Massachusetts Institute of Technology

Biological Engineering Division
Department of Mechanical Engineering
Department of Electrical Engineering and Computer Science

BEH.410/2.798J/6.524J/10.539J
Molecular, Cellular & Tissue Biomechanics, Spring 2003

Problem Set # 6 (Poroelastic materials)

Issued: Mon., 4/7/03
Due: Wed., 4/16/03

READING ASSIGNMENT:
Grodzinsky, Chapter 8

PROBLEMS:
Do problems 1-2 attached.

Continued on next page....

Problem 1: Linear, isotropic, homogeneous, poroelastic material

Consider a Poroelastic tissue specimen subjected to confined compression.

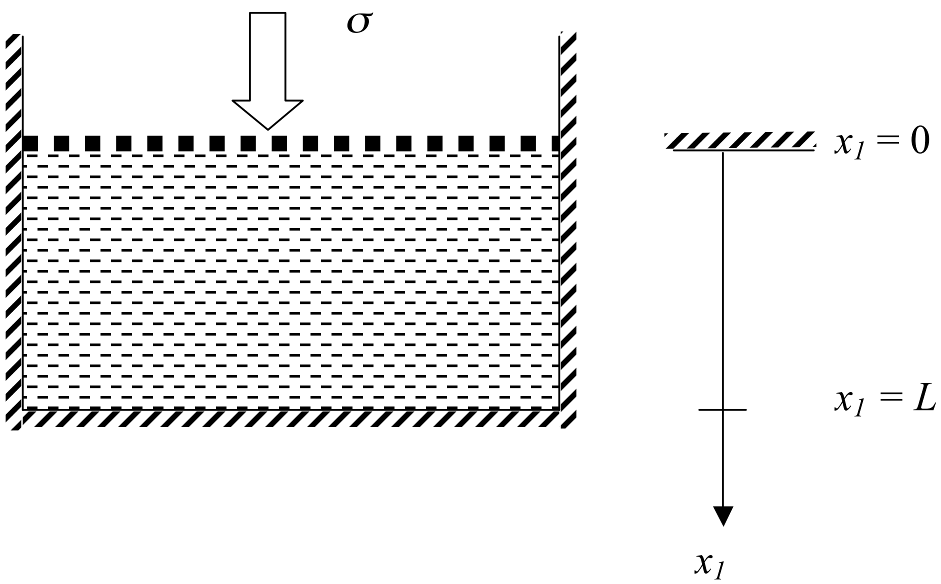
(a) Extending what was done in class, show that the displacement $u_1(x_1,t)$ is described by a partial differential equation having the form of a diffusion equation with equivalent "diffusivity" equal to Hk, the product of the confined compression modulus $H = (2G + \lambda)$ and the hydraulic permeability k.

(b) Derive an analogous diffusion equation that describes the spatial and temporal dependence of the fluid pressure p. What is the equivalent "diffusivity"?

(c) A step in displacement is applied at $x_1 = 0$ having amplitude u_0. State the boundary conditions on $u_1(x_1=0,t)$ and $u_1(x_1=L,t)$ and the initial condition $u_1(x_1,t=0)$ that would be used to solve for the displacement $u_1(x_1,t)$ occurring during this "stress relaxation". ***(Do not solve.)***

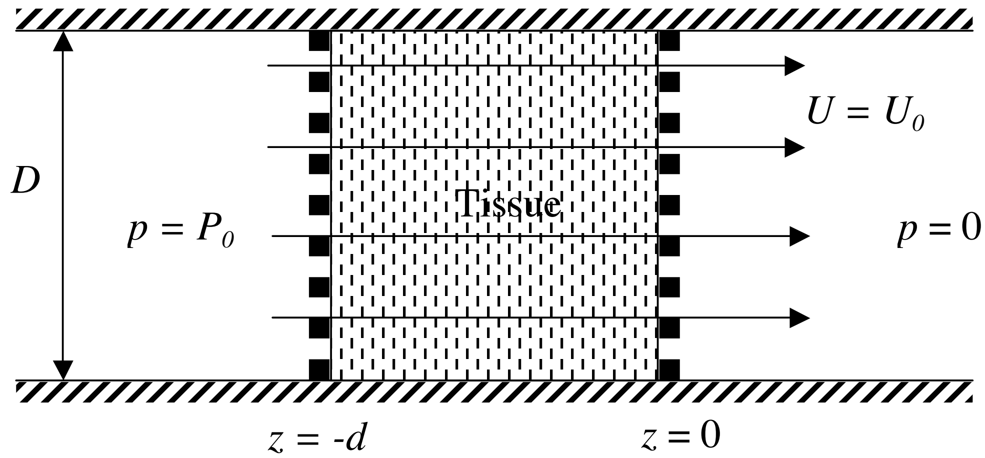
(d) A step in stress is applied at $x_1 = 0$ of amplitude σ_0. State the boundary conditions on the displacement (or its slope) and the initial condition on $u_1(x_1,t=0)$ that would be used to solve for the creep displacement $u_1(x_1,t)$. ***(Do not solve.)***

(e) For the stress relaxation example of part (c), solve the diffusion equation for the displacement $u_1(x_1,t)$ for all (x_1,t) given the initial and boundary conditions you provided above. (Using separation of variables, your answer will involve a sum of terms that are each periodic in space and exponentially decaying in time. Find the Fourier coefficients and the relaxation times.) What is the expression for the slowest ("n=1") decay time; i.e., the *stress relaxation* time, in terms of material and geometric constants?



Problem 2 Perrmeability measurements

This problem emphasizes a general issue concerning the measurement of hydraulic permeability of hydrated soft tissues and other poroelastic media: applied pressure gradients and the resulting fluid flow can cause consolidation or deformation of the tissue. This might give rise to a *nonlinear permeability* that may ultimately be important to include in a model.



A cylindrical disk of porous, hydrated tissue has thickness d and diameter D, and is held within a chamber that confines the disk at its radial periphery. the tissue is supported by a rigid, porous filter located at the position $z = 0$. A constant pressure drop P_0 is applied across the tissue from left to right, resulting in a constant fluid flow velocity U_0 with respect to the tissue (the rigid filter prevents the tissue from moving, but does not impeded fluid flow).

The applied pressure drop and resulting fluid flow cause a compresssion of the tissue against the rigid filter. You are to find the resulting steady state, z-dependent strain and displacement profiles using the 1-dimensional poroelastic model for tissue derived in class.

(a) As done in class, write expressions for (1) conservation of momentum, (2) Darcy's law, and (3) total stress versus strain (including hydrostatic pressure) in terms of the total stress σ_{zz}, strain ε_{zz}, displacement u_z, and pressure p.

The fourth equation, mass conservation, is given here as:

$$U = \frac{-\partial u_z}{\partial t} + U_0$$

where the term U_0 corresponds to the possibility of a constant flow of fluid even when $\partial u_z / \partial t = 0$, as is the case here.

(b) Combine your equations (1)-(3) with the above equation to find the differential equation for u_z in terms of the constant velocity U_0, the hydraulic permeability k, and the confined compression modulus $H = (2G + \lambda)$.

(c) For the case of steady flow ($\partial / \partial t = 0$), integrate your differential equation to find an expression for the displacement u_z in terms of two integration constants. Find the two constants from the boundary conditions:

(i) zero displacement at $z = 0$

(ii) zero strain ($\partial u_z / \partial z$) at $z = -d$

(d) Write your final expressions for $u_z(z)$ and $\varepsilon_{zz}(z)$ within the tissue. Sketch u_z and ε_{zz} as functions of z within the tissue $(-d<z<0)$.