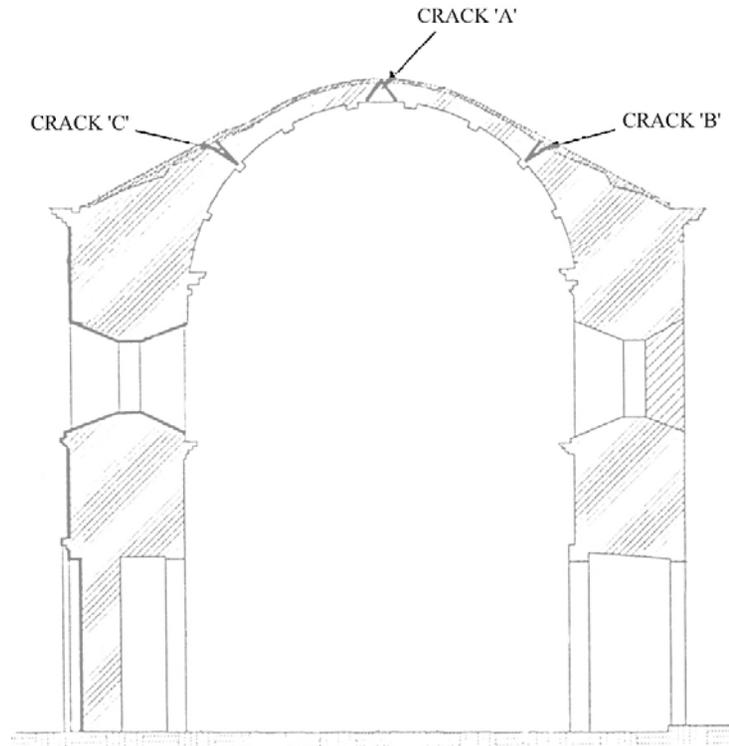


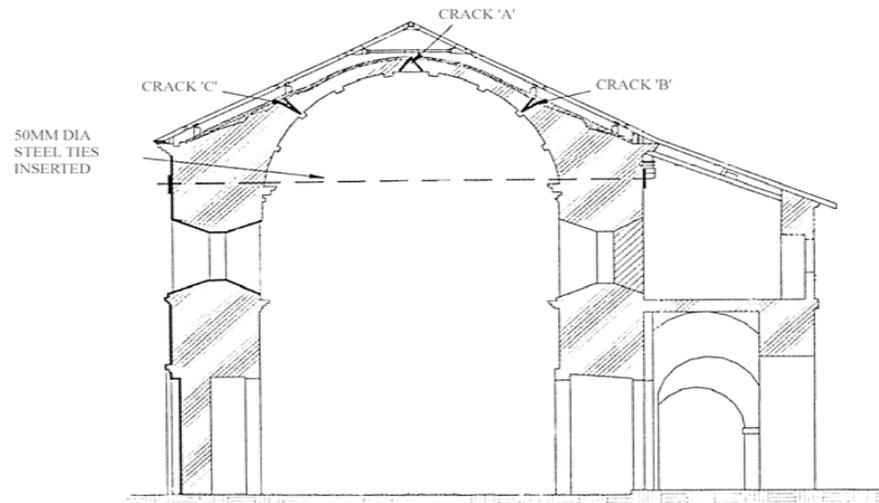
Lecture 4



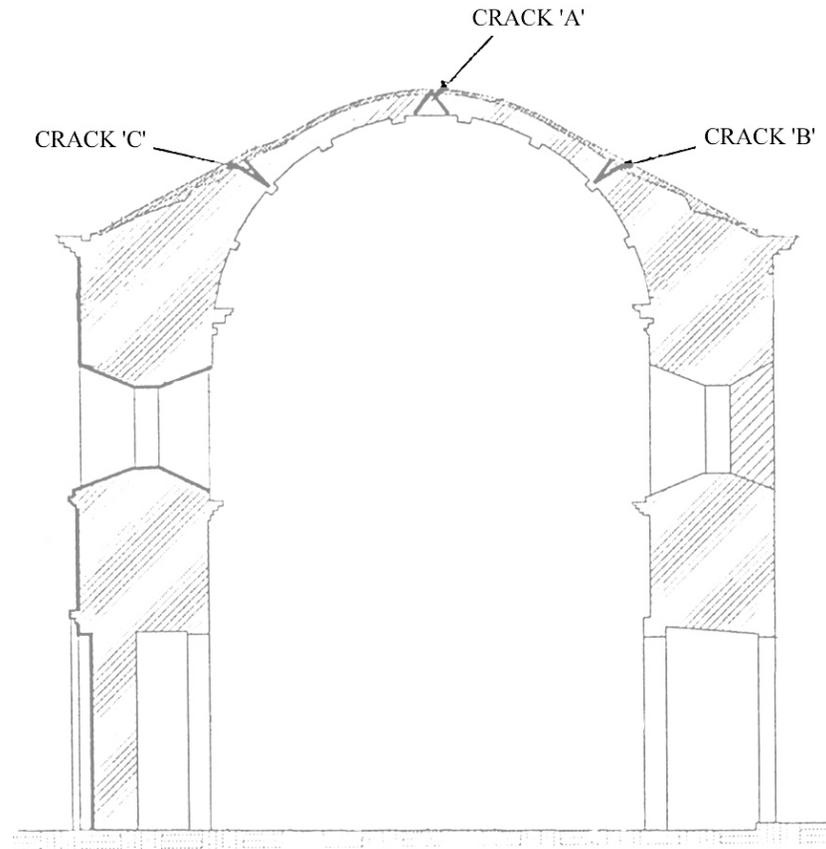
Case Study of 16th C Church in Old Goa

Case Study:

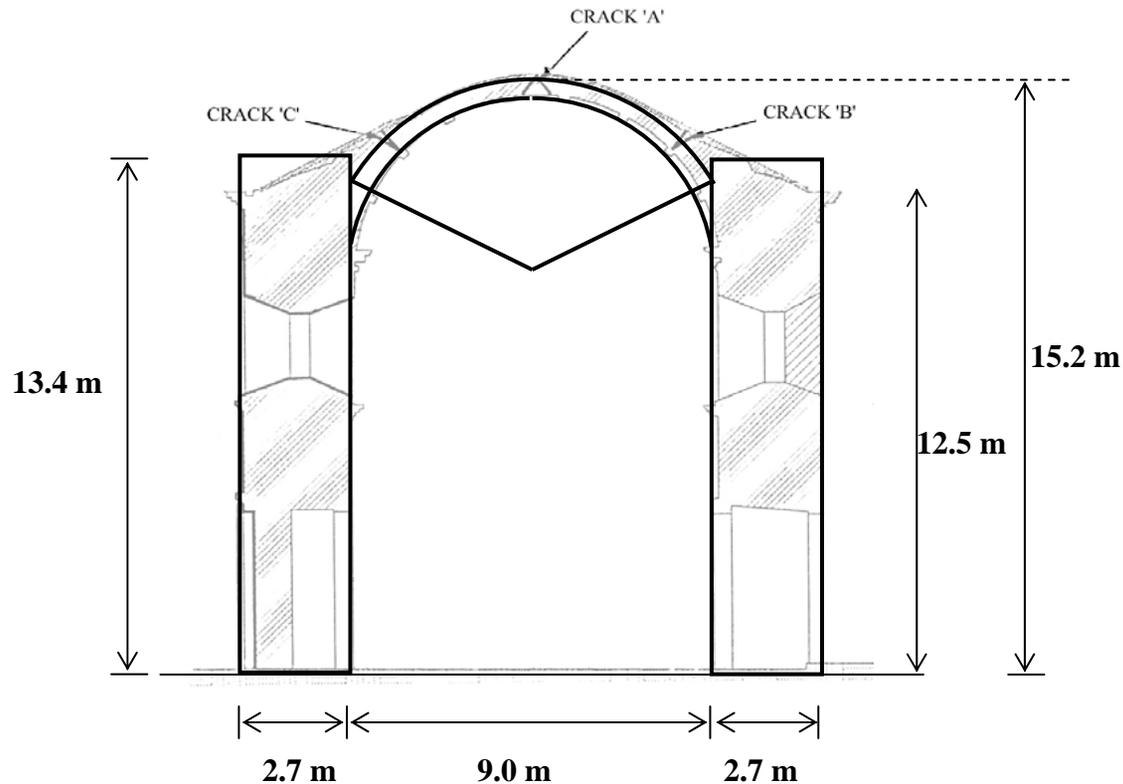
16th C. Church in Goa, India



16th C. Church in Goa, India



16th C. Church in Goa, India



Unit weight of material 25 kN/m^3 – analyze 1 m strip

Solid Buttress at Goa

Weight of the buttress is 905 kN (per metre width)

Vertical reaction of the arch, $V = 64$ kN

Overturning of solid buttress, $H_s = 112$ kN

The reduction in thrust capacity as the buttress leans is approximately 10 kN per degree of leaning.

For the buttress leaning by 0.4 degrees, the overturning force is reduced to 108 kN

Fractured Buttress at Goa

Weight of the buttress is 905 kN (per metre width)

Vertical reaction of the arch, $V = 64$ kN

Overturning of solid buttress, $H_s = 112$ kN

The maximum horizontal thrust for the vertical buttress is approximately $H_u = 69$ kN, corresponding to a fracture height of $e = 8.7$ m. (nearly a 40% reduction)

Therefore, in the existing state with a lean of 0.4° , the buttress capacity has been reduced to $H_f = 65$ kN from 69 kN.

To prevent failure of the buttress by sliding, the weight of the buttress above the springing is 61 kN and the weight of the arch provides a vertical force of 64 kN. Assuming a static coefficient of friction of 0.7, sliding will occur for horizontal forces greater than 88 kN. Therefore the buttress will fail by overturning before sliding.

Arch at Goa

Exists in state of *minimum* thrust

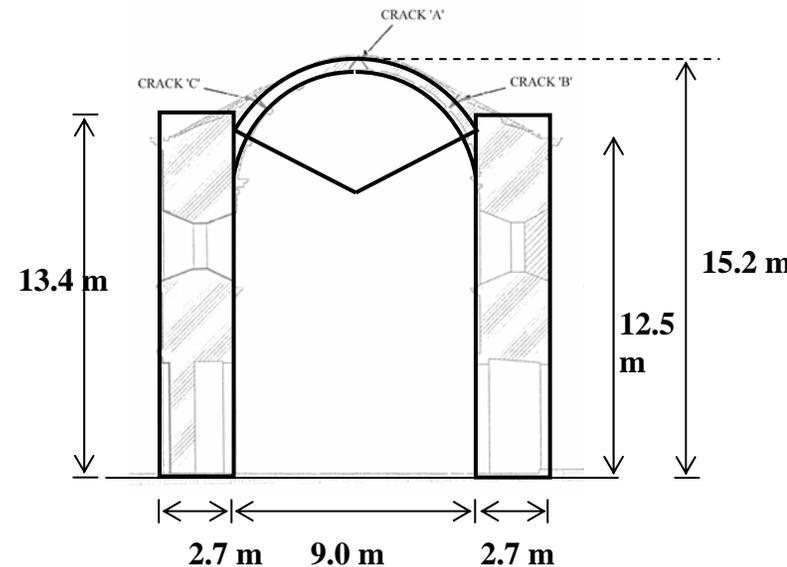
Horizontal force 39 kN and vertical reaction is 64 kN

Span increases by 0.1 m from 9.0 m to 9.1 m

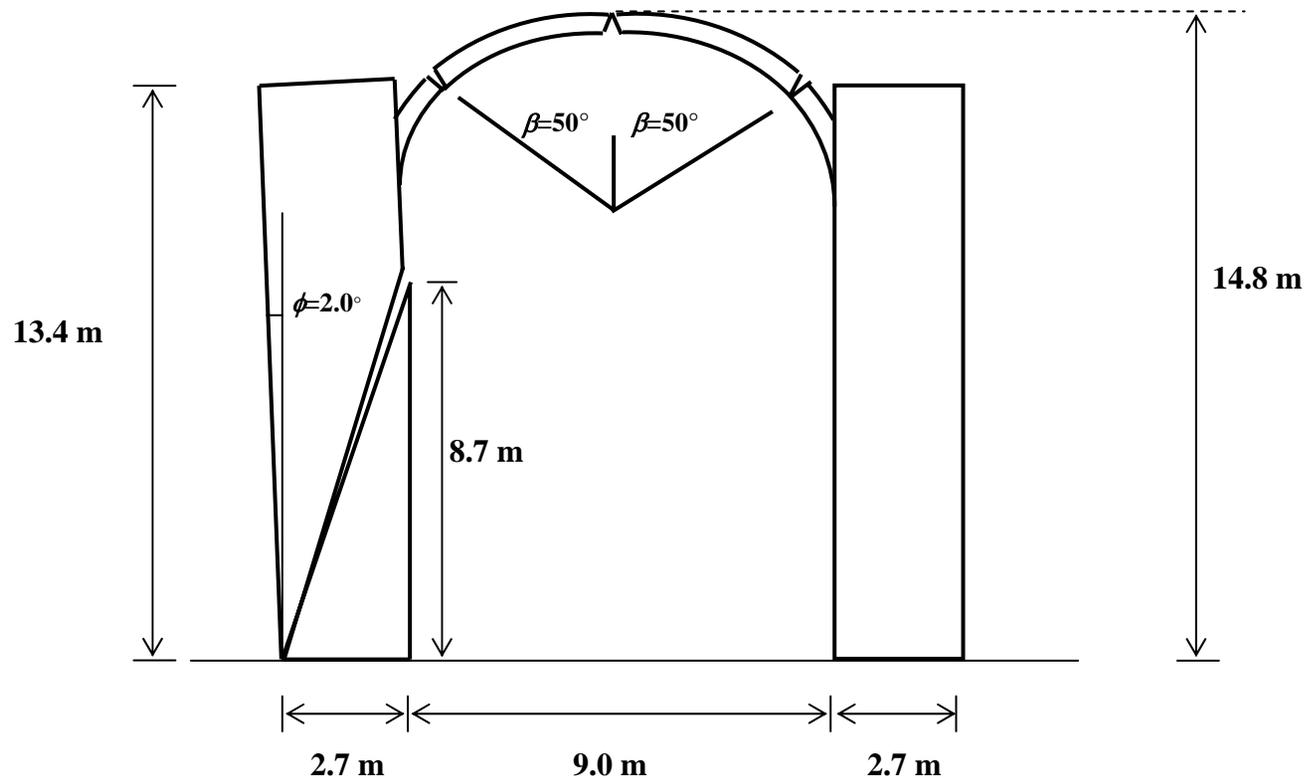
Arch thrust increases to 41 kN

Initial thrust capacity is 69 kN for horizontal thrust, so safety factor is $69 \text{ kN} / 39 \text{ kN} = 1.8$ initially

With leaning, it is reduced to $65 \text{ kN} / 41 \text{ kN} = 1.6$

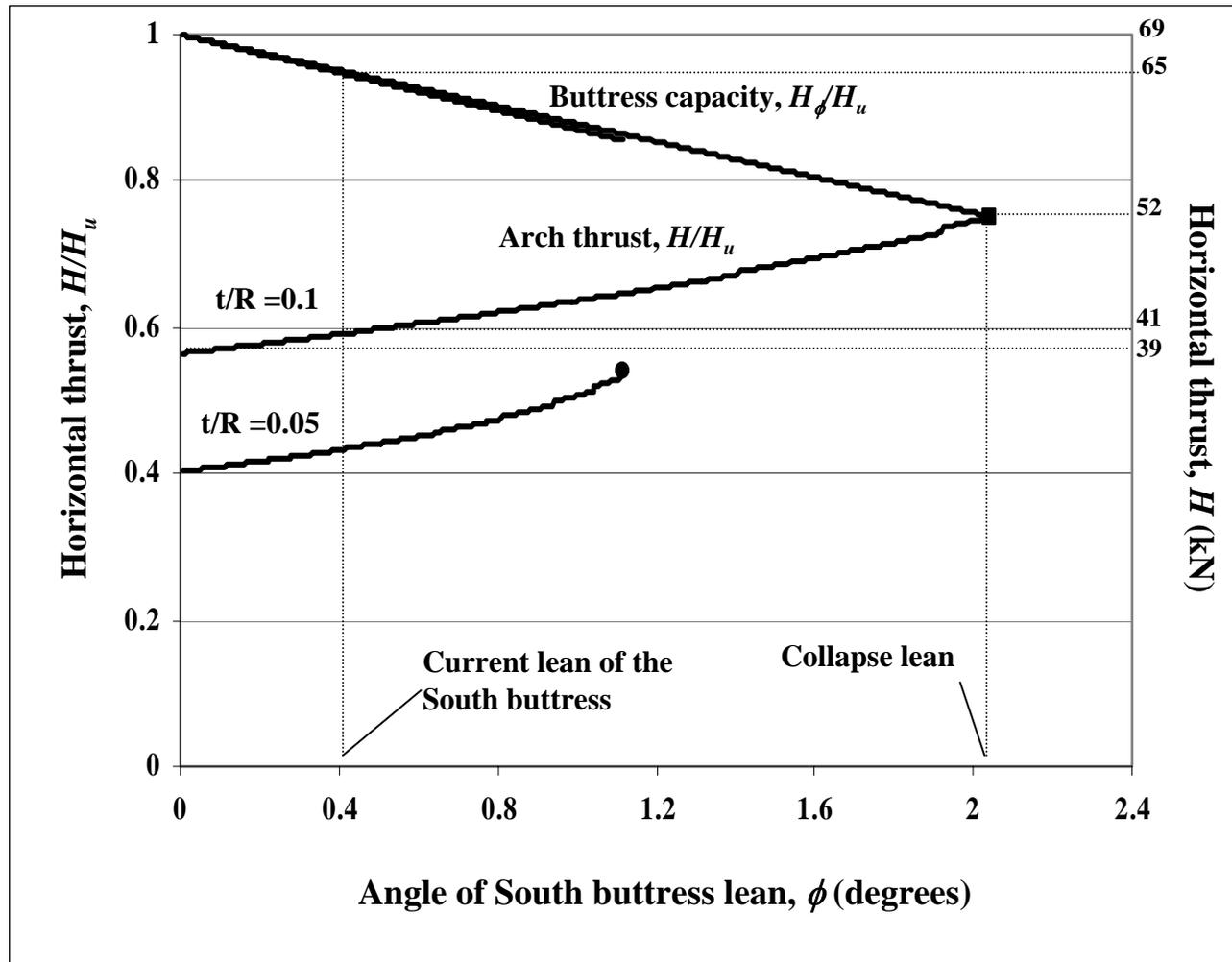


16th C. Church in Goa, India



Collapse state of the church at Goa due to leaning of the South buttress. When the buttress has leaned outward by 2° , the thrust from the distorted vault will exceed the thrust capacity of the buttress and the vault will collapse. At this point, the crown of the vault has descended by 0.4 m and the thrust of the arch will have increased from 41 kN to 52 kN.

16th C. Church in Goa, India



Arch at Goa

Exists in state of *minimum* thrust

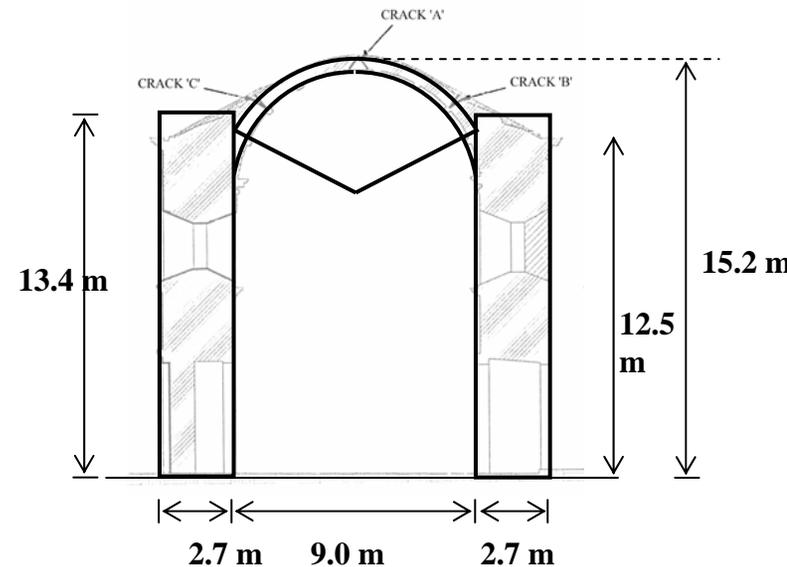
Horizontal force 39 kN and vertical reaction is 64 kN

Span increases by 0.1 m from 9.0 m to 9.1 m

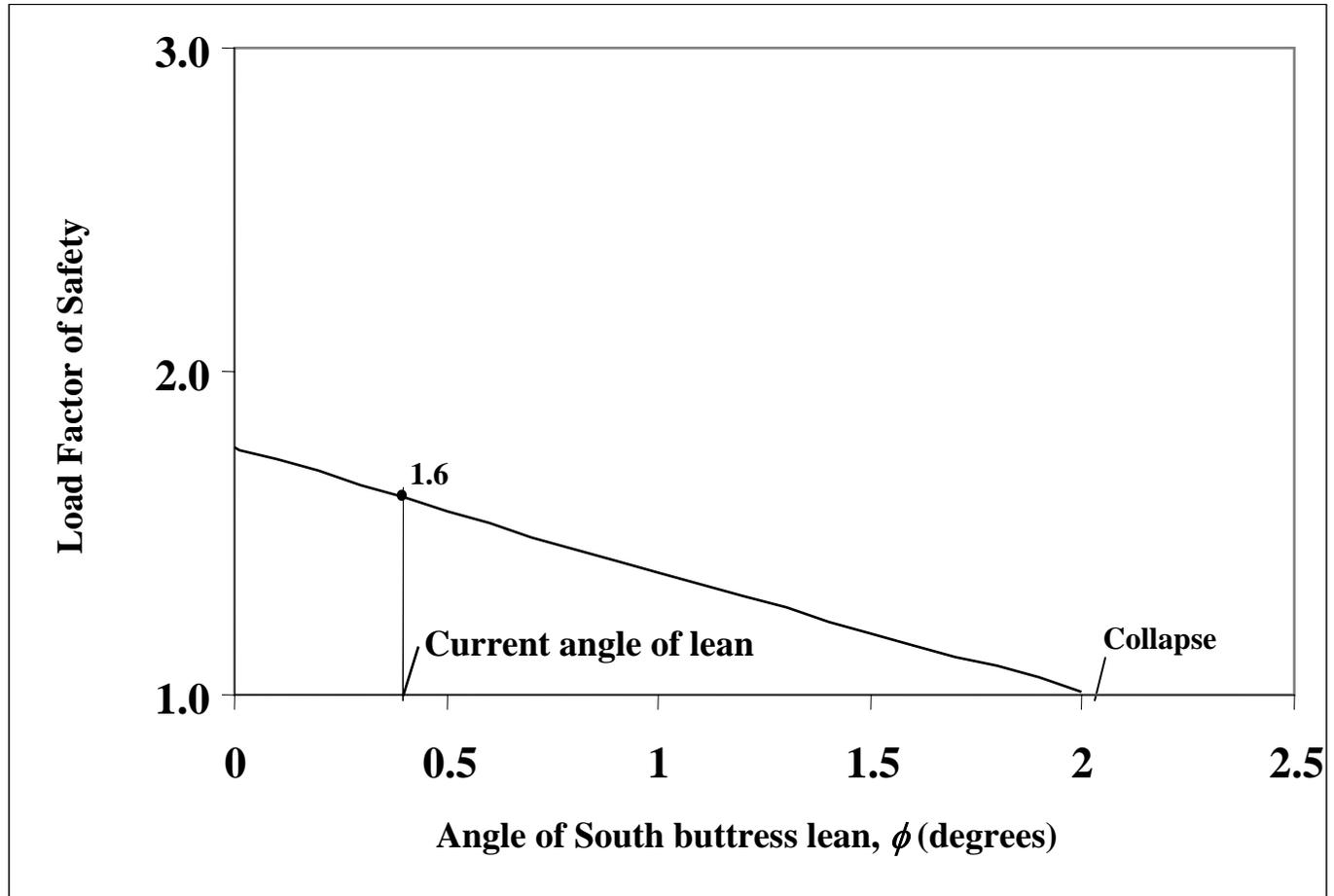
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With leaning, it is reduced to $65 \text{ kN} / 41 \text{ kN} = 1.6$



Safety Factor of Goa Church



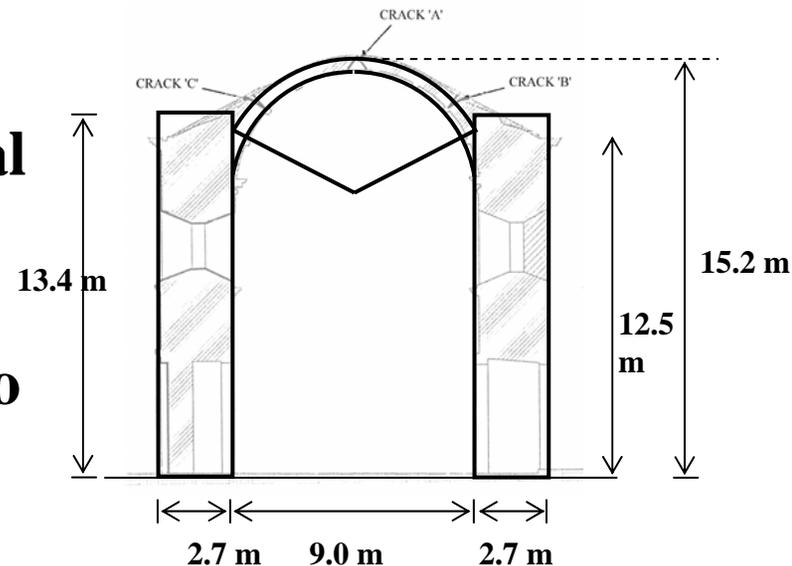
Conclusions from Case Study of Church at Goa, India

Overturning failure of buttress is more critical than a sliding failure

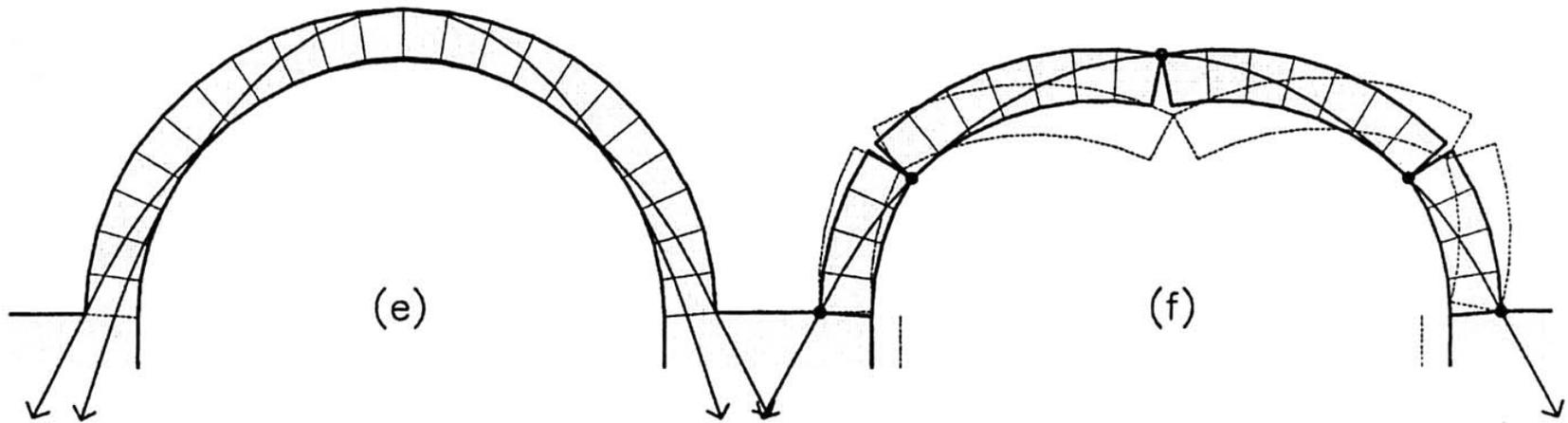
Safety factor against buttress failure is 1.6, compared to original value of 1.8

Leaning of buttress was leading to the collapse

Installed steel tie rod to carry horizontal thrust and to prevent collapse

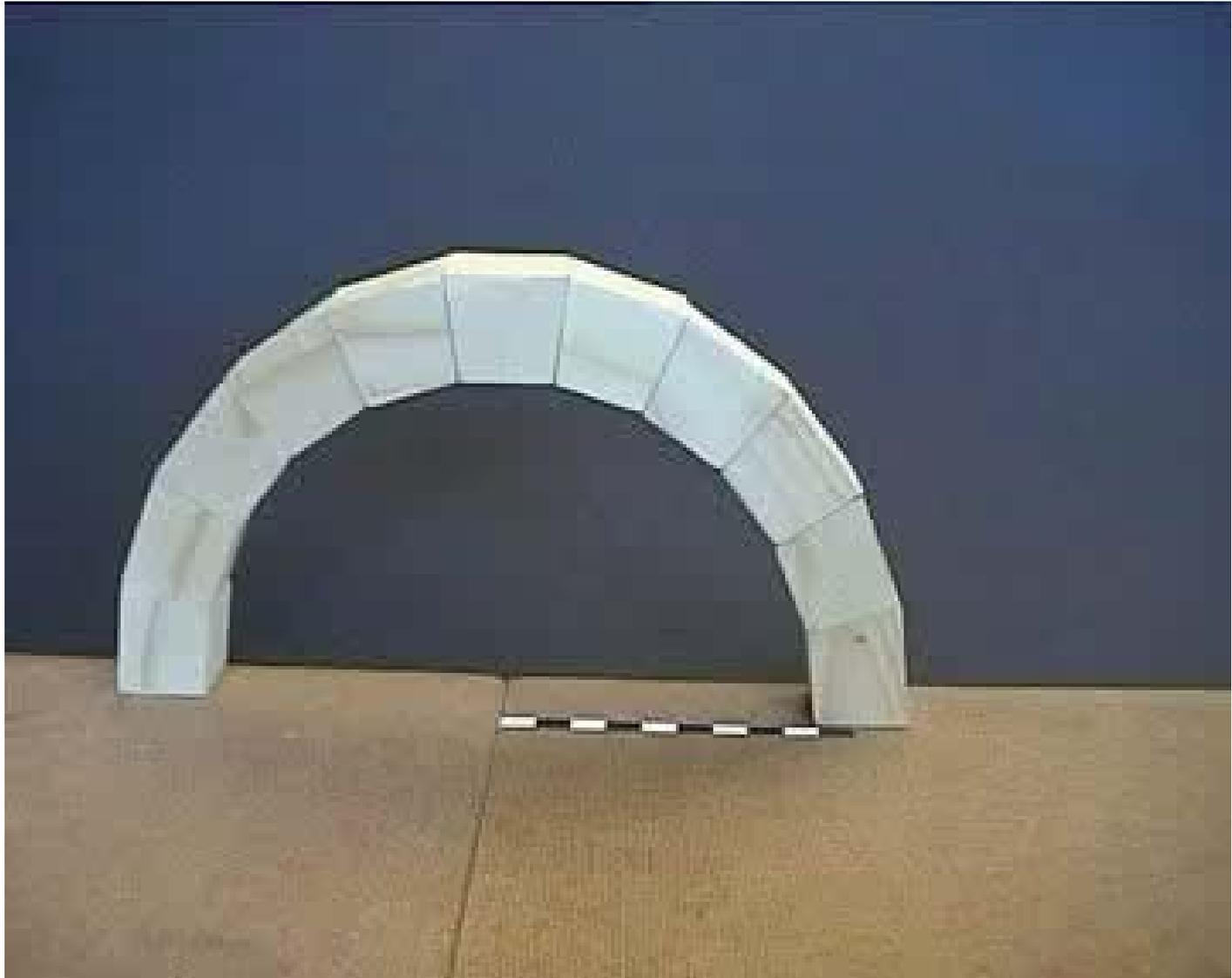


Range of Arch Thrust



Smars (2000)

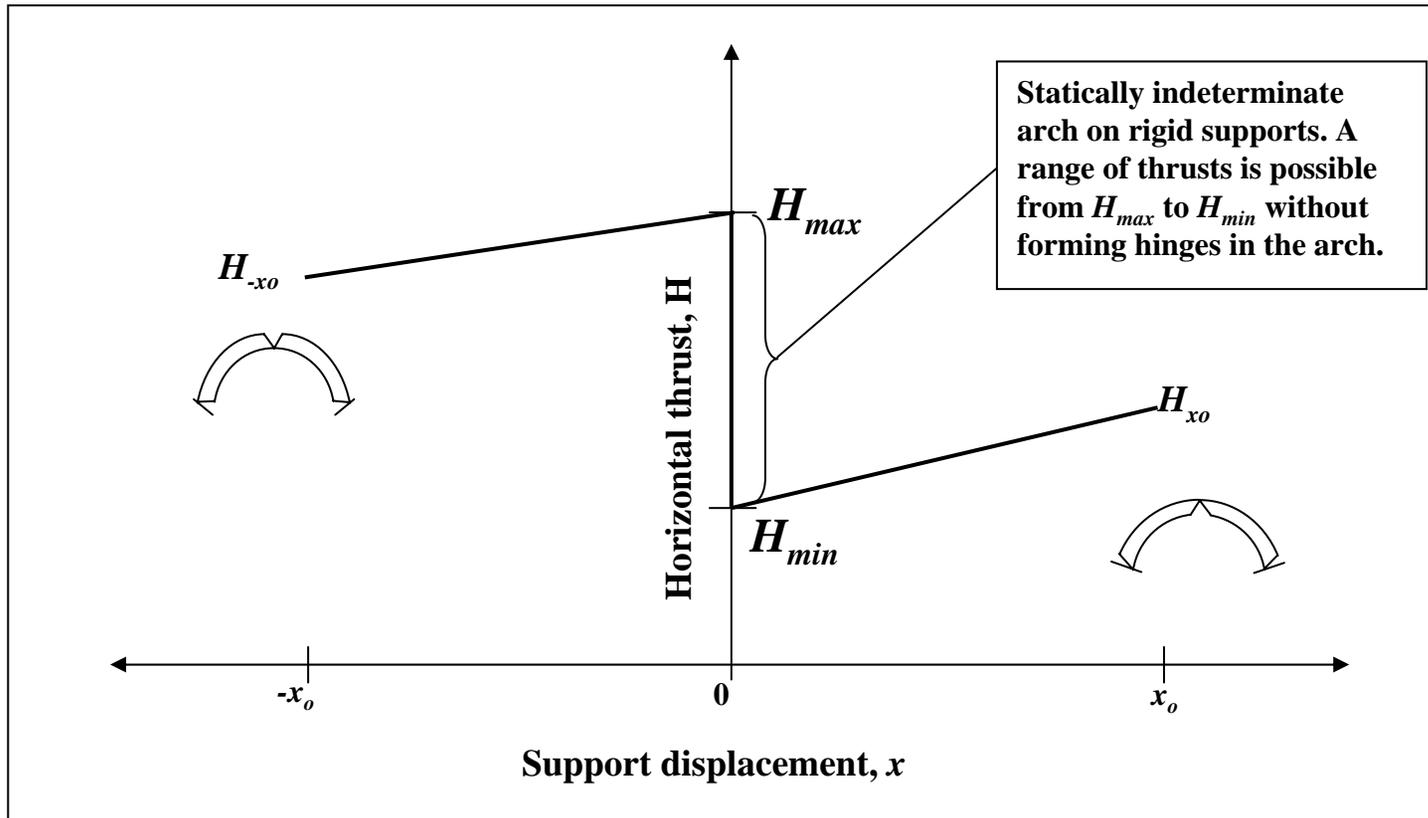
Arch on Spreading Supports



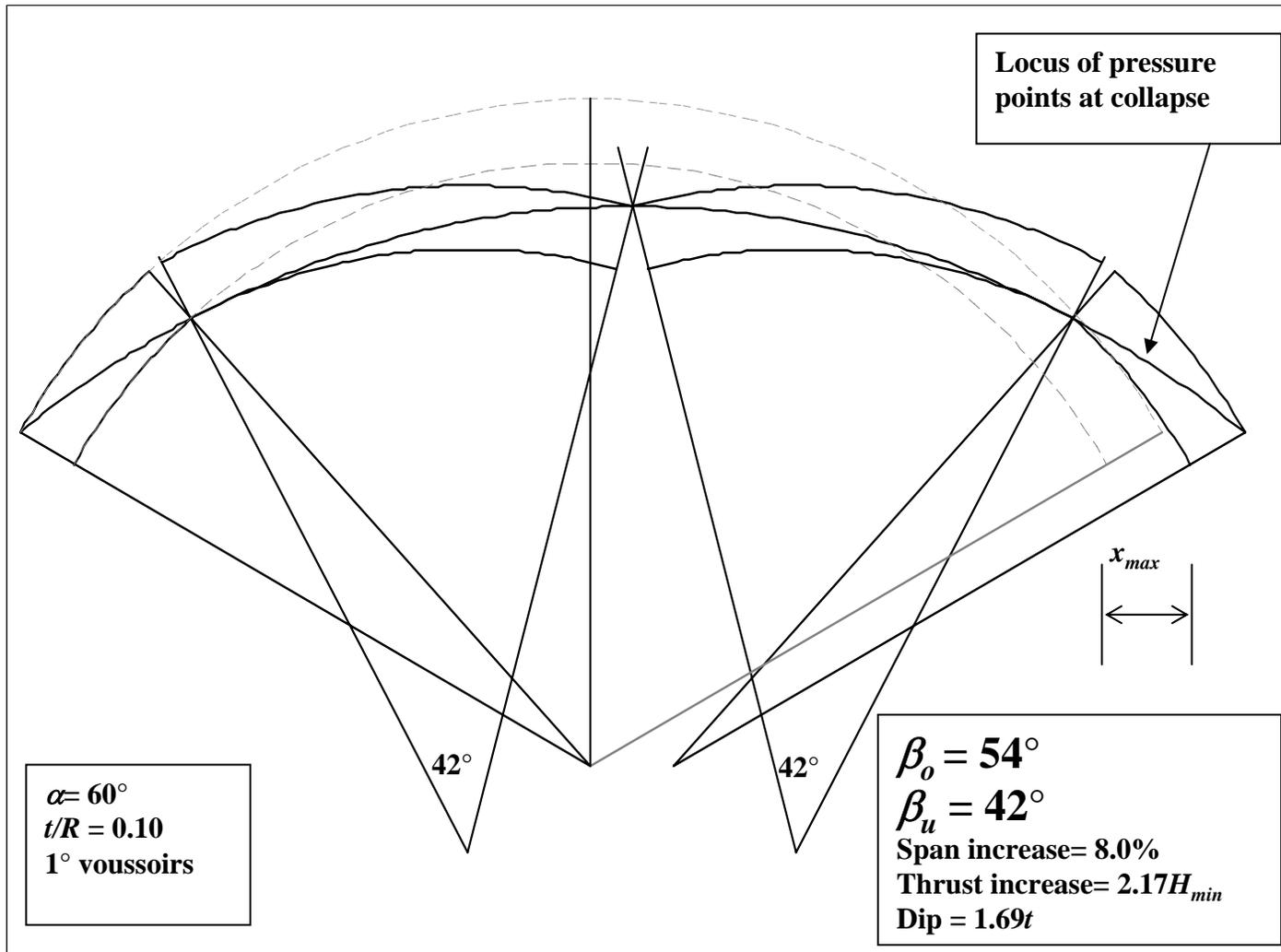
Arch on “Closing” Supports



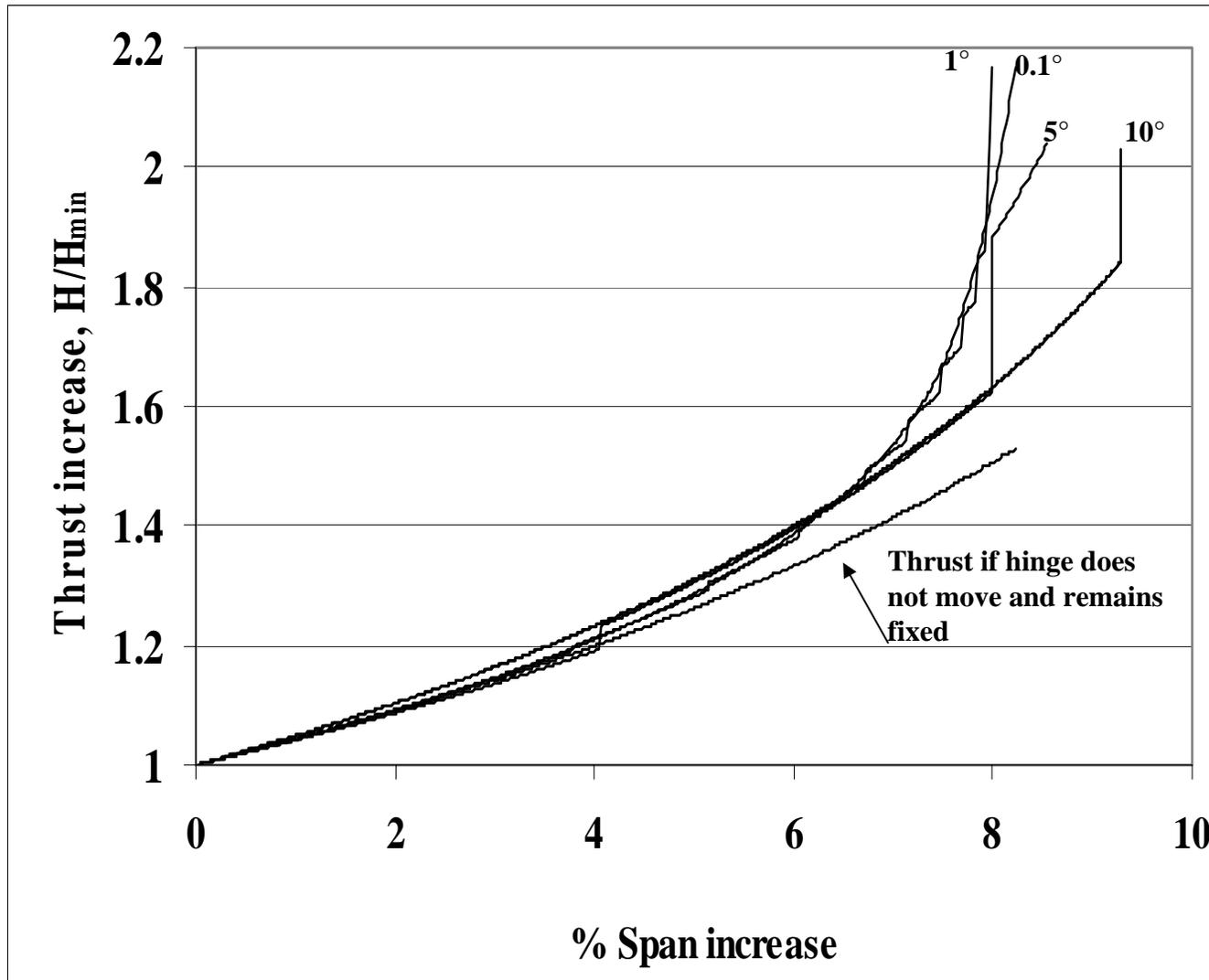
Range of Arch Thrust



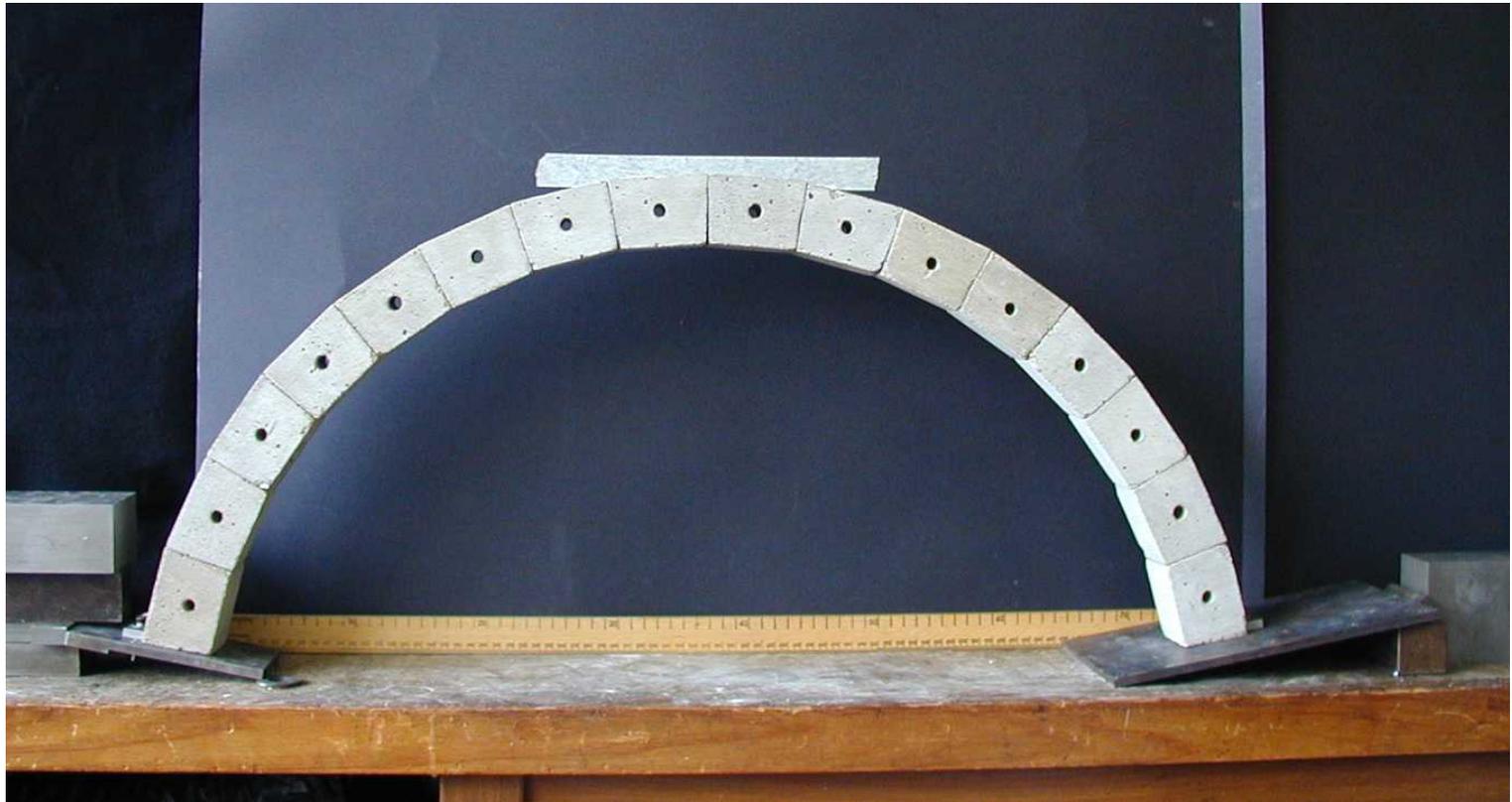
Arch at Collapse State



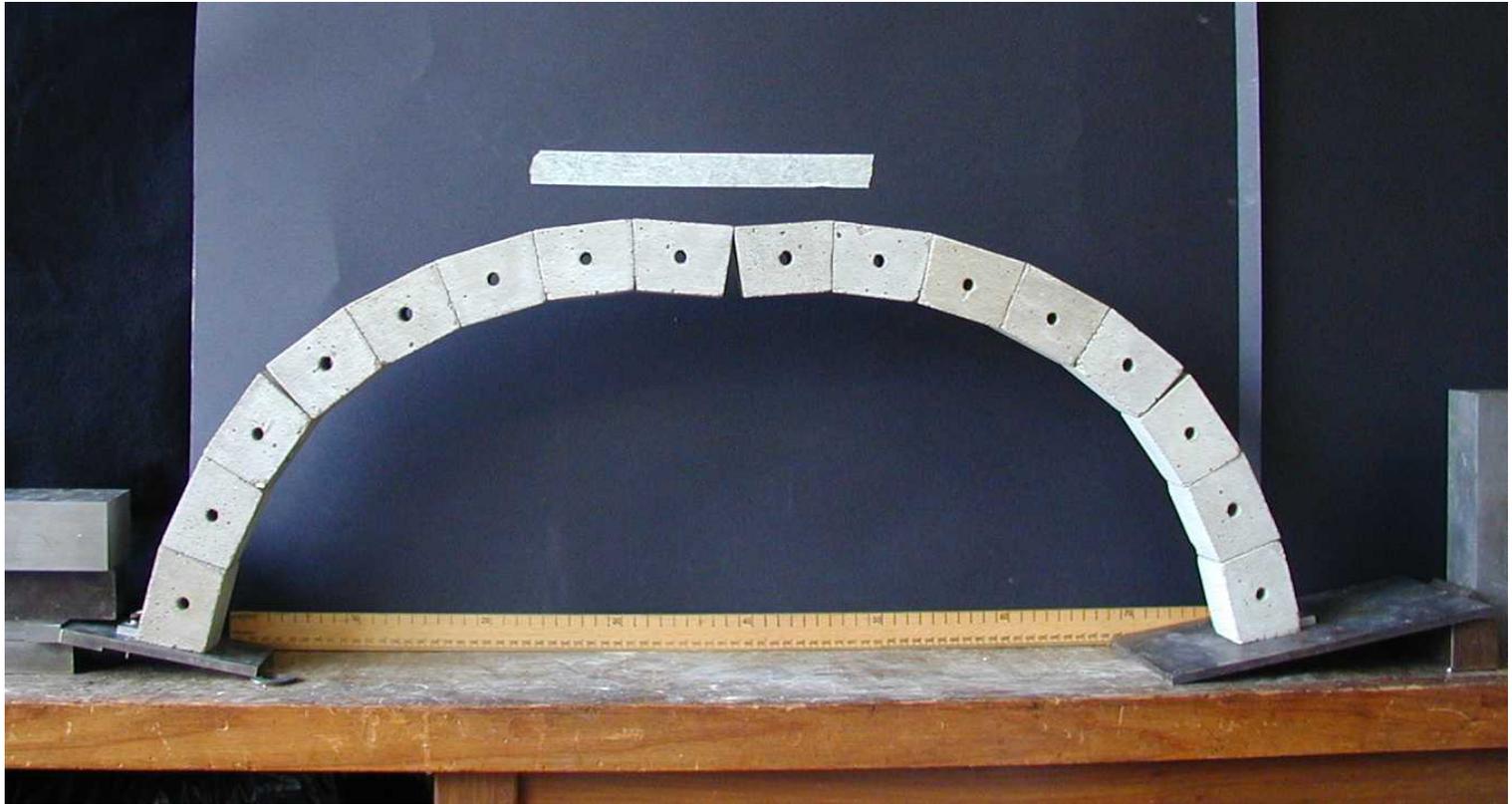
Increase in Arch Thrust



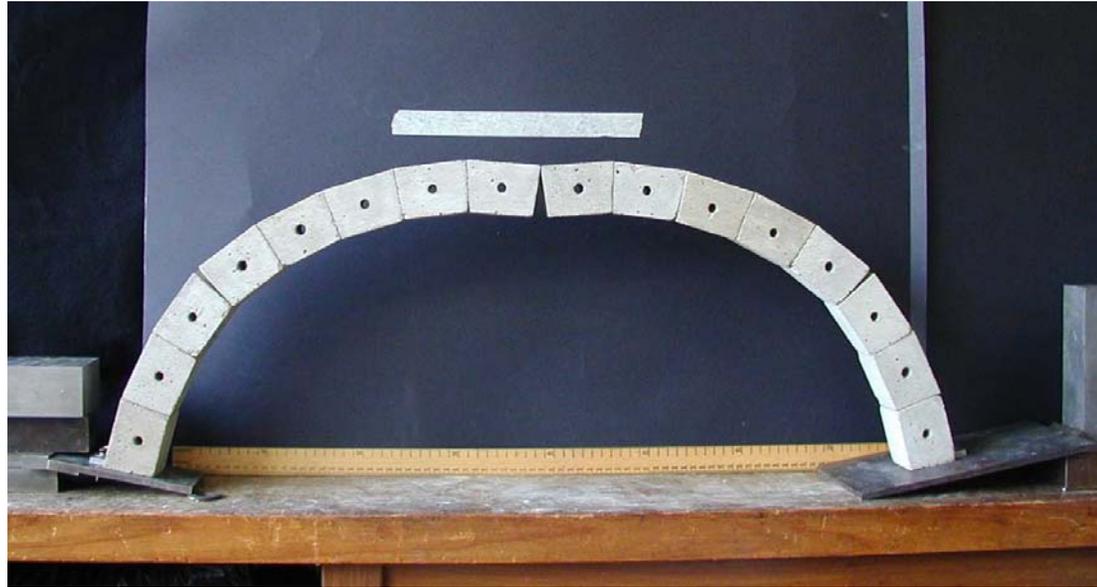
Model Arch Experiment



Model Arch Experiment

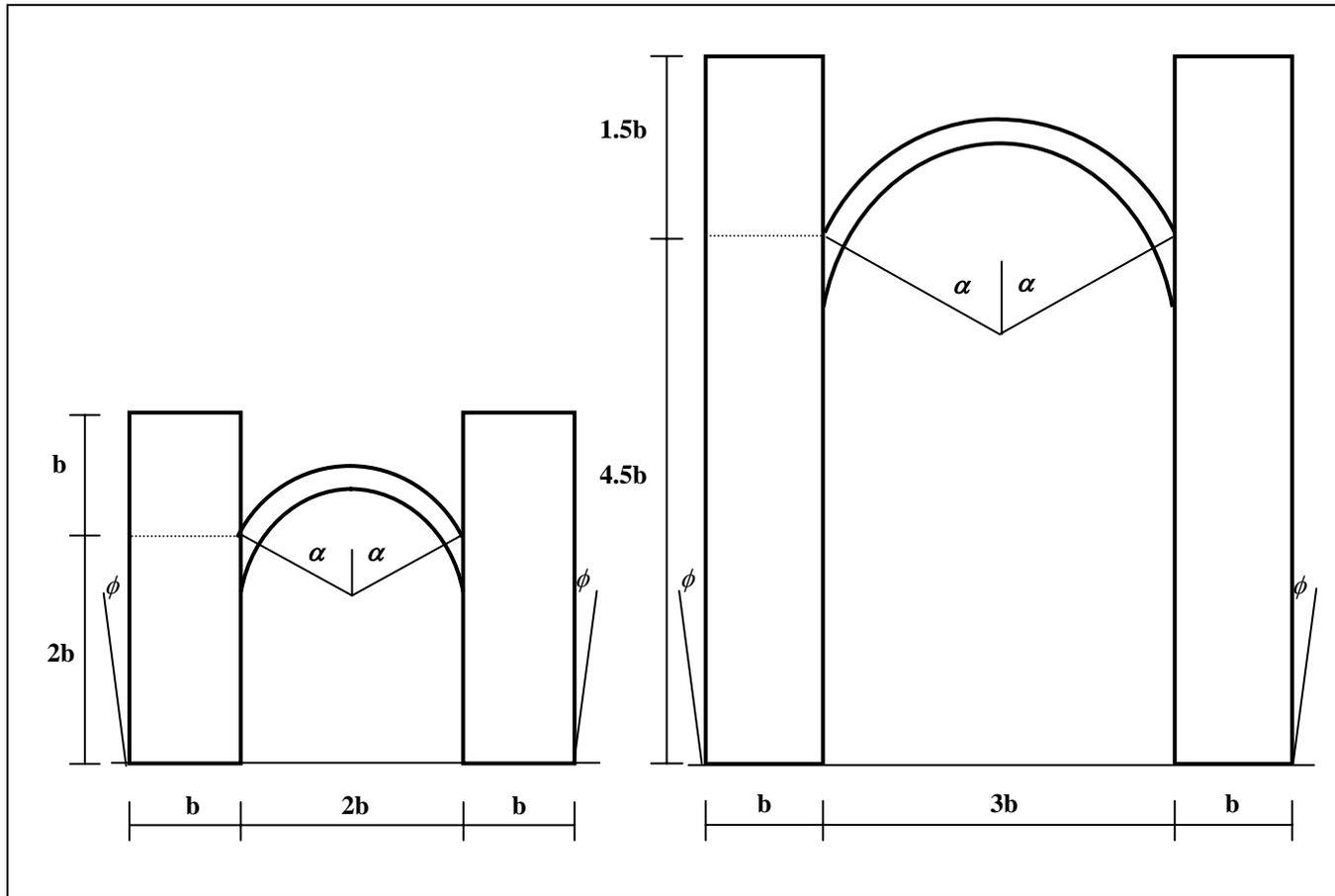


Arch on Spreading Supports



- **For a given span increase, multiple equilibrium states are possible (with different hinges).**
- **The collapse state can only be found by beginning from an assumed equilibrium state.**

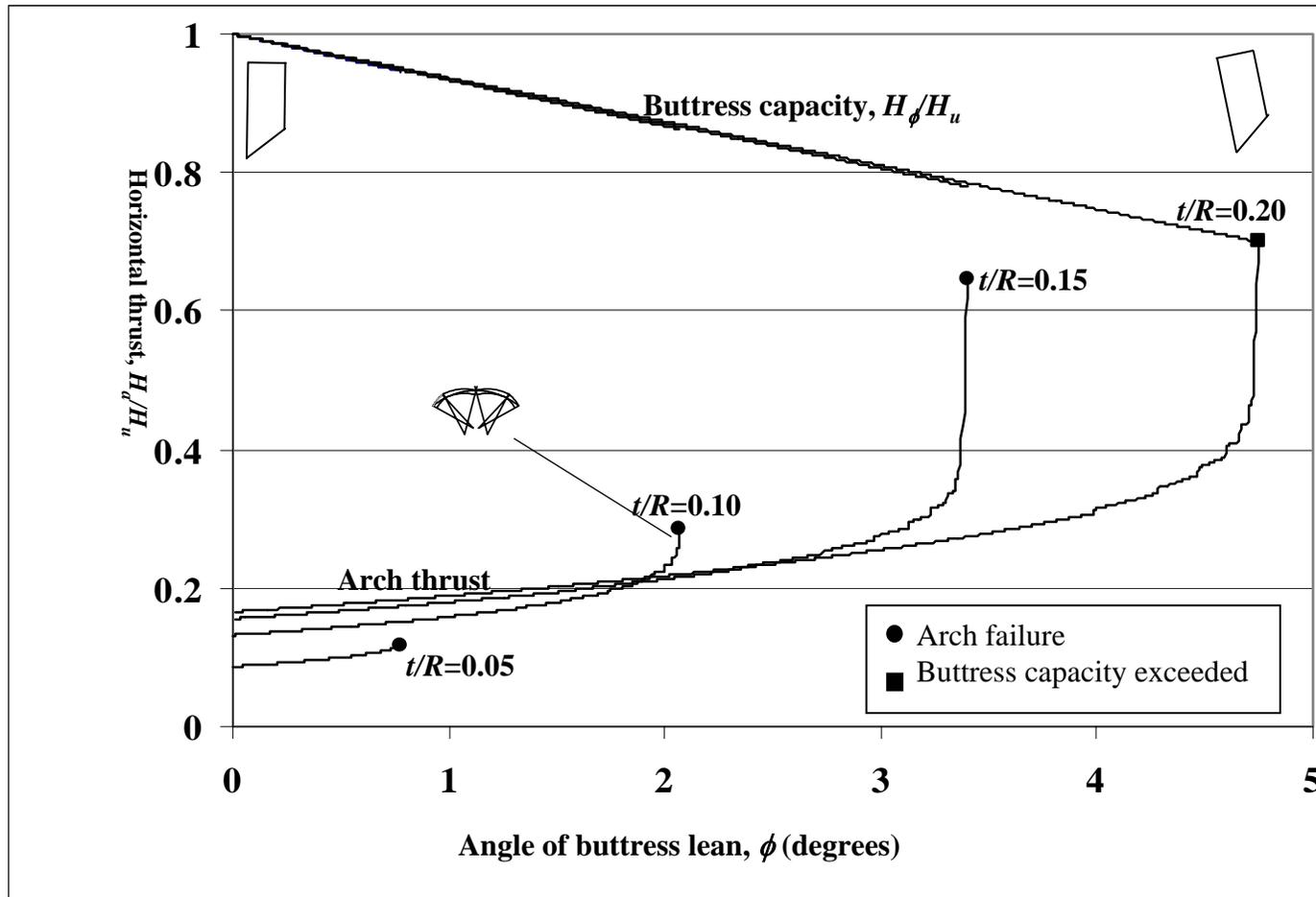
Arch on Buttresses: Case A and B



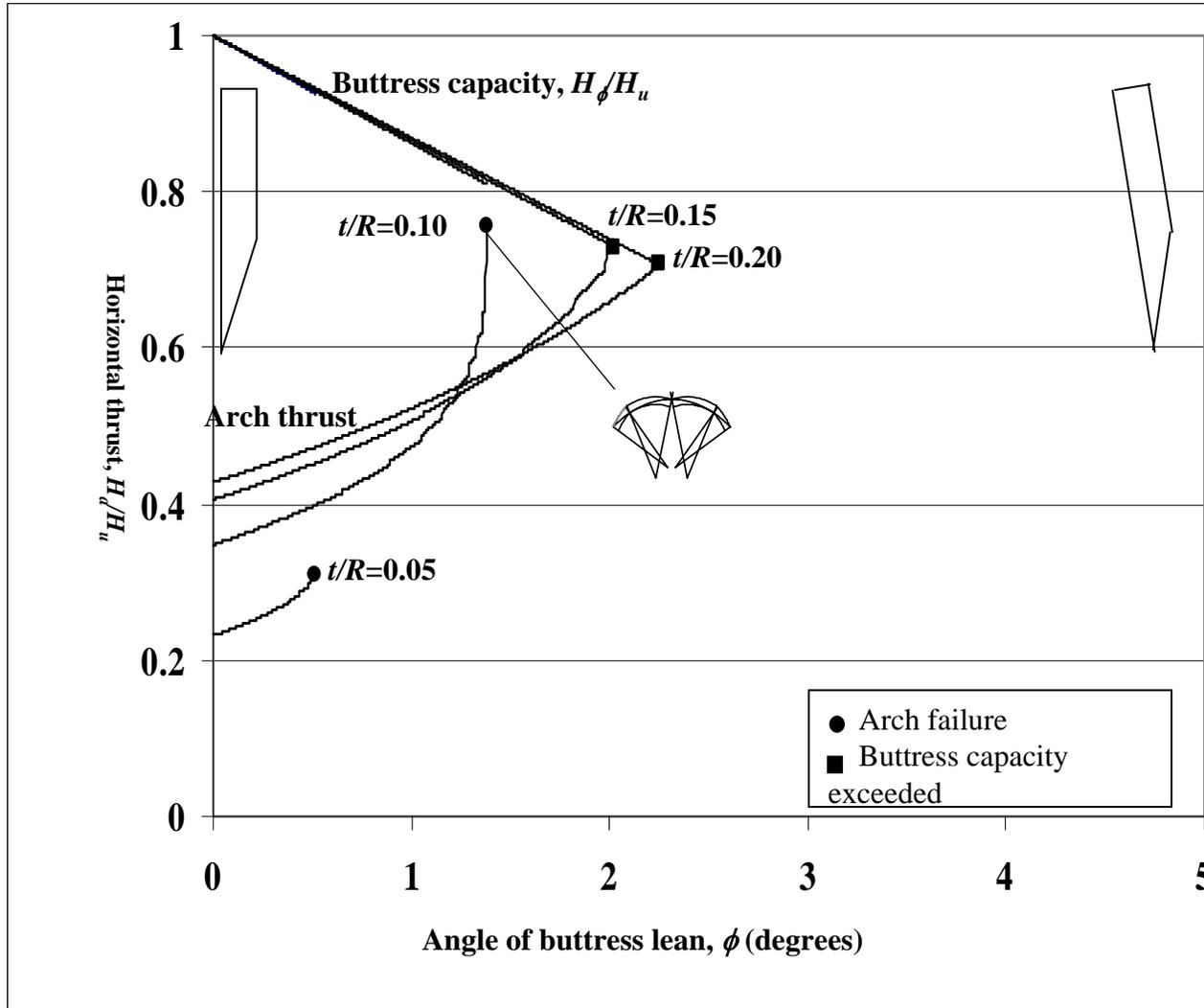
a) Buttressed arch Case A

b) Buttressed arch Case B

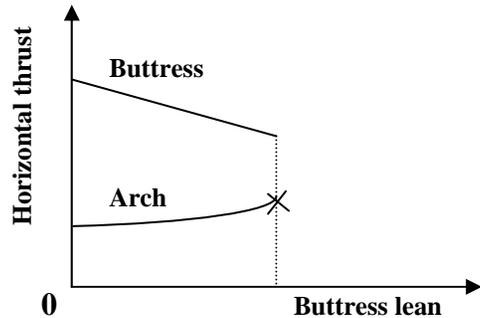
Case A at Collapse



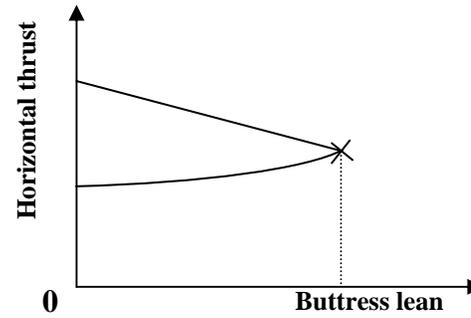
Case B at Collapse



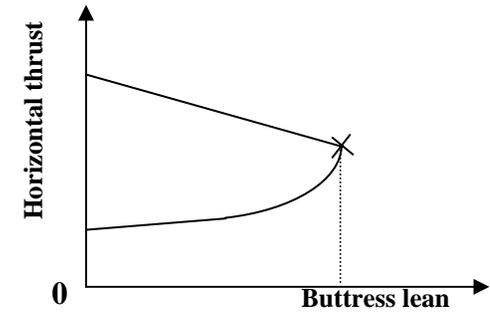
Collapse of Arch on Leaning Buttresses



(a) Strong-buttress failure



(b) Weak-buttress failure

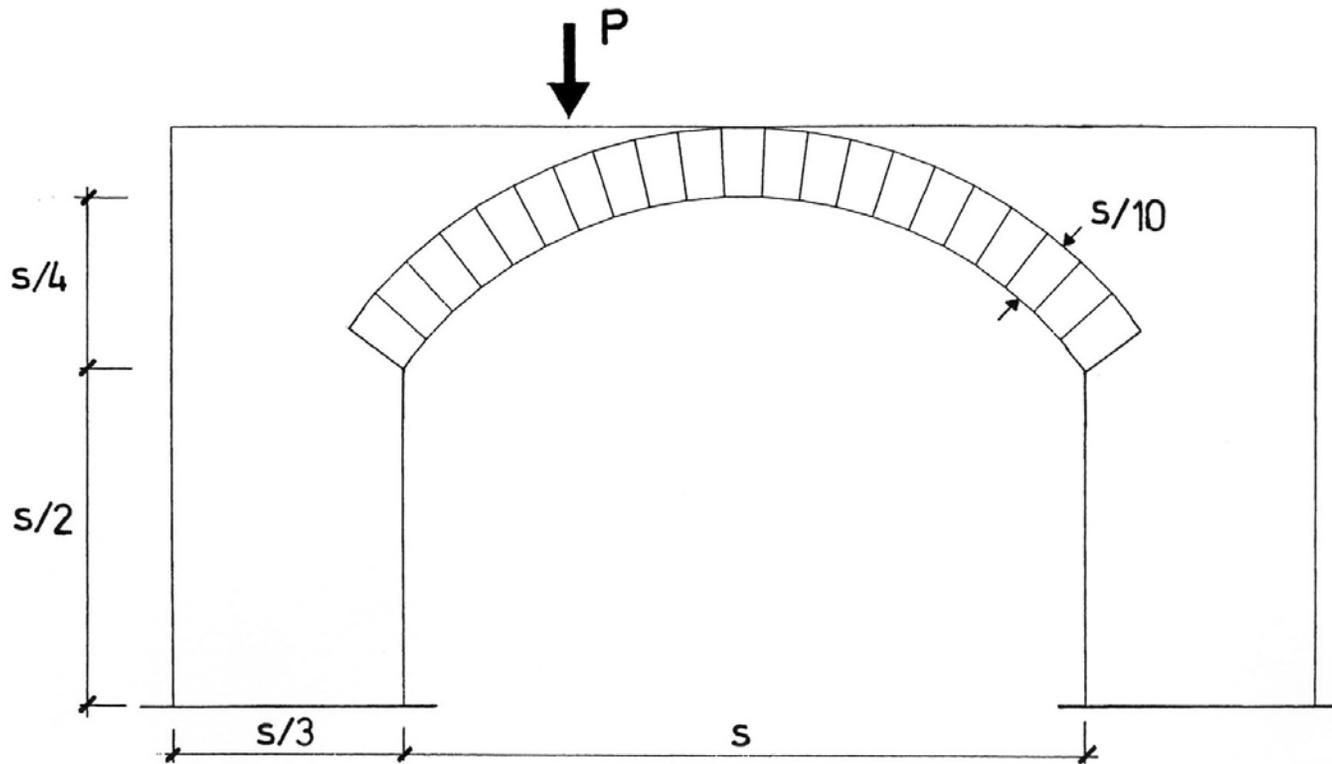


(c) Intermediate failure

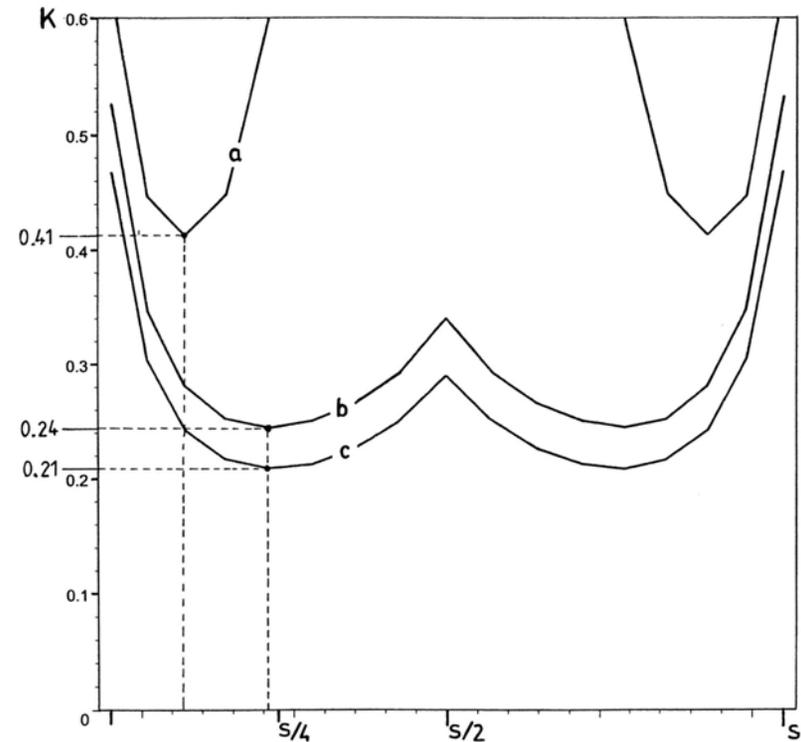
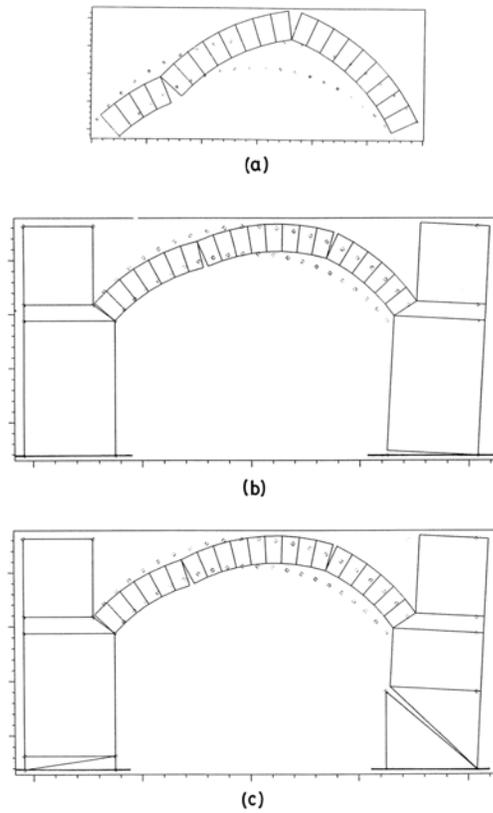
Three types of failure for an arch supported on leaning buttresses. The horizontal axis presents the inclination of the buttress and the vertical axis presents the change in horizontal thrust as the buttresses lean outwards. The arch collapse state is marked by "x".

In all cases, the arch collapses and the buttresses remain standing.

Bridge Collapse Under Point Load

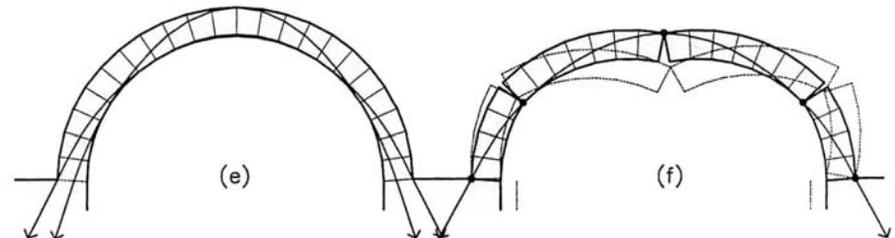


Bridge Collapse: Influence of Buttress Fracture



Problems with Elastic Analysis

- Tells us nothing about collapse state
- Stresses are low, so material failure is not a concern
- Actual displacements are an order of magnitude greater than elastic displacements
- Problem of *stability* not *strength*



Discussion

- **Masonry analysis**
- **HW #1**
- **Final projects**

Future Challenges

- **Assessment of structures is growing in importance and guidelines are required**
- **New analysis tools are necessary (elastic FEM are not appropriate)**
- **Static problems are not solved, and dynamic problems present a greater challenge**

Specific Problems

- **What is the “true” thrust of an arch?**
- **What is the load capacity of a buttress?**
- **How to assess the safety of an ancient structure?**
- **What is the influence of displacements on safety?**