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Molecular, Cellular & Tissue Biomechanics, Spring 2003

Problem Set #1 (Elastic materials)

Issued: 3/10/03

Due: 3/17/03

READING ASSIGNMENT:

Malvern, Chapter 6, pp. 278-282

Ward, Chapters 1 & 2

Papers on Extracellular Matrix Macromolecules

Section 3.5 on Connective Tissue in ALG text.

PROBLEMS:

Do problems 1-3 attached.

Continued on next page....

Problem 1

A small cubic tissue specimen of dimension $h = 1\text{cm}$ is tested to determine its elastic properties. If we assume that the material can be modeled as linear, isotropic and homogeneous, answer each of the following:

a) The sample is placed between two rigid surfaces and compressed by 5% of its initial height (Fig. 1a). If the material were incompressible and has a Young's modulus of 10^6 Pa, how much force would need to be applied?

b) Next, a shearing force is applied as in Fig. 1b. What is the shear modulus of the specimen, based on your measurements in (a) and the stated assumptions? How great a shear force would need to be applied to displace the top surface a distance of $0.1h$?

c) If, in the process of conducting the experiment in (a), it is observed that the lateral dimensions of the specimen increase from h to $1.02h$, what is the final volume of the specimen? Is the specimen incompressible? What is the Poisson ratio?

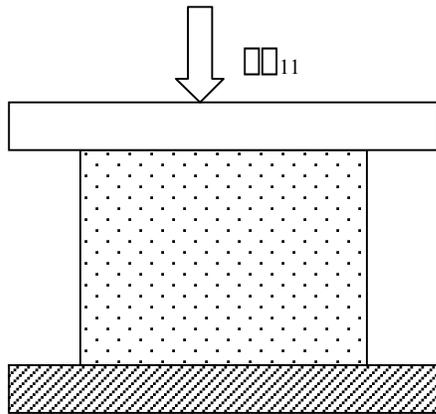


Fig. 1a

h

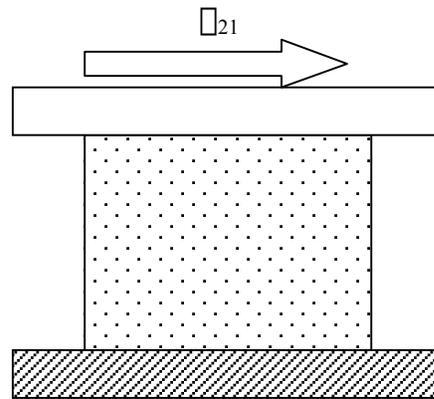


Fig. 1b

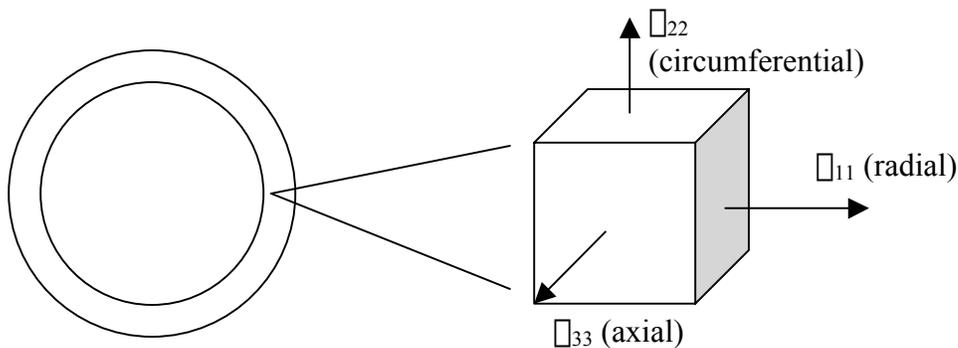
(c) Find an expression for the Young's modulus E in terms of the shear modulus G and Poisson's ratio ν by applying both forms of Hooke's law to the shear configuration shown in Fig. 1b.

Problem 2

An artery is subjected to an internal pressure of 100 mmHg (760 mmHg = 10^5 Pa). Assuming the wall is isotropic and linearly elastic and has a Poisson ratio of 0.45, an elastic modulus (E) of 10^5 Pa, initial radius (R) of 1 cm, and wall thickness (h) of 0.1 cm, answer the following:

a) What is the average circumferential stress in the wall? Making the thin-wall approximation so that the circumferential stress can be assumed uniform, what is the radial dependence of the radial compressive stress?

Now consider a small cubic element located at a position halfway through the wall where we consider the circumferential (“1”), radial (“2”), and axial (“3”) coordinates as shown in the sketch and revert to planar coordinates (for algebraic simplicity). The radial and compressive stresses are assumed known from (a).



b) Taking the artery to be axially-constrained so that $\epsilon_{33} = 0$, find σ_{33} , σ_{22} , and σ_{11} at a location halfway through the wall ($r = R+h/2$).

c) What is the change in volume of the small element shown in the figure? What is the bulk compressive modulus? Recognizing that the tissue is comprised primarily of water and other essentially incompressible constituents, describe how such a volume change might take place.

d) What condition must be satisfied between the three strains (ϵ_{11} , ϵ_{22} , ϵ_{33}) for an incompressible material ($\nu = 0.5$)? Show that this leads to a bulk modulus ($K = \nu + 2G/3$) that is infinite. If the cube of material depicted in the figure above is initially h on each side, and is $1.1h \times 0.8h \times 0.8h$ following deformation, what is ν ? Does this make physical sense? How might this be possible given that the material is a biological, hydrated specimen?

Problem 3

The biomechanical properties of a tissue matrix are determined by the tissue's randomly oriented collagen network enmeshed in a gel of negatively charged proteoglycans. For the experiments described below, this tissue is well described in equilibrium by a linear, homogenous, isotropic ("Hookean") constitutive law for the porous network.

a) This tissue section is held between two glass slides to view the matrix and cells during deformation (Fig. 1). The specimen is subjected to an applied stress σ_1 , which produces a small compressive strain ϵ_1 in the x_1 -direction. The rigid, fluid impermeable glass slides prevent any elongation in the x_2 -direction, but there are no constraints in the x_3 -direction (i.e., unconfined in the x_3 -direction).

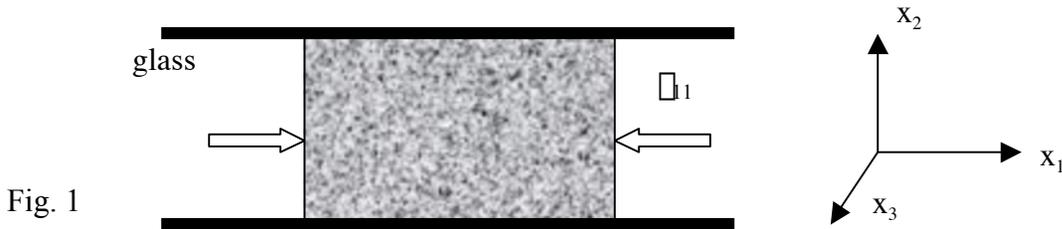


Fig. 1

Use the conditions in the x_2 and x_3 directions in the equilibrium compressed state to find an expression for the apparent equilibrium modulus E_a in the x_1 -direction (defined by $E_a = \sigma_1 / \epsilon_1$) in terms of the tissue's equilibrium Young's modulus E and the Poisson ratio ν .

b) The charge-titration behavior of the tissue is graphed in Fig. 2. (The net charge is due to ionization of sulfate, carboxyl, and amino acid groups on the various matrix macromolecules, which varies with pH).

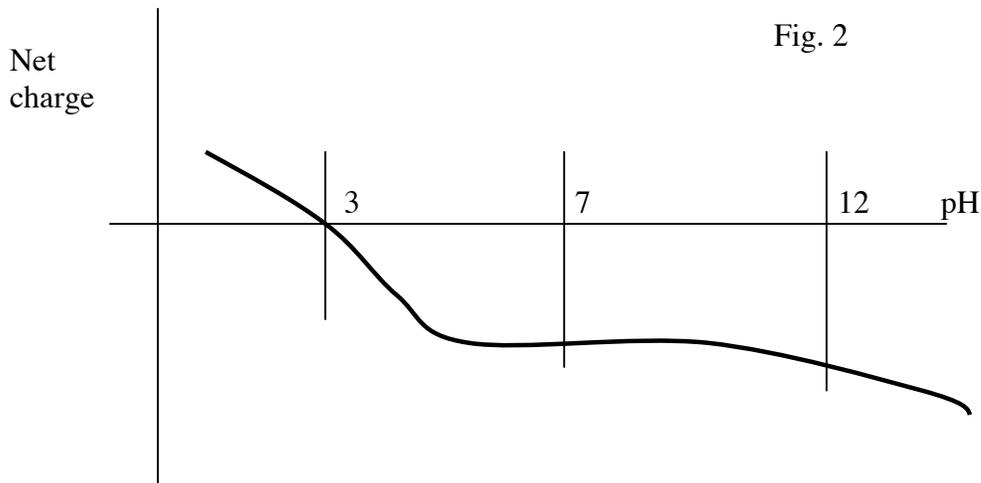


Fig. 2

The known magnitude of the modulus E_a in equilibrium at physiological pH ~ 7 is known. On a graph similar to that in Fig. 2, sketch the relative variation of E_a with pH in the range of pH = 2-12. The important result here is the relative variation with pH, not the exact magnitude at any specific pH.