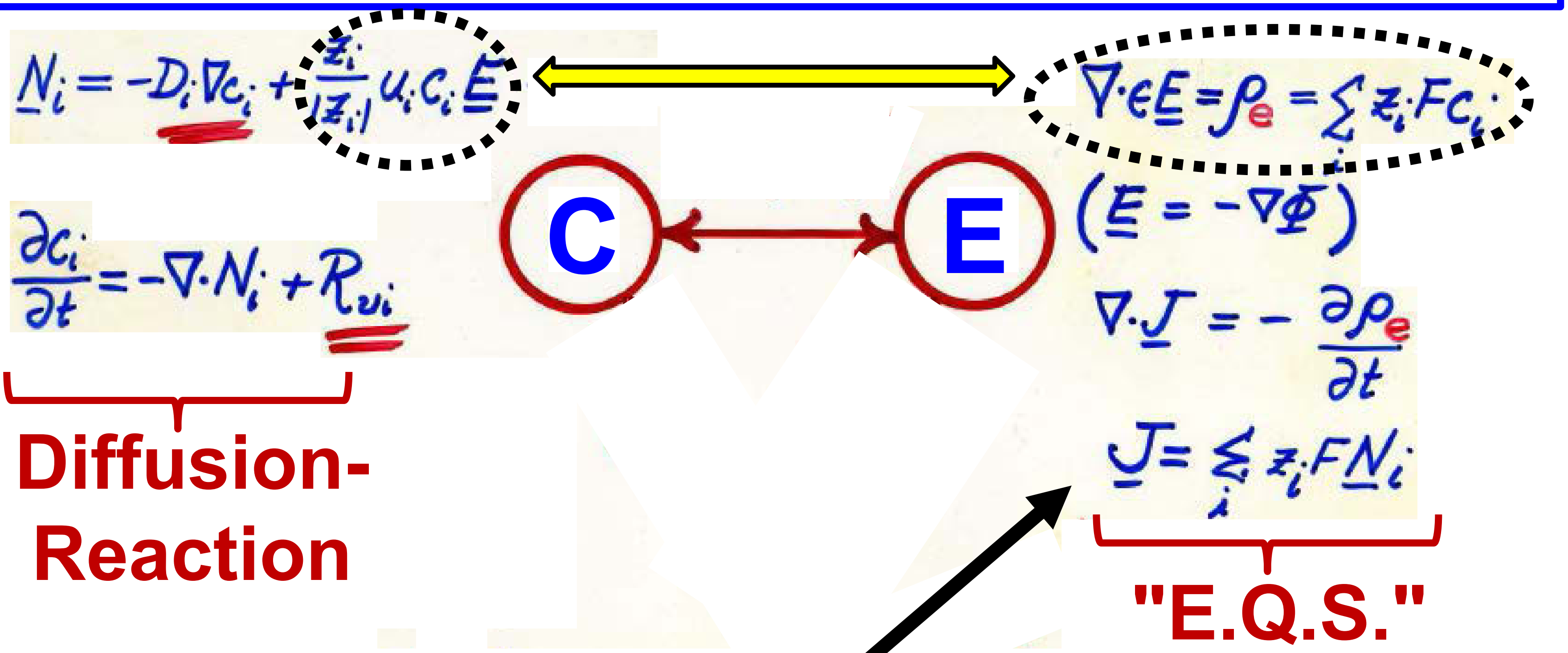
x = L

PSet 4, P3: “Gradient” Gel Electrophoresis



Splice solutions of Laplace’e Eq. together via appropriate **Boundary Conditions**

FFF: Complete Description of Coupled Transport and Biomolecular Interactions



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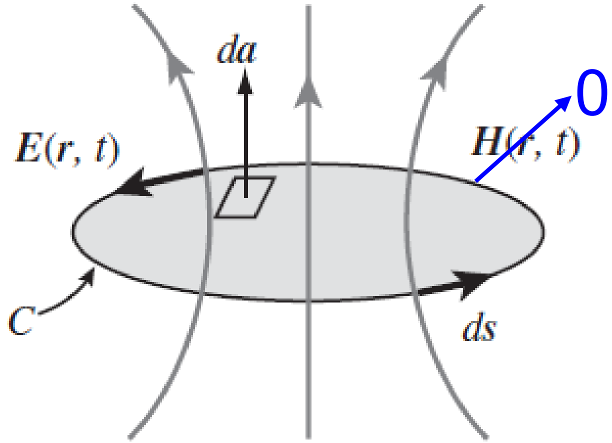
Diffusion-Reaction

"E.Q.S."

Current density $$\underbrace{F\left(\overset{(10^5)}{\frac{\text{coul}}{\text{mol}}}\right)}_{\text{Faraday's constant}} N_i\left(\frac{\text{mol}}{(\text{m}^2\bullet\text{s})}\right) = J\left(\frac{\text{A}}{\text{m}^2}\right)$$

Quasistatic Approximation:



$$\oint_C \mathbf{E}\cdot d\mathbf{s} = \cancelto{0}{-\frac{d}{dt}\int_S \mu_0 \mathbf{H}\cdot d\mathbf{a}} \approx 0$$

- ***E*** is a "Conservative Field"
- Can define an "electrical potential" ϕ
- $\underline{\boldsymbol{E}} = -\nabla\phi$

PSet 4, Prob 1

(1) From EM Waves to Quasistatics (a 3-line derivation...)

For slow enough time rates of change ($\partial/\partial t \rightarrow 0$), we can neglect the ($\partial \mu H/\partial t$) term in **Faraday's law** and arrive at the **quasistatic form, $\nabla \times \boldsymbol{E} \approx \mathbf{0}$**.......Show that this quasistatic limit corresponds to the case where the **wavelength λ** of the EM wave is **>> characteristic length L of the system** (e.g., a tissue, cell, etc.)....use scaling analysis with Maxwell's eqns…..

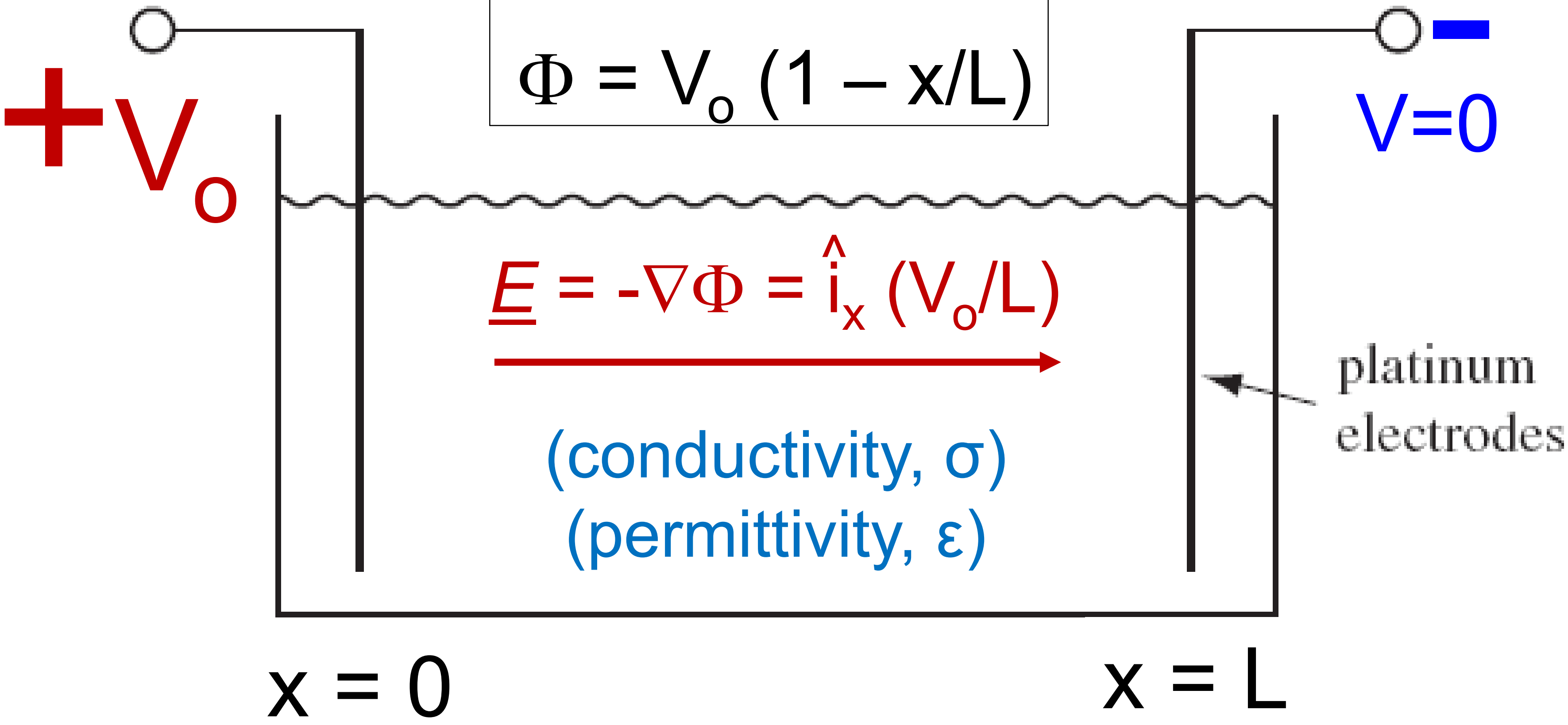
ElectroStatics: $\nabla \bullet \underline{J} = -(\partial \rho_e / \partial t) \equiv 0$

$\nabla \bullet J = 0 = \nabla \bullet \sigma E = \sigma[\nabla \bullet (-\nabla \Phi)] = 0 \rightarrow \nabla^2 \Phi = 0$ Laplace

$(J = \sigma E)$

$$\frac{\partial^2 \Phi}{\partial x^2} = 0$$

$$\Phi = V_o (1 - x/L)$$



$+V_o$

$V=0$

$\underline{E} = -\nabla \Phi = \hat{i}_x (V_o/L)$

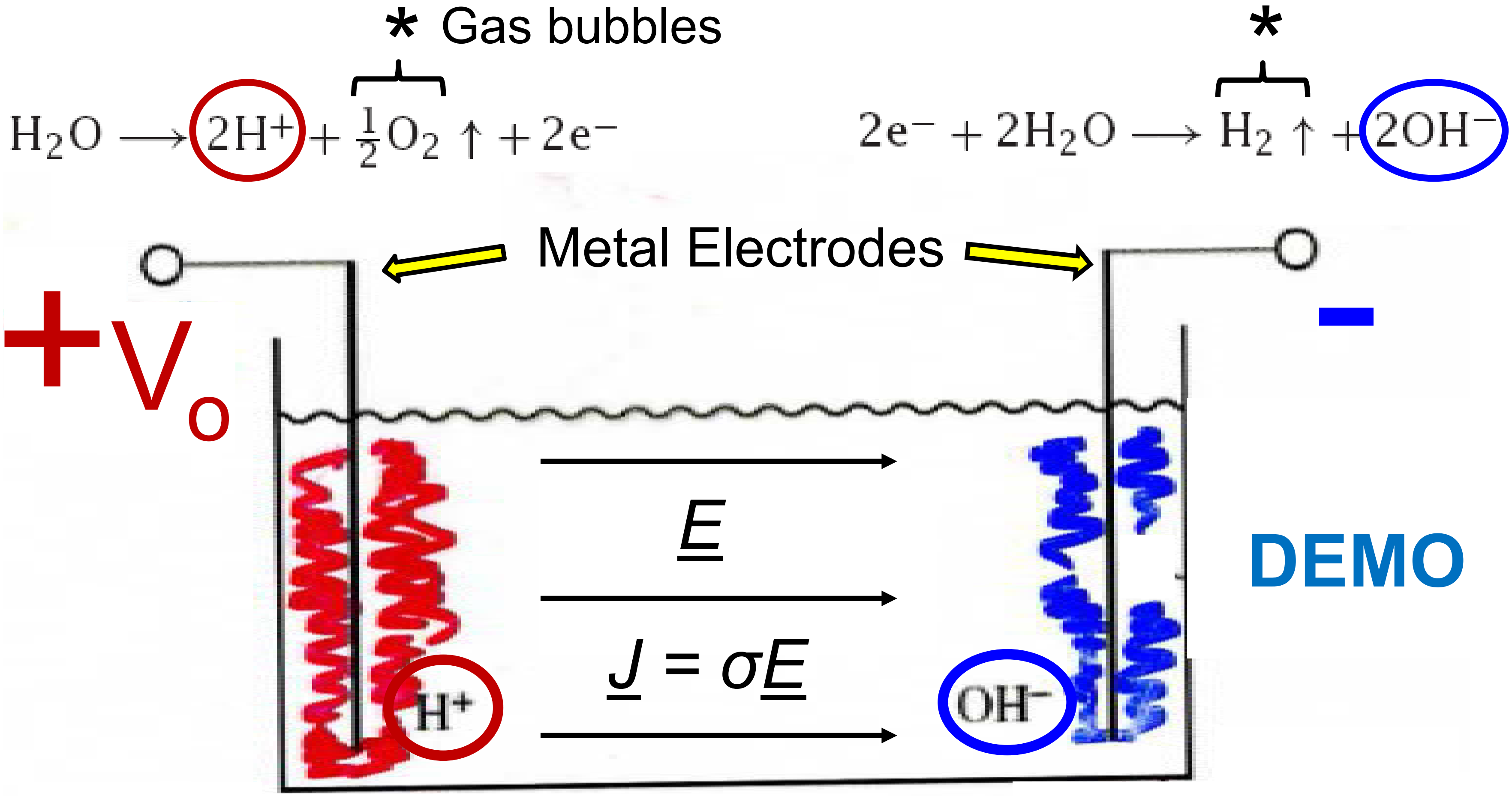
(conductivity, σ)

(permittivity, ε)

$x = 0$

$x = L$

But: Electrolysis Reactions at Electrodes



$$H_2O \longrightarrow 2H^+ + \overbrace{\tfrac{1}{2}O_2}^{*} \uparrow + 2e^- \qquad 2e^- + 2H_2O \longrightarrow \overbrace{H_2}^{*} \uparrow + 2OH^-$$

DEMO

Really: ***J*** **= σ*E* +** ***diffusion + convection***

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20.430J / 2.795J / 6.561J / 10.539J Fields, Forces, and Flows in Biological Systems
Fall 2015

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