MASSACHUSETTS INSTITUE OF TECHNOLOGY

Molecular, Cellular and Tissue Biomechanics
BEH.410 / 2.978J / 6.524J / 10.537J

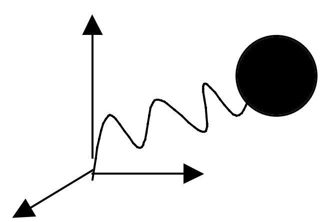
Problem Set #1 **Issued: 2/13/03**
Due: 2/19/03 (in class)

Please staple each problem separately as we will have different graders grading each problem. Also please put your name on each problem!

Problem #1: Boltzmann's Relation and Cell Micromechanics

Recently D. Discher studied the mechanical behavior of the red blood cell cytoskeleton (we will learn soon that this is a polymer network attached to the cell plasma membrane). Discher attached a bead of 40 nm to this network (see figure A below) and tracked the center of mass of the bead over time (figure B). The network acts as a *spring* constraining the motion of the bead.

Image removed due to copyright considerations.



a) Consider a simple system consisting of a bead attached to a spring (with spring constant κ) at constant N,V,T. Calculate the mean squared displacement relative to the average position $\left\langle \left(x - \langle x \rangle\right)^2 \right\rangle$. (Note: you should start by first proving the general statistical relationship: $\left\langle \left(x - \langle x \rangle\right)^2 \right\rangle = \langle x^2 \rangle - \langle x \rangle^2$)

b) From the data above, estimate the cytoskeleton's spring constant.

c) Your analysis in part a) can be generalized to prove the Principle of Equipartition of Energy: the mean value of each independent quadratic term in the energy of a system is equal to 1/2 kT. Prove that in general if the energy of a molecule depends on the square of a parameter (such as position), then the mean energy $\langle U \rangle$ associated with the parameter is equal to 1/2 kT.

Problem #2: Microrheology and the Langevin Equation

In problem #1 we used equilibrium Statistical Mechanics to relate the mean squared displacement of the bead to the spring constant of the cytoskeleton. Here we will employ a simplified 1-dimensional form of the Langevin equation:

$$\zeta \frac{dx}{dt} = -\kappa x + f(t)$$

$$\langle f(t) \rangle = 0$$

$$\langle f(t) f(t') \rangle = 2\zeta kT \delta(t - t)$$

You will notice that we have neglected inertia and the Brownian force is delta correlated. (Note: the prefactor of 2kTξ is obtained using the inertial form of the Langevin equation without spring forces to solve for the velocity correlation function $\langle v(t+\tau) v(t) \rangle = \frac{F}{2m\zeta} \exp(-\tau\zeta / m)$ then using equipartition of energy at τ=0.)

a) Show that the position autocorrelation function for the 1-D model above is $\langle x(t+\tau) x(t) \rangle = \frac{kT}{\kappa} \exp(-\tau\kappa / \zeta)$. Note: you can start with the fact that

$$x(t) = \int_{-\infty}^{t} \exp\left[\frac{-\kappa (t - t')}{\zeta}\right] dt' .$$

b) In a particle tracking experiment one usually measures $\Delta(\tau) = \langle (x(t+\tau) - x(t))^2 \rangle$. Use your result from part a) and equipartion of energy to arrive at a simple expression for $\Delta(\tau)$.

c) Calculate the limits of $\Delta(\tau)$ at short τ and long τ.

d) Sketch your answer from part c) and describe how you would calculate the viscosity and spring constant of a complex medium (such as a cell) from a plot of $\Delta(\tau)$ versus τ.

Problem #3: Collapse of a Macromolecule

Problem #11 in chapter 10 of Dill and Bromberg