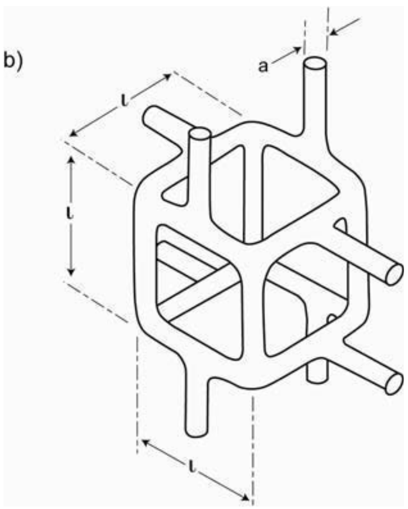
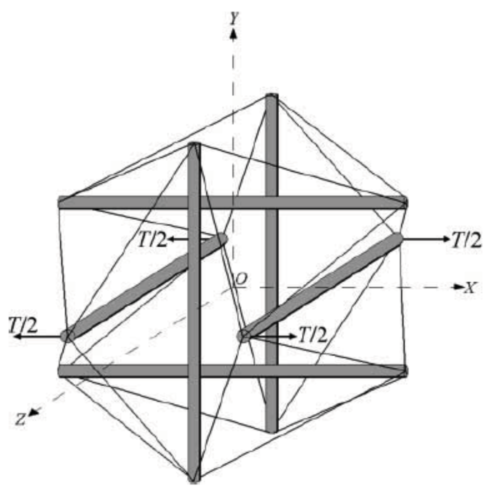
Three microstructural models for the cytoskeleton

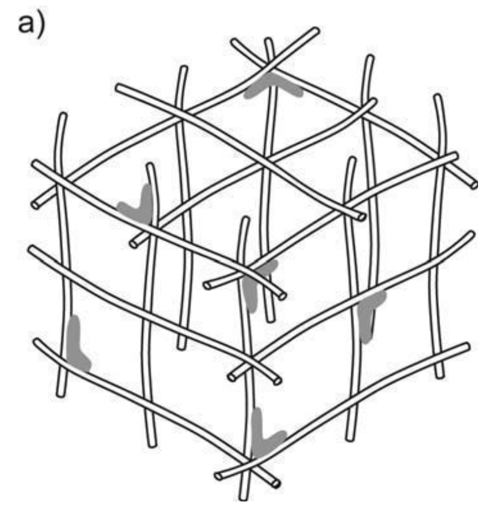


Cellular solids

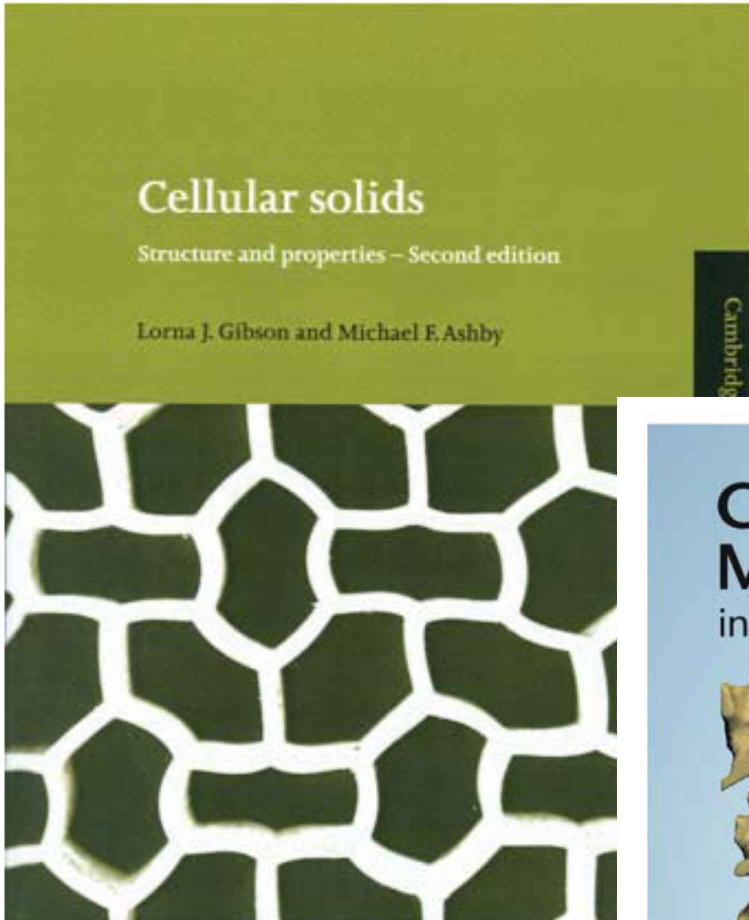


Tensegrity

Source: Stamenović, D., and Donald E. Ingber. "Models of Cytoskeletal Mechanics of Adherent Cells." *Biomechanics and Modeling in Mechanobiology* 1, no. 1 (2002): 95-108.

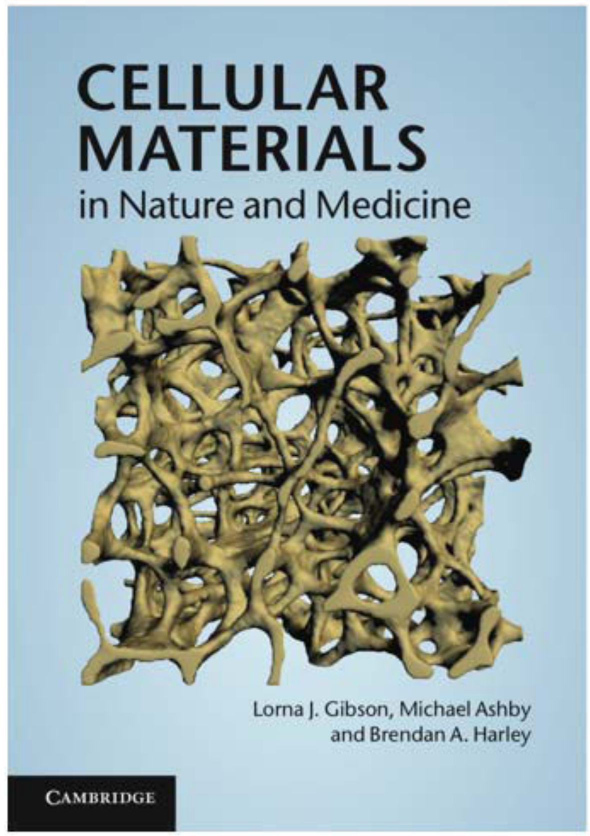


Biopolymer



Gibson & Ashby, 1988

Photographs of Lorna Gibson, and C. Forbes Dewey removed due to copyright restrictions.



The cytoskeleton as a homogeneous, isotropic, elastic material.

Figure removed due to copyright restrictions.

Fig. 1. The cytoskeleton of a macrophage lamellipodium as seen by electron microscopy. The fibrous structure is mainly comprised of actin filaments. (John Hartwig, http://expmed.bwh.harvard.edu)

Cellular Solids Model

(Gibson & Ashby, 1988, Satcher & Dewey, 1997)

Figures removed due to copyright restrictions.

$\Phi \sim (a/L)^2$ (solid fraction)

$\delta \sim FL^3/(E_f I)$ from bending analysis where $I \sim a^4$

$\sigma \sim F/L^2$

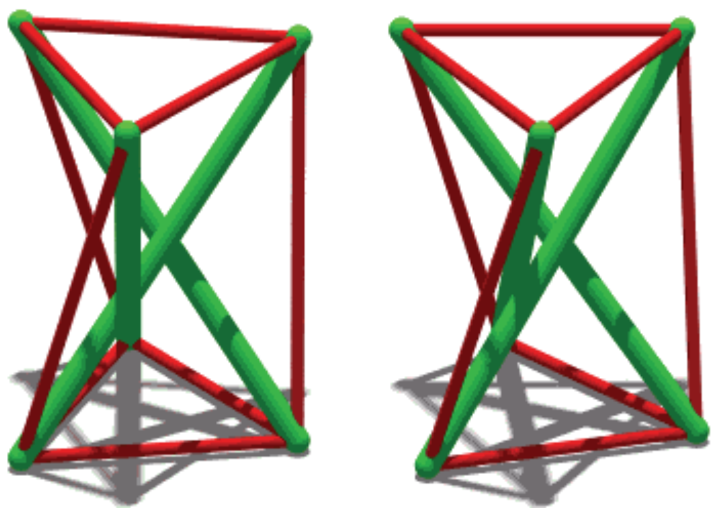
$\varepsilon \sim \delta/L$

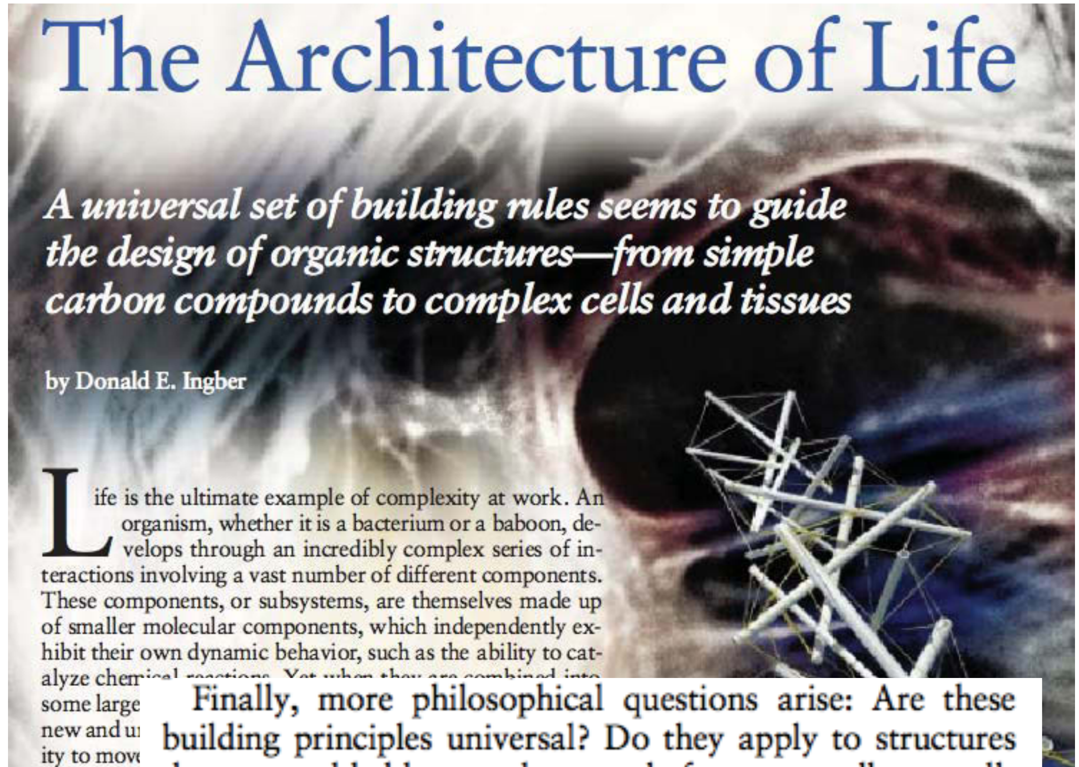
$E_n = \sigma/\varepsilon = c_1 E_f I/L^4$ (network modulus)

$E_n/E_f = c_1\,\Phi^2$ or $G_n \sim E_f\,\Phi^2$

a = radius of filaments

Don Ingber, Scientific American





The Architecture of Life

A universal set of building rules seems to guide the design of organic structures—from simple carbon compounds to complex cells and tissues

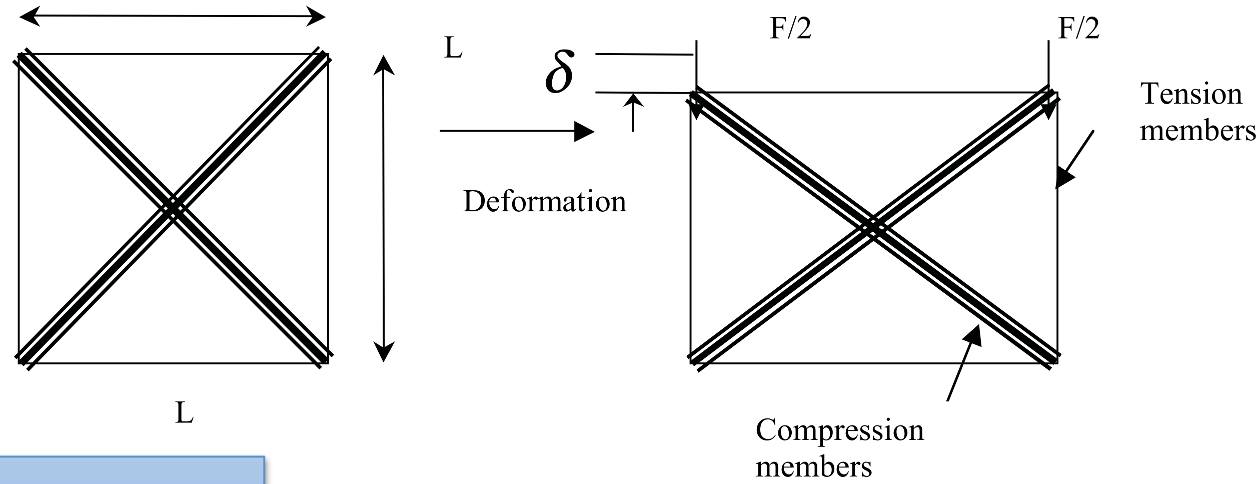
by Donald E. Ingber

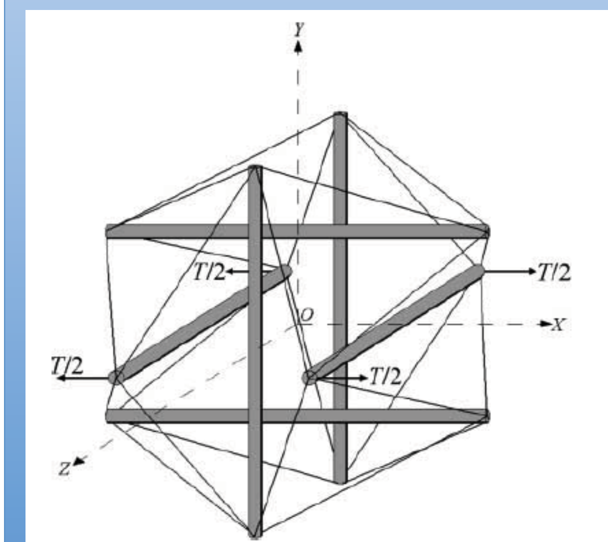
Life is the ultimate example of complexity at work. An organism, whether it is a bacterium or a baboon, develops through an incredibly complex series of interactions involving a vast number of different components. These components, or subsystems, are themselves made up of smaller molecular components, which independently exhibit their own dynamic behavior, such as the ability to catalyze chem… some large… new and u… ity to mov…

Finally, more philosophical questions arise: Are these building principles universal? Do they apply to structures that are molded by very large scale forces as well as small-scale ones? We do not know. Snelson, however, has proposed an intriguing model of the atom based on tensegrity that takes off where the French physicist Louis de Broglie left off in 1923. Fuller himself went so far as to imagine the solar system as a structure composed of multiple nondeformable rings of planetary motion held together by continuous gravitational tension. Then, too, the fact that our expanding (tensing) universe contains huge filaments of gravitationally linked galaxies and isolated black holes that experience immense compressive forces locally can only lead us to wonder. Perhaps there is a single underlying theme to nature after all. As suggested by early 20th-century Scottish zoologist D'Arcy W. Thompson, who quoted Galileo, who, in turn, cited Plato: the Book of Nature may indeed be written in the characters of geometry.

Source: Ingber, Donald E. "The Architecture of Life." *Scientific American* 278, no. 1 (1998): 48-57.

Tensegrity Model





See Stamenovic for a full derivation.

$$U \sim \int_0^{L_1} \sigma_{f1}\varepsilon_{f1} a^2\, dx + \int_0^{L_2} \sigma_{f2}\varepsilon_{f2} a^2\, dx$$

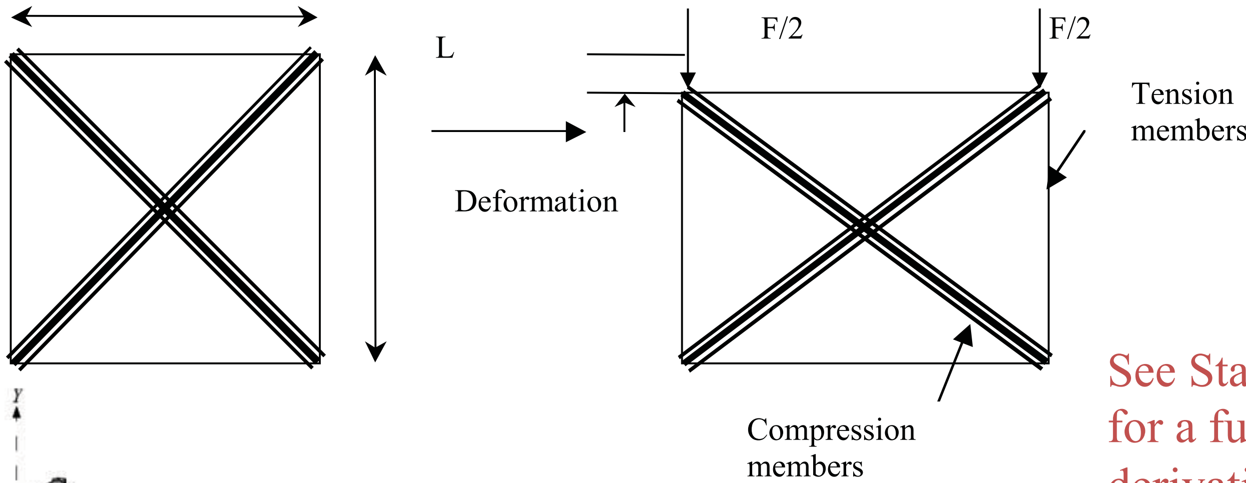
Work done = Δ stored elastic energy

$$F\delta \sim La^2 \left(\frac{\delta}{L}\right)^2 \left(2\sigma_{f0} + E_f\right)$$

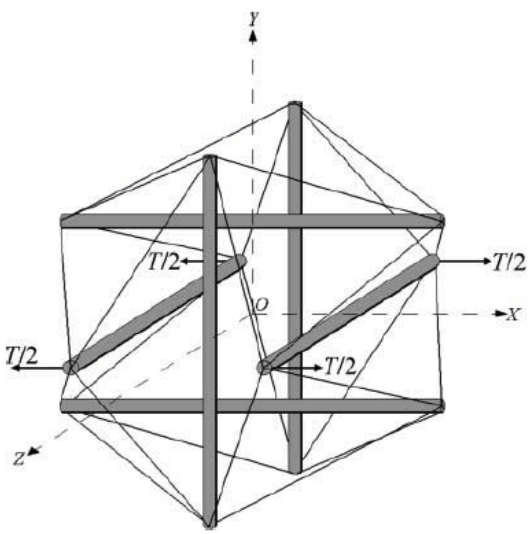
$$E_n \sim \frac{\sigma_n}{\delta/L} \propto \left(2\sigma_{f0} + E_f\right)\left(\frac{a}{L}\right)^2 \propto \left(2\sigma_{f0} + E_f\right)\Phi$$

Source: Stamenović, D., and Donald E. Ingber. "Models of Cytoskeletal Mechanics of Adherent Cells." *Biomechanics and Modeling in Mechanobiology* 1, no. 1 (2002): 95-108.

Tensegrity Model



See Stamenovic for a full derivation.



$$E = \frac{\sigma_c \Phi}{3} \frac{1+4\varepsilon}{1+12\varepsilon}$$

Where σ_c is the prestress in the individual tensile elements and ε is the initial strain in each.

Or, in the limit of ε--> 0,

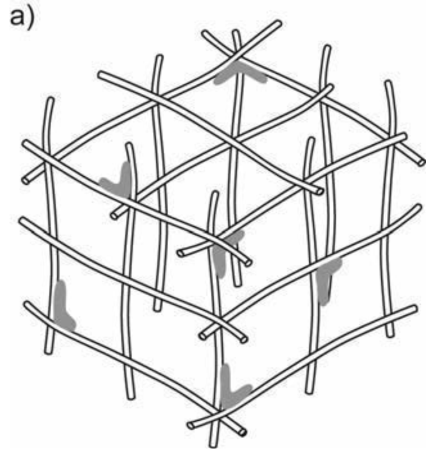
$G_n \sim P/3$

where P is the pre-stress in the tensile elements per unit total cross-sectional area ($P=\pi\sigma_c a^2/L^2$).

Source: Stamenović, D., and Donald E. Ingber. "Models of Cytoskeletal Mechanics of Adherent Cells." *Biomechanics and Modeling in Mechanobiology* 1, no. 1 (2002): 95-108.

Biopolymer Models

a)



For a single segment of polymer between cross-links (Isambert and Maggs, 1997, Maggs, 1999, Storm, et al., 2005)

$$F = \frac{l_p}{l^4} K_b \delta$$

$$\varepsilon_n = \frac{\delta}{l}$$

$$\sigma_n \sim F \cdot \frac{filaments}{area} \sim \frac{F}{\xi^2}$$

Low cross-link density
$$E_n = \frac{\sigma_n}{\varepsilon_n} \sim \frac{l_p K_b}{l^3 a^2} \Phi$$

Maximum cross-link density ($l \sim \xi$)
$$E_n = \frac{\sigma_n}{\varepsilon_n} \sim \frac{l_p K_b}{a^5} \Phi^{5/2}$$

l_p = persistence length

l = distance between entanglements or cross-links

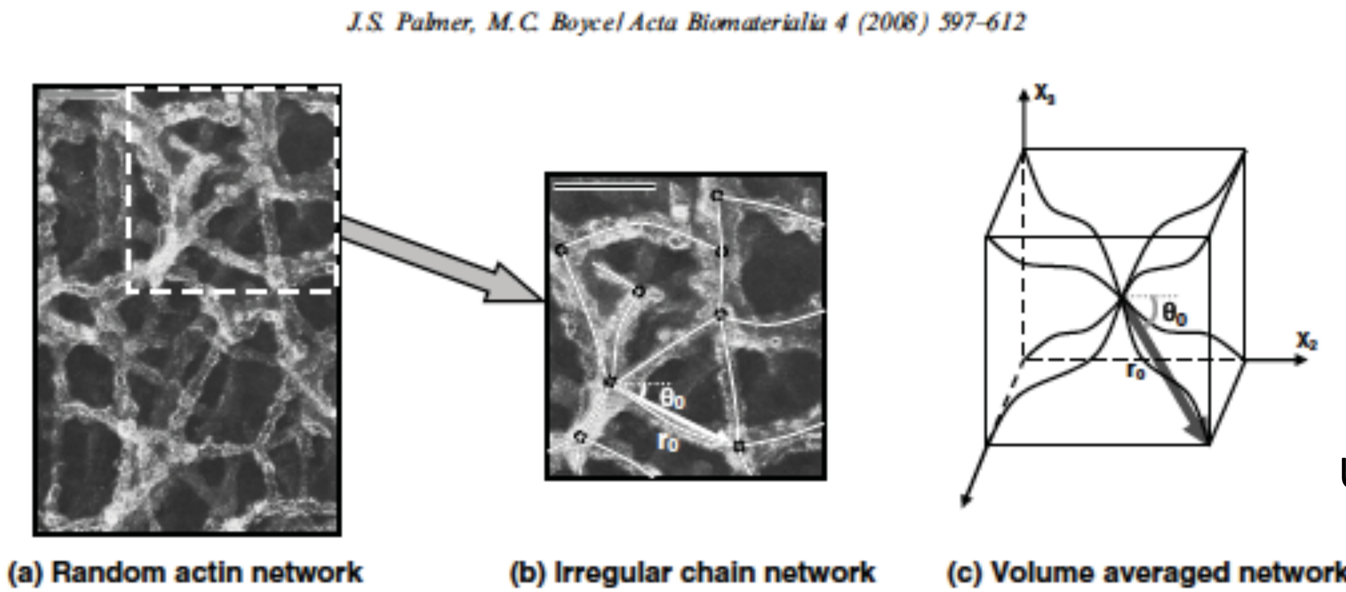
ξ = filament spacing

ε_n = network strain

E_n = network elastic modulus

δ = change in distance between entanglements/cross-links

Φ = solid fraction



Unit cell

Photograph of Mary Boyce removed due to copyright restrictions.

Mary Boyce, MIT

The initial shear modulus is given by

$$G_0 = \frac{nk_B T r_0}{3l_p}\left(\frac{1}{4(1-r_0/L_c)^2}\right)\left(\frac{L_c/l_p - 6(1-r_0/L_c)}{L_c/l_p - 2(1-r_0/L_c)}\right)$$

n = filament density
l_p = persistence length
r_0 = rest junction-to-junction distance
L_c = contour length

n = # filaments/vol = $\Phi/(a^2 L_c)$

Scaling behaviors for the three models

Tensegrity

Predicts a linear dependence on prestress (alone!)

Athermal

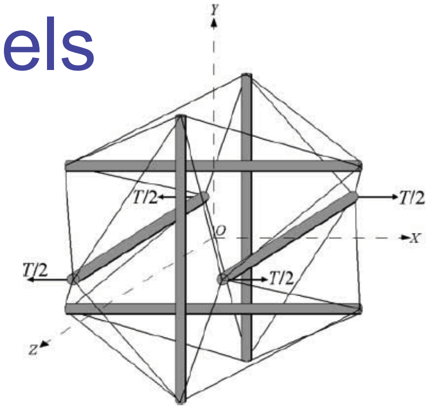
No ability to change cross-link density

No role for cross-link mechanics

Viscoelasticity?

Not valid in the limit of zero prestress

$$G' \sim \sigma_n$$



Source: Stamenović, D., and Donald E. Ingber. "Models of Cytoskeletal Mechanics of Adherent Cells." *Biomechanics and Modeling in Mechanobiology* 1, no. 1 (2002): 95-108.

Cellular Solids

Filament bending stiffness dominates

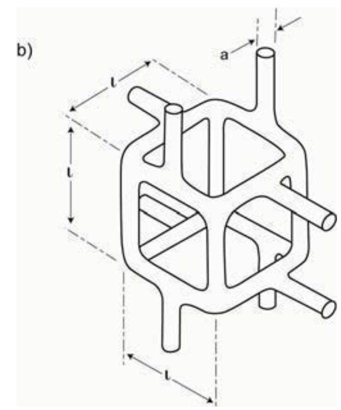
Maximal cross-link density

Athermal

No role for cross-link mechanics

Viscoelasticity?

$$G' \sim E_f \Phi^2$$



Biopolymer

Thermal (WLC at high extensions)

Viscoelastic. Captures ¾ power law at high frequency

Cross-link density and mechanics?

$$G' \sim K_b^2 \Phi^1 \rightarrow K_b^2 \Phi^{5/2}$$

a)