Homogeneous??

Cells in 3D matrix

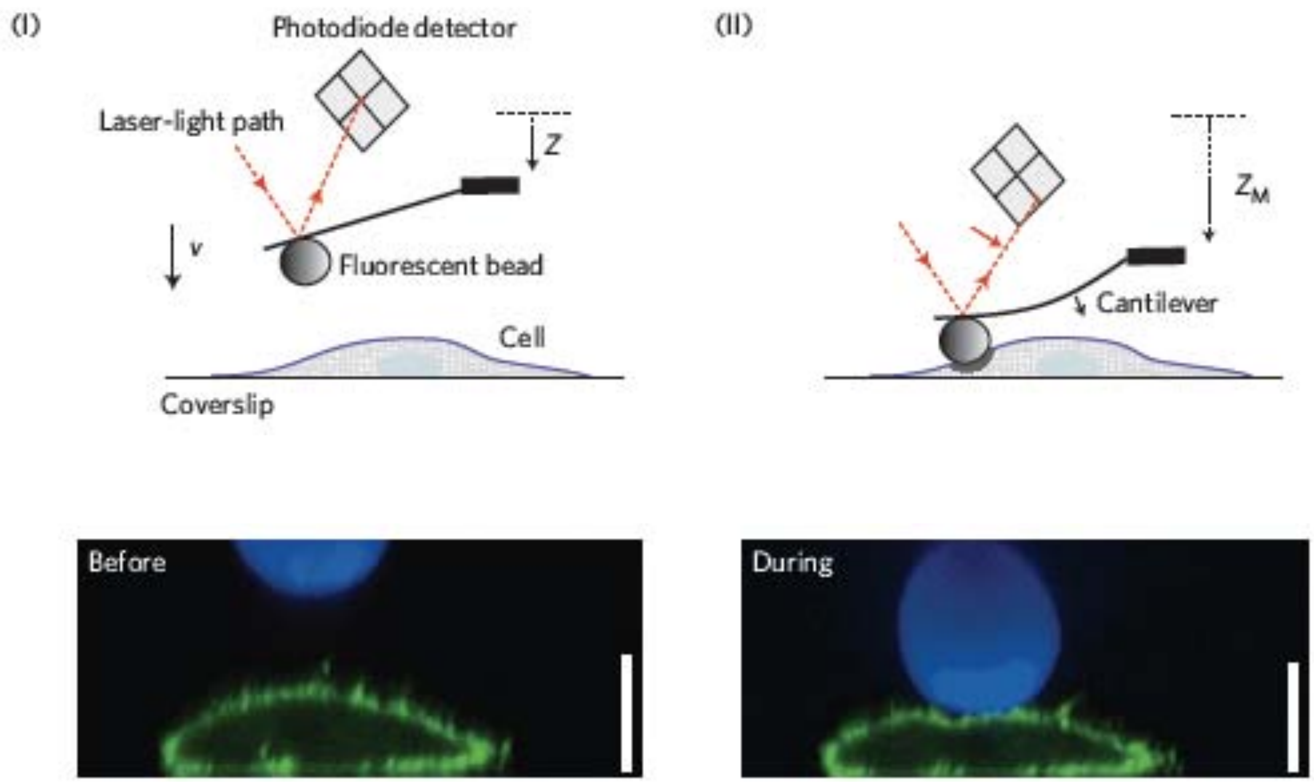
)LJXUHV UHPRYHG GXH WR FRS\ULJKW UHVWULFWLRQV

MDA-MB-231 breast cancer cells migrating inside a collagen gel.

- Dense cortical actin with myosin.
- Cross-linkers more homogeneously distributed

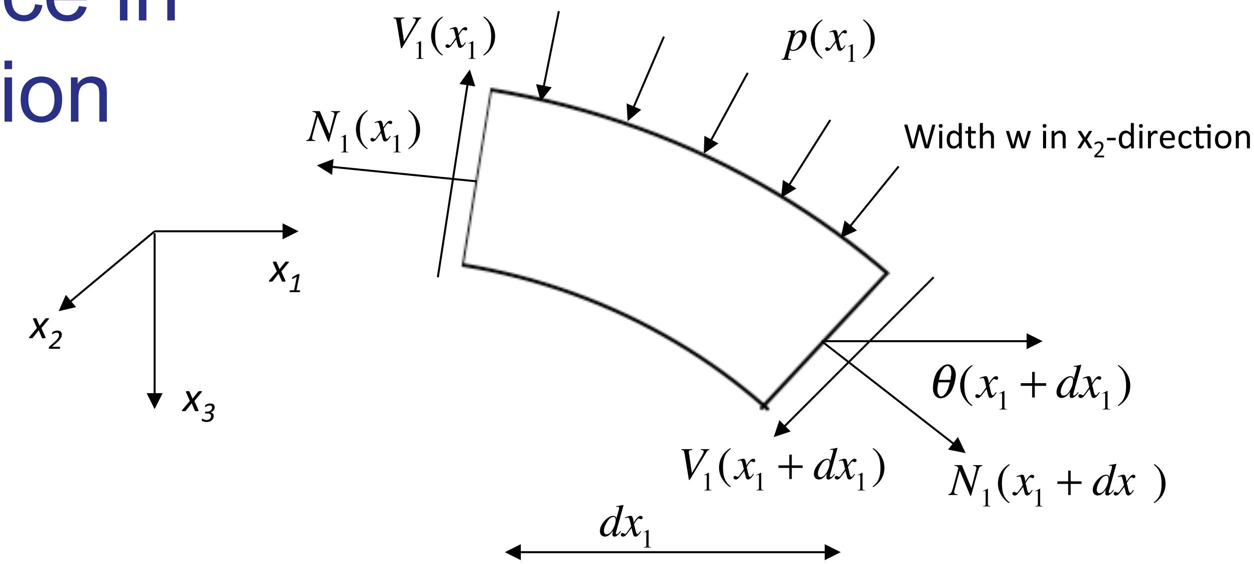
Rajagopalan, unpublished

Indentation by a microsphere
Importance of the cortex



Source: Moeendarbary, Emad, et al. "The Cytoplasm of Living Cells Behaves as a Poroelastic Material." *1DWXUH 0DWHULDOV* 12, no. 3 (2013): 253-61.

Force balance in the x_3 direction



$$\theta(x_1) \approx \partial u_3 / \partial x_1$$

$$V = \int_0^h \sigma_{13} dx_3$$

$$p dx_1 - V_1(x_1) - N_1(x_1)\theta(x_1) + V_1(x_1 + dx_1) + N_1(x_1 + dx_1)\theta(x_1 + dx_1) = 0$$

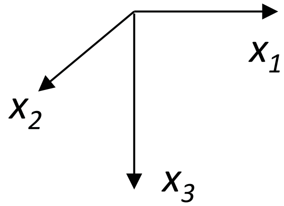
Use Taylor expansions for V_1 and $N_1\theta_1$

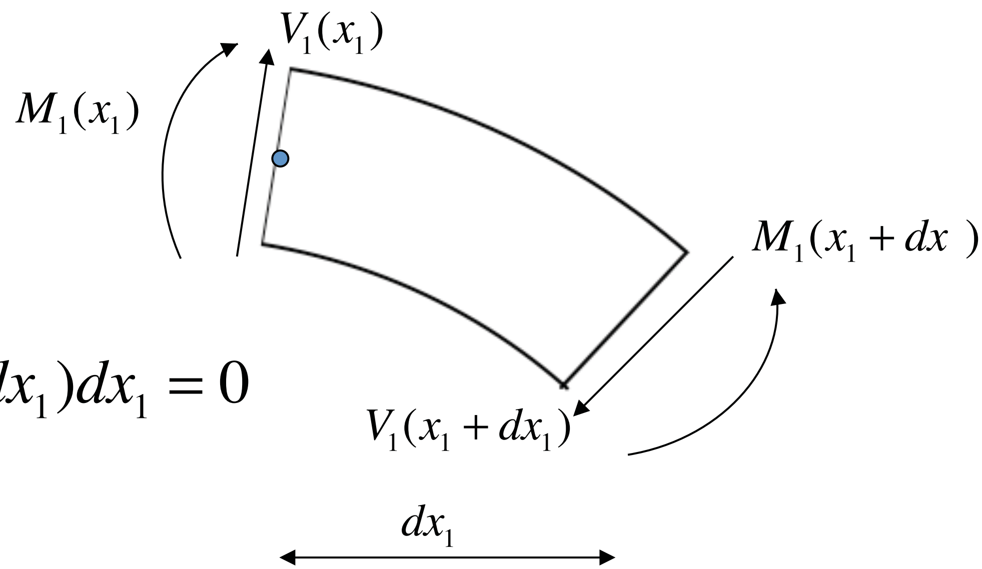
$$V_1(x_1 + dx_1) = V_1(x_1) + \frac{\partial V_1}{\partial x_1} dx$$

Combine and divide by dx_1:

$$p(x_1) + \frac{\partial V_1}{\partial x_1} + \frac{\partial}{\partial x_1}\left(N_1\theta_1\right) = p(x_1) + \frac{\partial V_1}{\partial x_1} + \frac{\partial}{\partial x_1}\left[N_1\left(\frac{\partial u_3}{\partial x_1}\right)\right] = 0$$

Moment (torque) balance about the x_2 axis





$$-M_1(x_1) + (M_1 + \frac{\partial M_1}{\partial x_1} dx_1) - V_1(x_1 + dx_1)dx_1 = 0$$

$$\frac{\partial M_1}{\partial x_1} = V_1(x_1)$$

$$M_1(x_1) = -K_b^{'} \frac{\partial^2 u_3}{\partial x_1^2}$$

$$p - K_b^{'} \frac{\partial^4 u_3}{\partial x_1^4} + \frac{\partial}{\partial x_1}\left(N_1 \frac{\partial u_3}{\partial x_1} \right) = 0$$

in 1D

Full governing equations for linear deformations, and the reduced forms for bending or tension dominance

Bending stiffness — Membrane tension

$$K_b'\left(\frac{\partial^4 u_3}{\partial x_1^4}\right) - N\left(\frac{\partial^2 u_3}{\partial x_1^2}\right) - p = 0$$

$$\frac{Bending}{Tension} \propto \frac{K_b'\bar{u}/\lambda^4}{N\bar{u}/\lambda^2} \propto \frac{K_b'}{N\lambda^2} \gg 1$$

$$\frac{K_b'}{N\lambda^2} \ll 1$$

$$K_b'\left(\frac{\partial^4 u_3}{\partial x_1^4}\right) = p$$

$$p = -N\left(\frac{\partial^2 u_3}{\partial x_1^2}\right) \cong N\left(\frac{1}{R}\right)$$

u = displacement

p = pressure difference

N = membrane tension

R = radius of curvature

x = spatial coordinate

λ = characteristic length

20.310J / 3.053J / 6.024J / 2.797J Molecular, Cellular, and Tissue Biomechanics
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