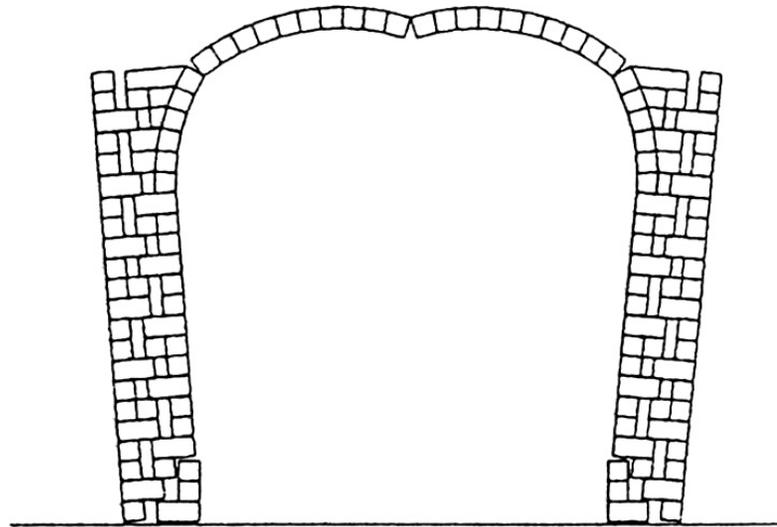


# Lecture 3

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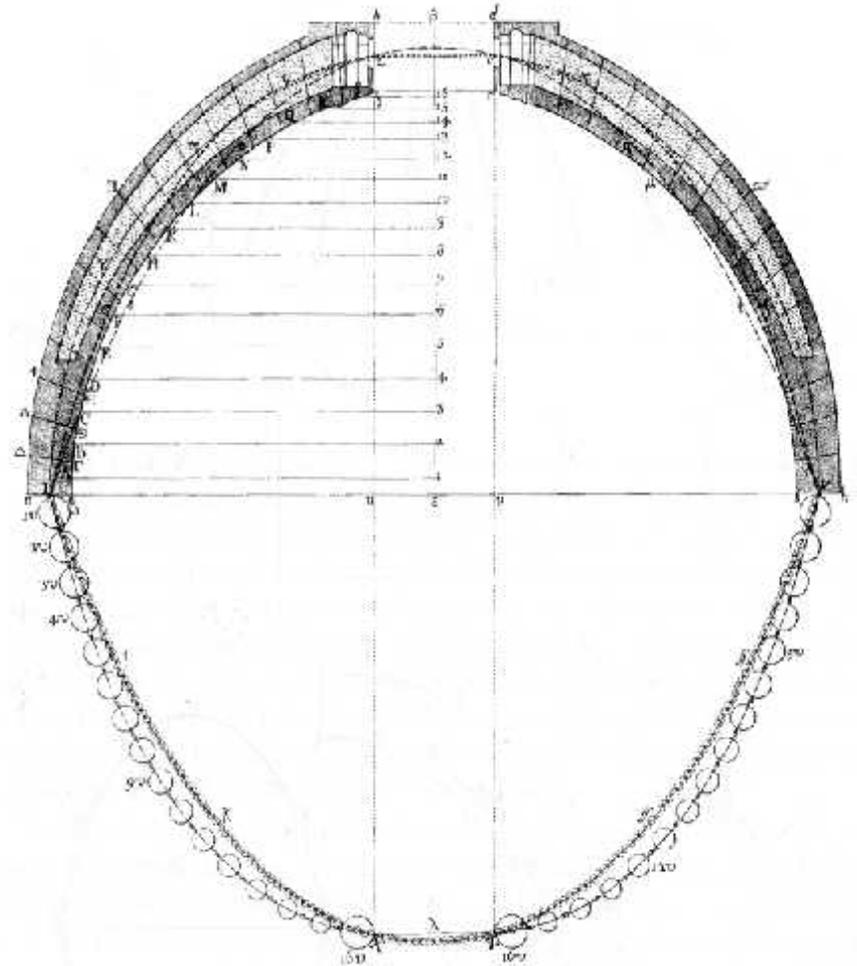
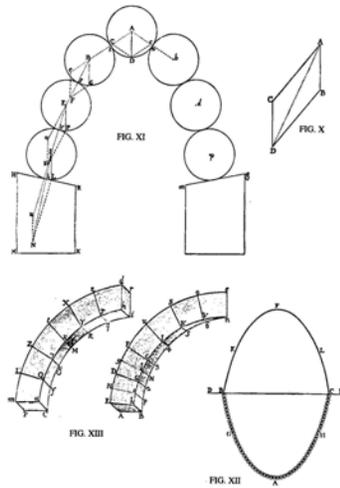
**Analysis of Masonry Structures:  
Arches, Vaults, and Buttresses**

# Limit Analysis of Masonry

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- **Lower Bound Theorem**
  - Seeks permissible line of compressive force for the given loading
- **Upper Bound Theorem**
  - Seeks critical load which results in a failure mechanism

# Poleni (1748) applies lower bound to St. Peter's of Rome



# Selby Abbey, 12<sup>th</sup> C, England

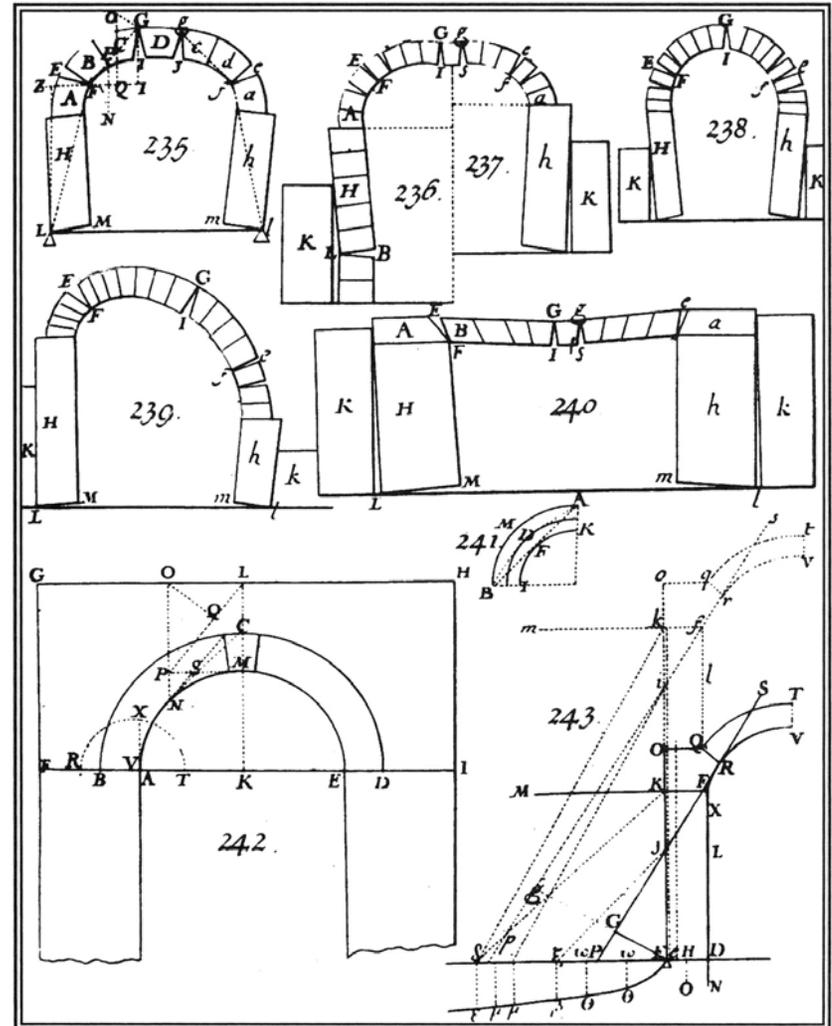
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- **Tower construction “punches” through**
- **Arches deform to accommodate support movements**
- **Stable because a line of thrust can be found within the deformed arch**

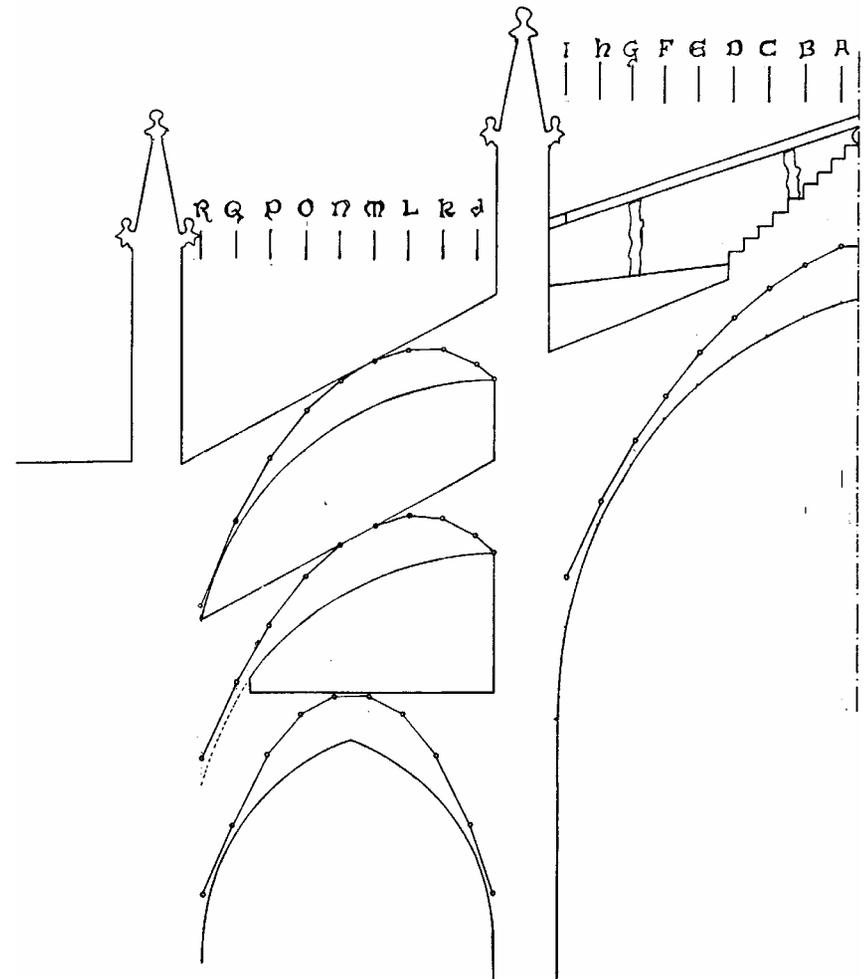
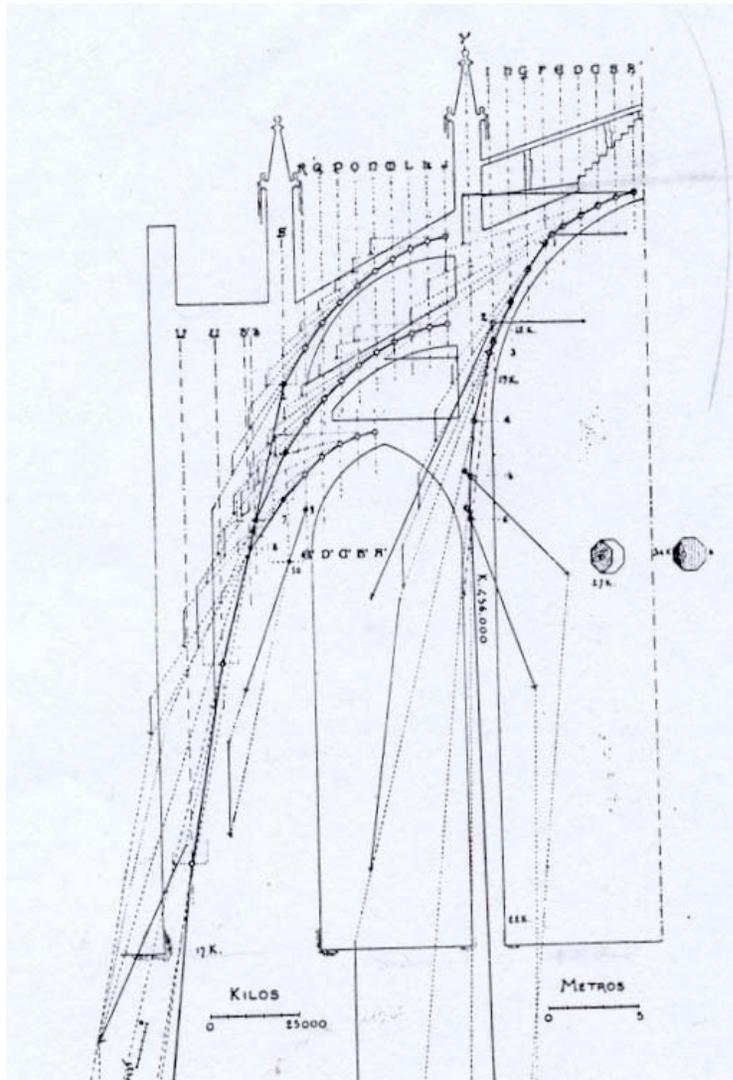
# Experiments by Danyzy (1732)

**Collapse occurs by hinging between blocks, when a load path can not be contained within the masonry**

**Safety is a question of geometry and stability, not crushing of stone**



# Cathedral in Palma de Mallorca, Analysis by Joan Rubio (1912)

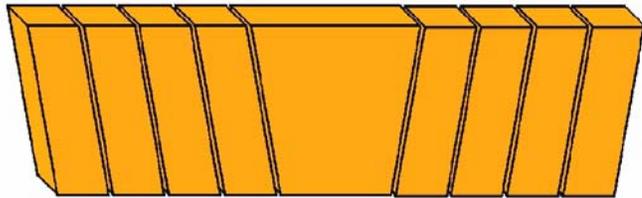


# Analysis of a Flat Arch



Image courtesy of Denis Y. Yu, [structurae.de](http://structurae.de)

# Analysis of a Flat Arch

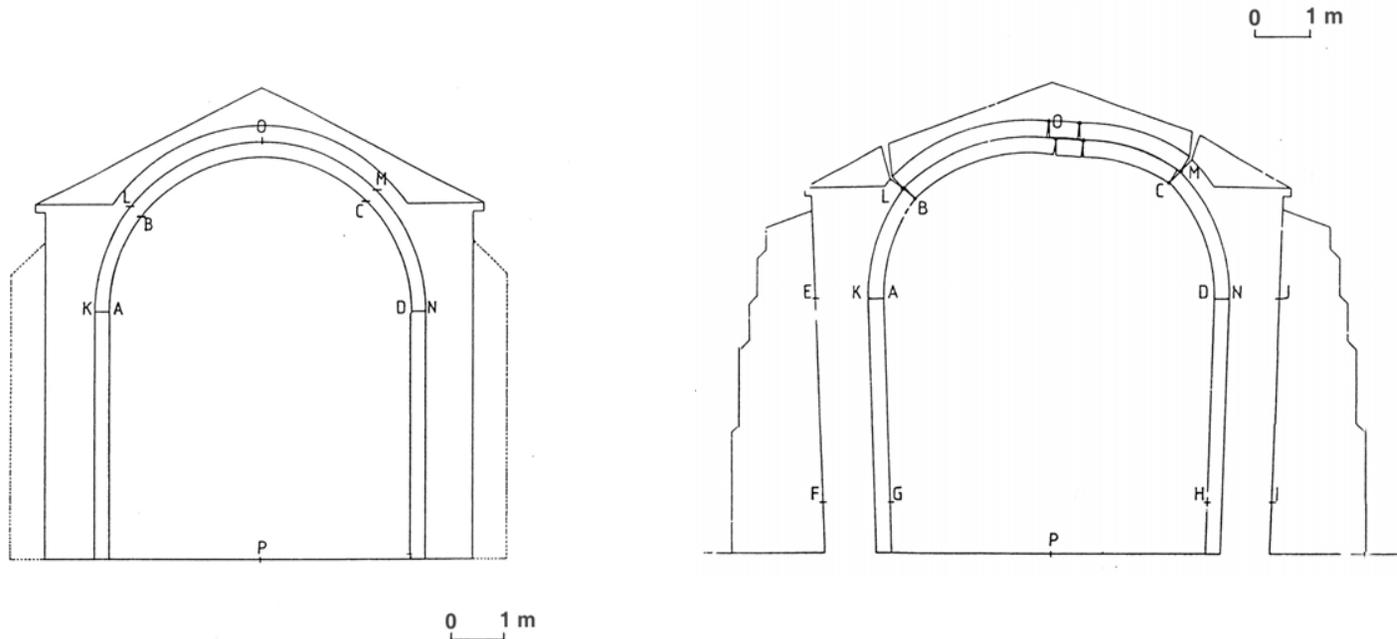


**Thrust of a flat arch?**



Flat Arch

# Safety of an Arch on Buttresses



**Must solve three problems:**

- 1. Load capacity of buttress (and influence of lean)**
2. Collapse state of arch on spreading supports
3. Analysis of arch supported on leaning buttresses

# Buttress Analysis, DuPuit (1870)

Fig. 86.

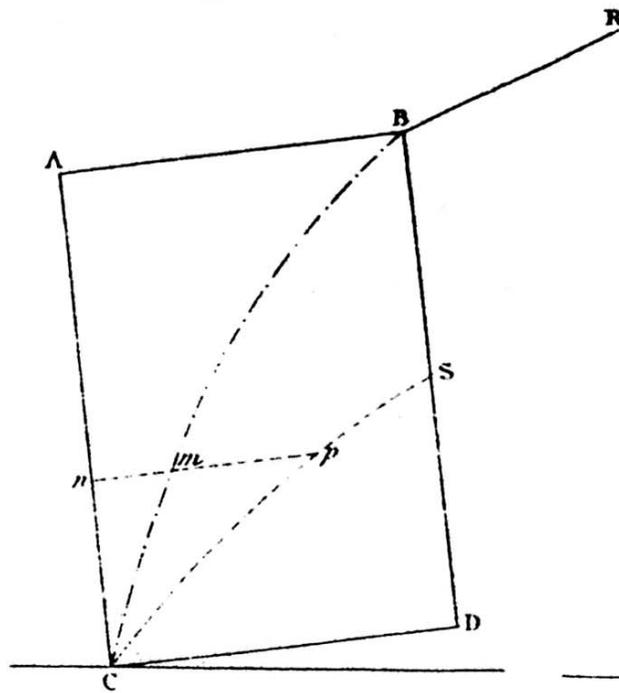
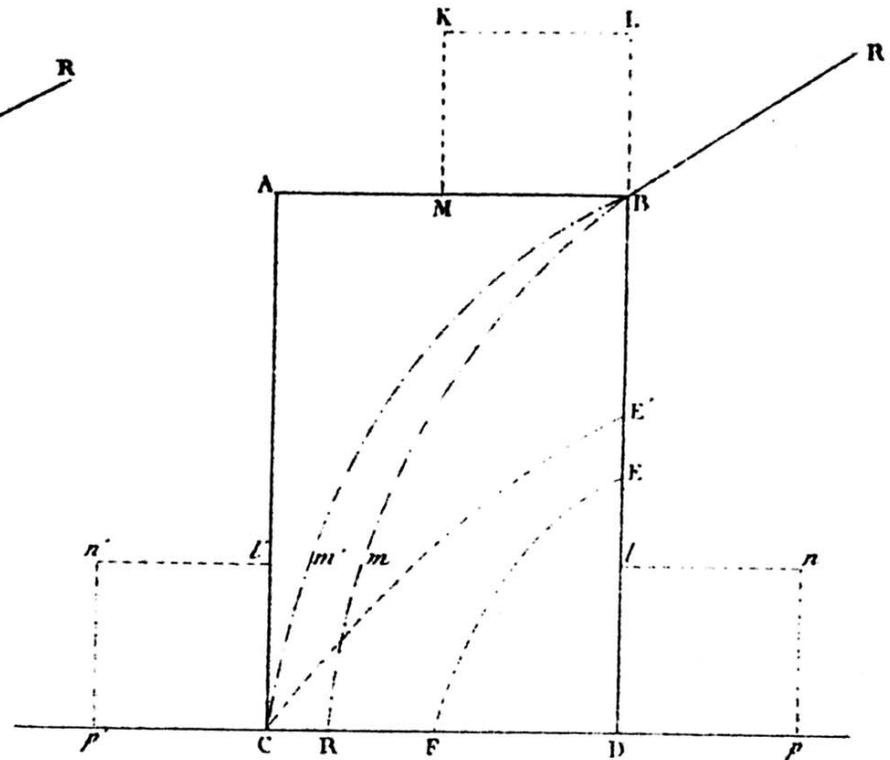
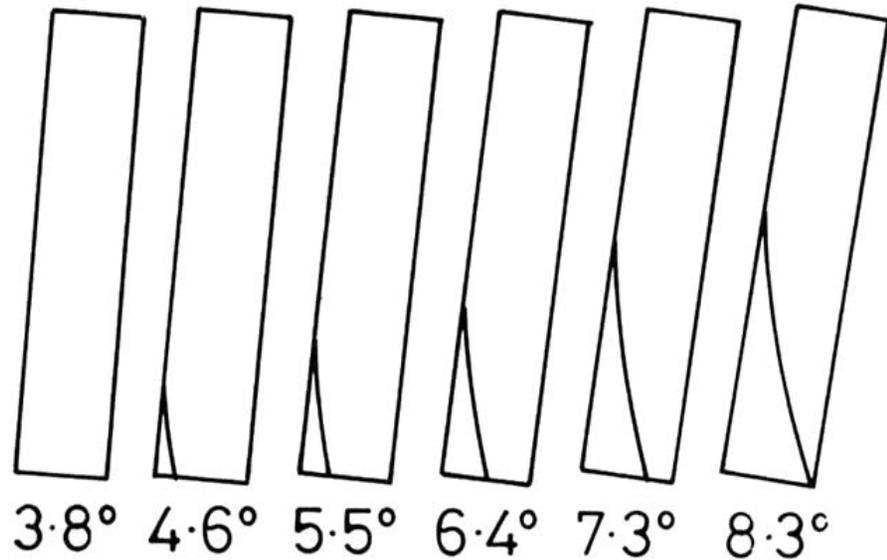
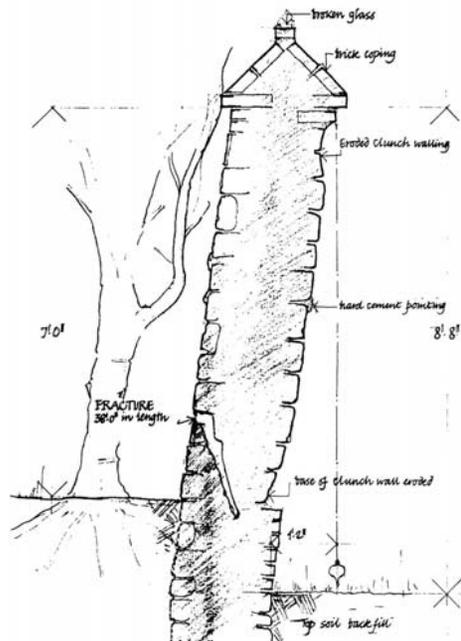


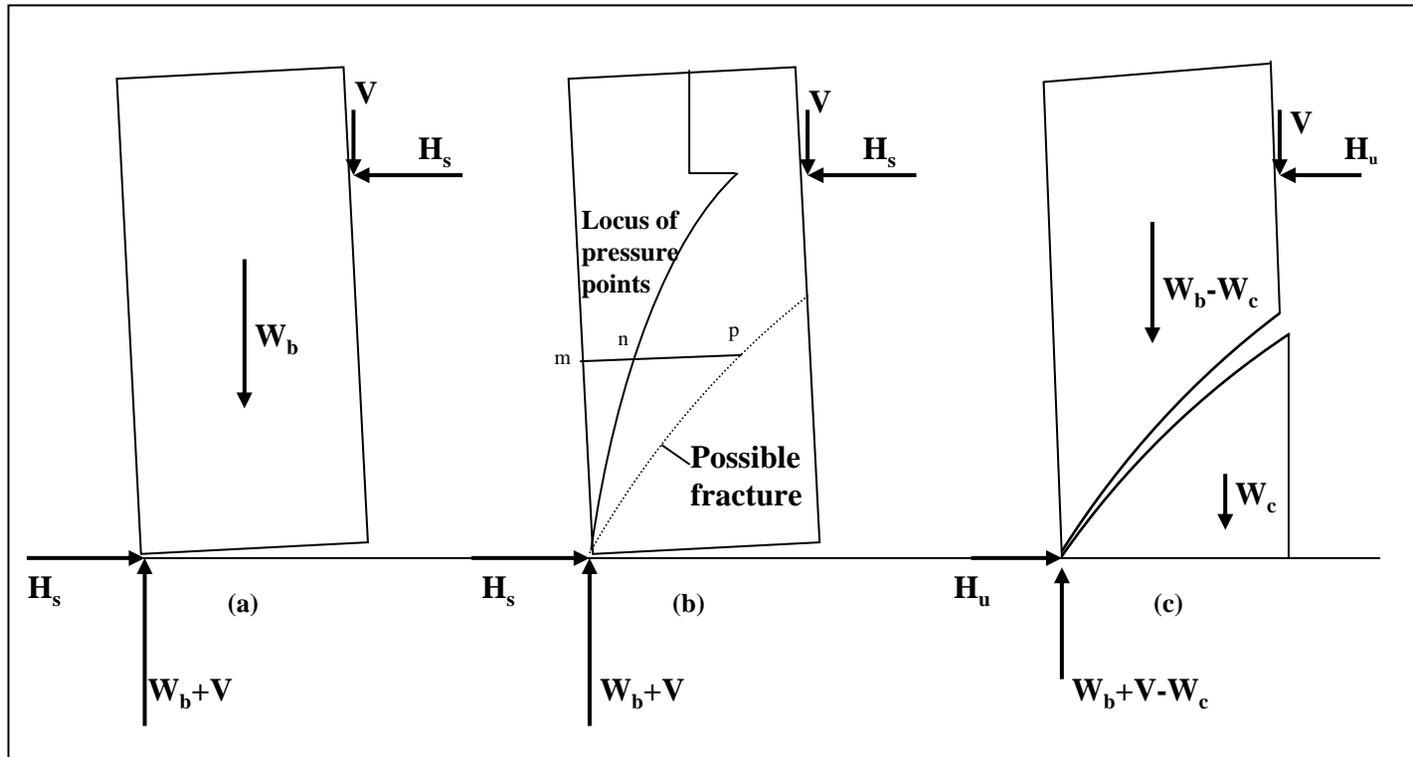
Fig. 87.



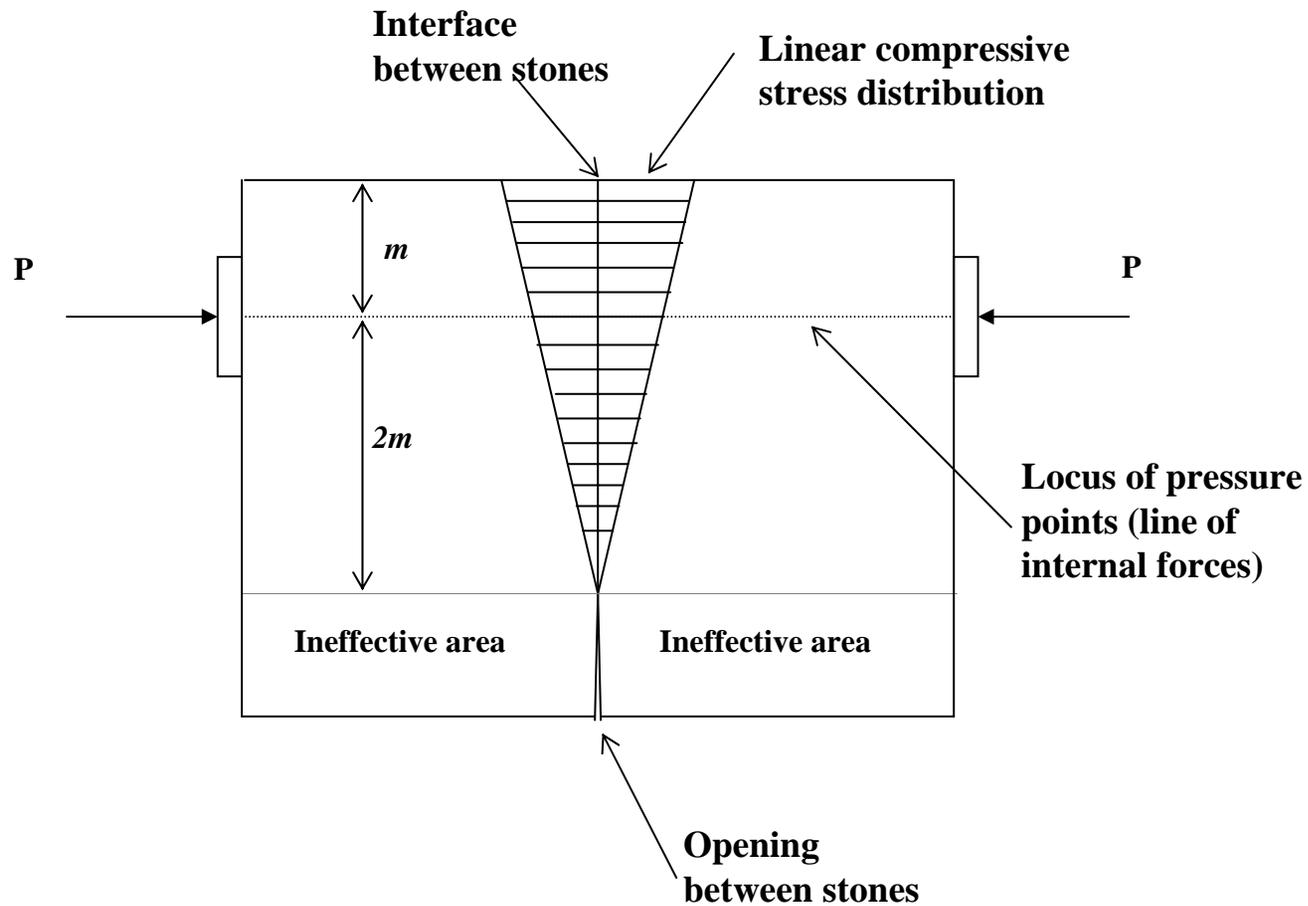
# Heyman on Leaning Walls (1992)



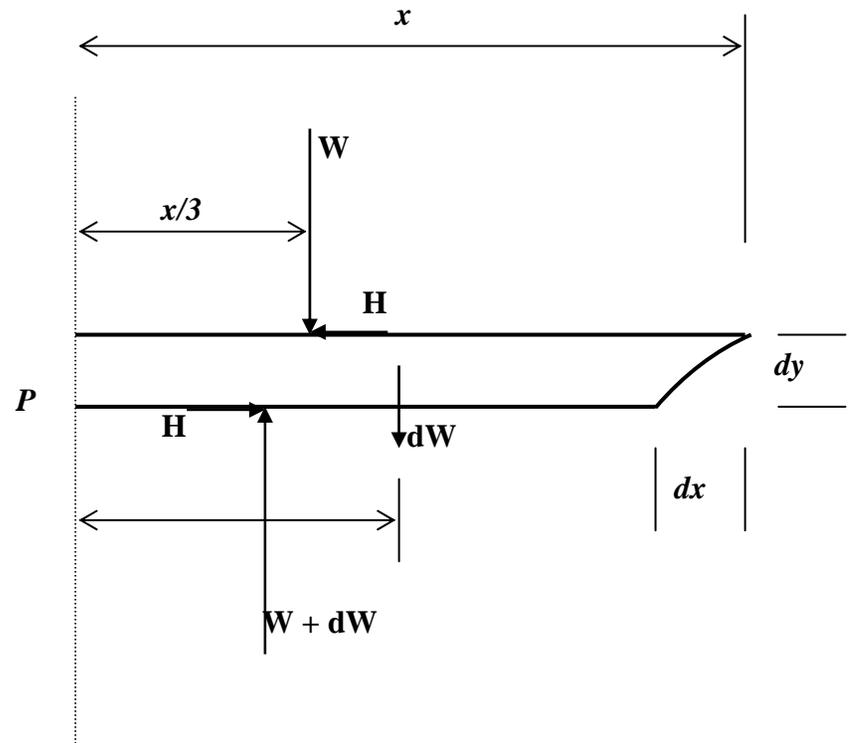
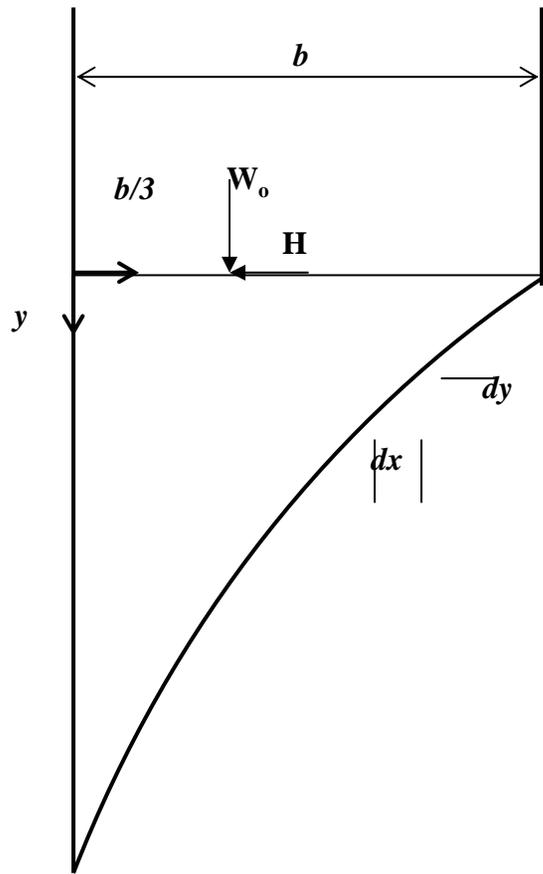
# Buttress Collapse



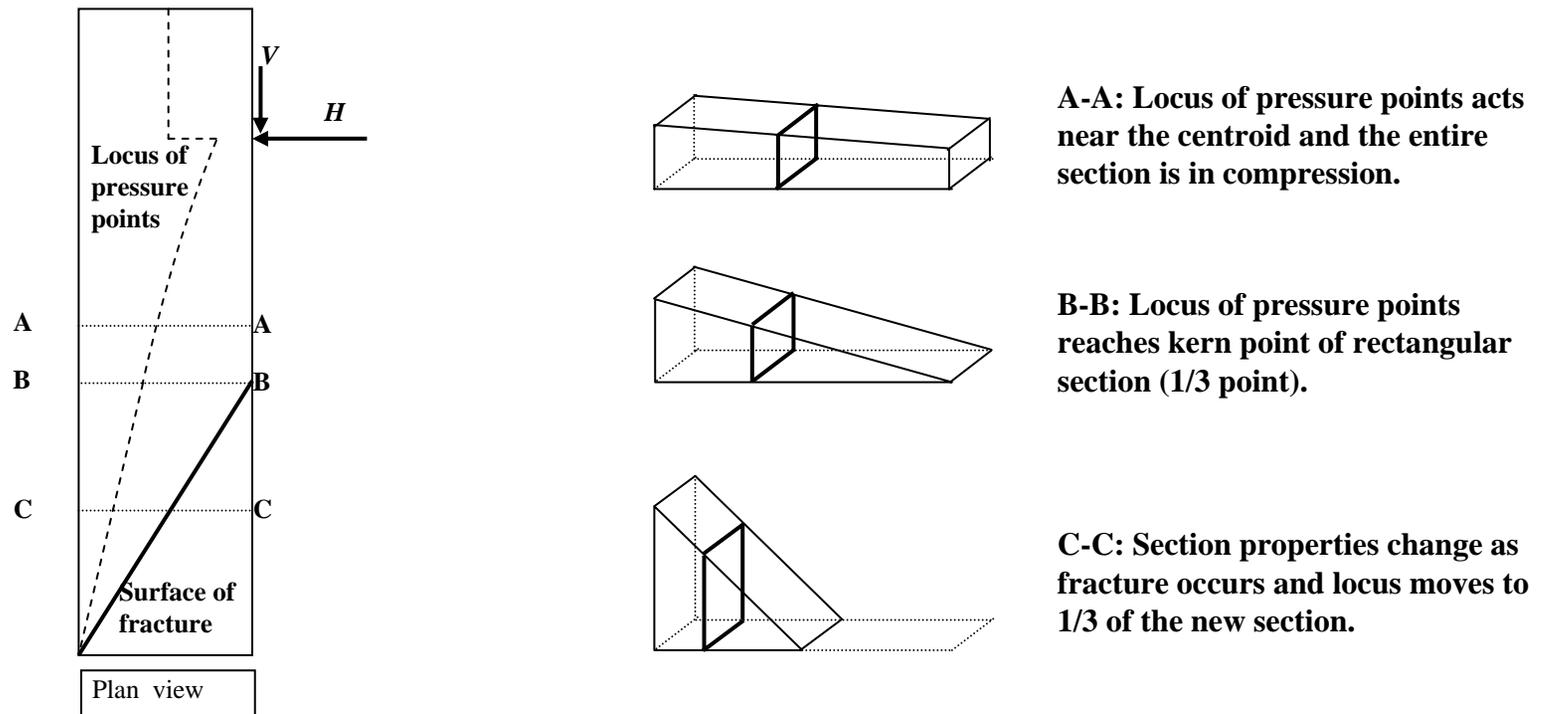
# Assumed Compressive Stress Distribution



# Determine Shape of Fracture



# Assumed Buttress Stress Distribution at Collapse

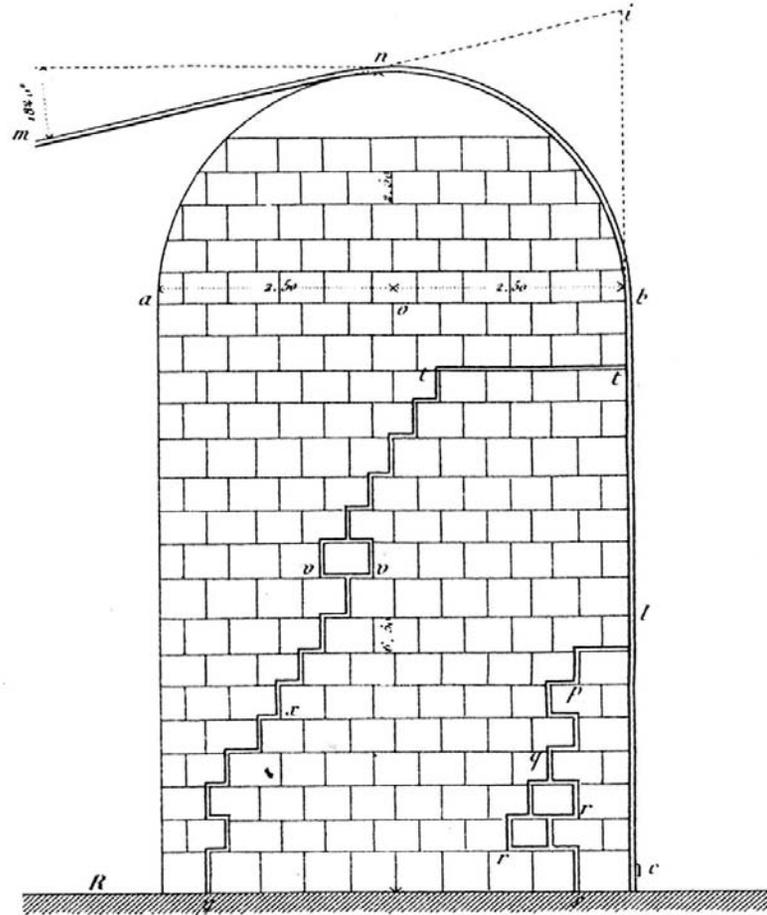


**Fracture reduces thrust capacity by >30% in many cases**

# Vicat Experiments on Suspension Bridge Towers (1832)

*Expériences sur des piliers à paremens verticaux.*

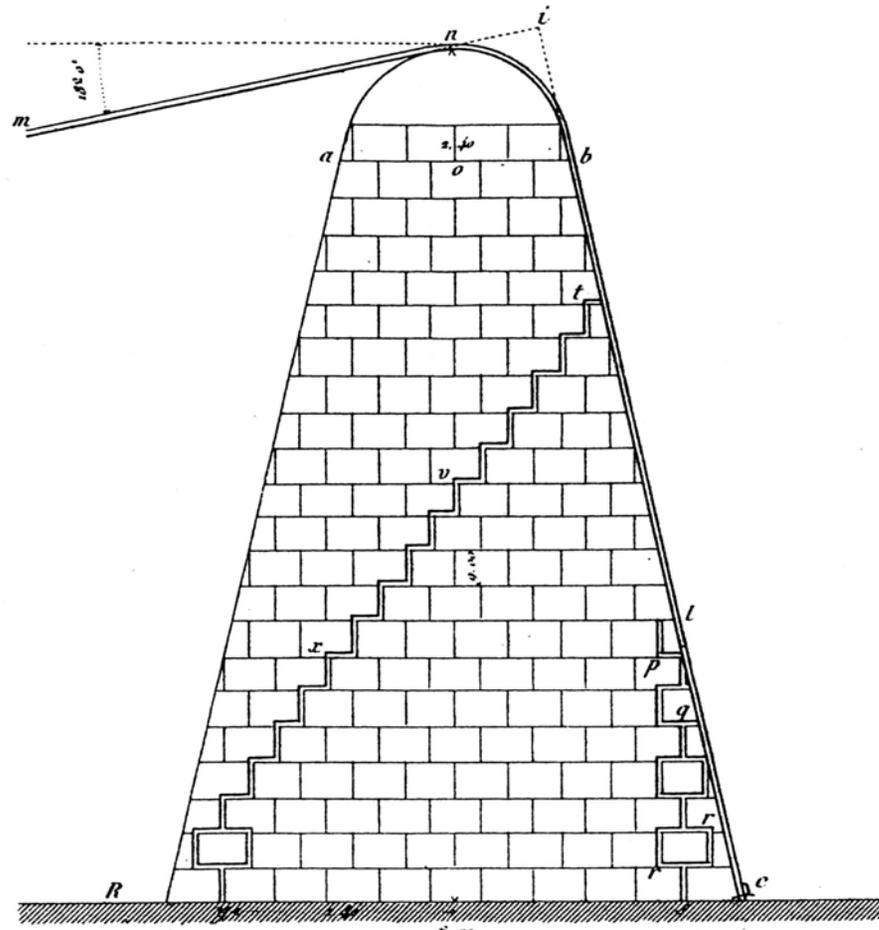
*A. 1<sup>o</sup> avec des attaches enveloppantes. Fig. 1.*



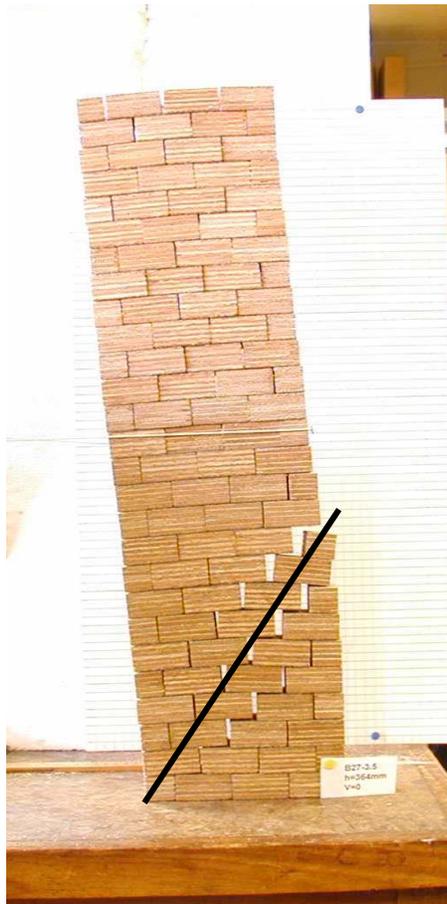
# Vicat Experiments on Suspension Bridge Towers (1832)

*Expériences sur les piliers à paremens inclinés.*

*A. 1. avec des attaches enveloppantes. Fig. 2.*

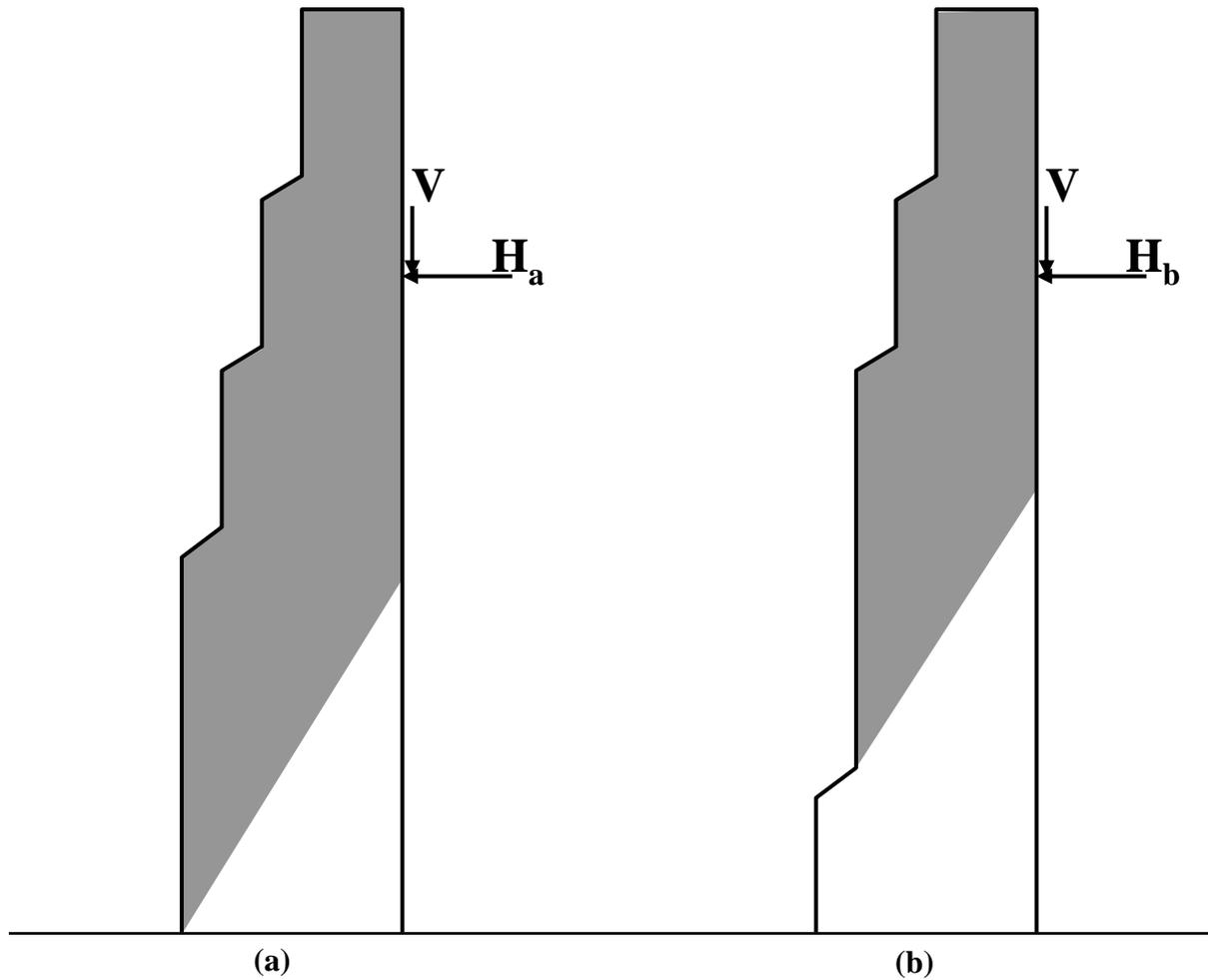


# Model Buttress Experiments

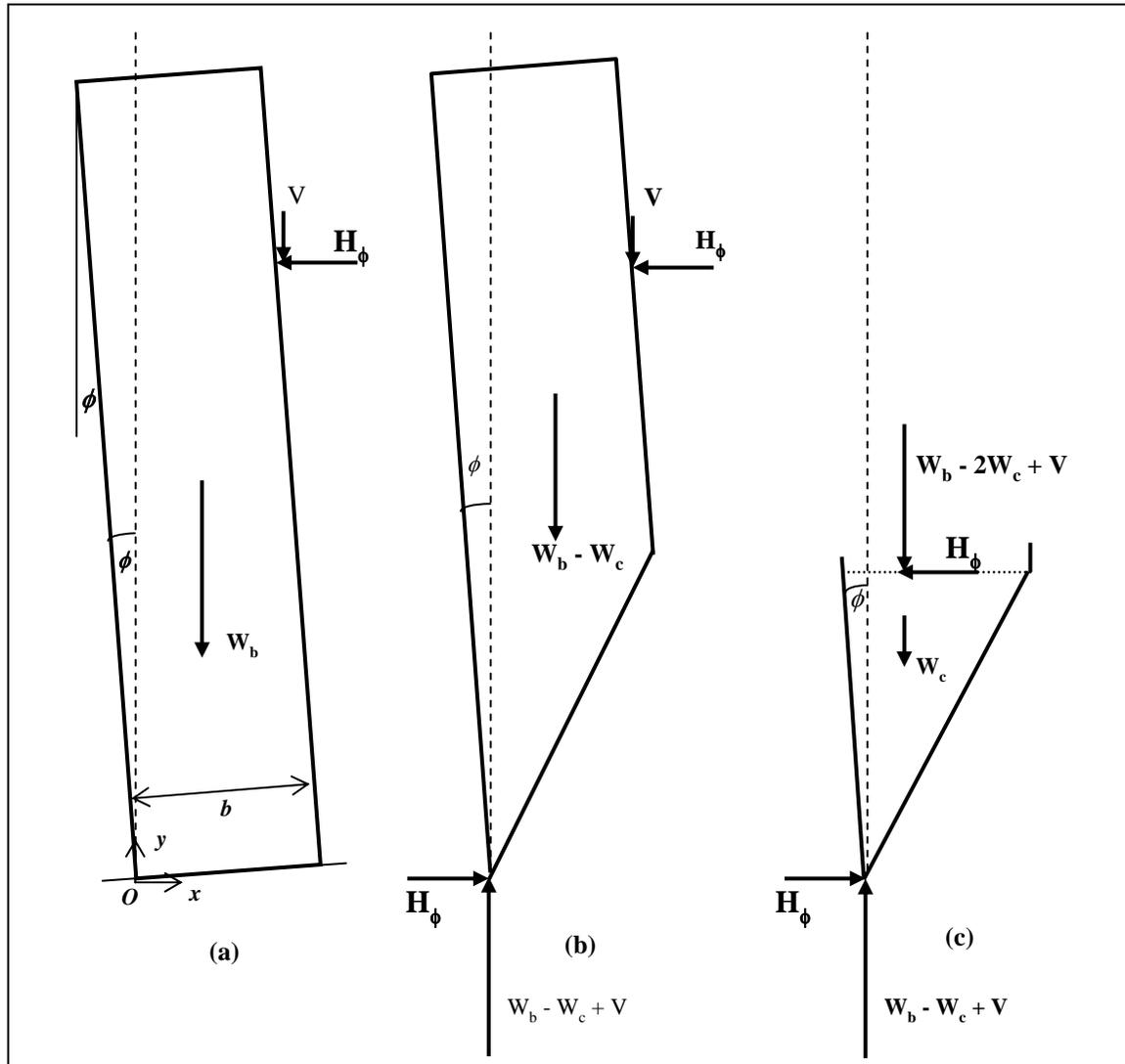


**Fracture reduces thrust capacity by 20% to 30%**

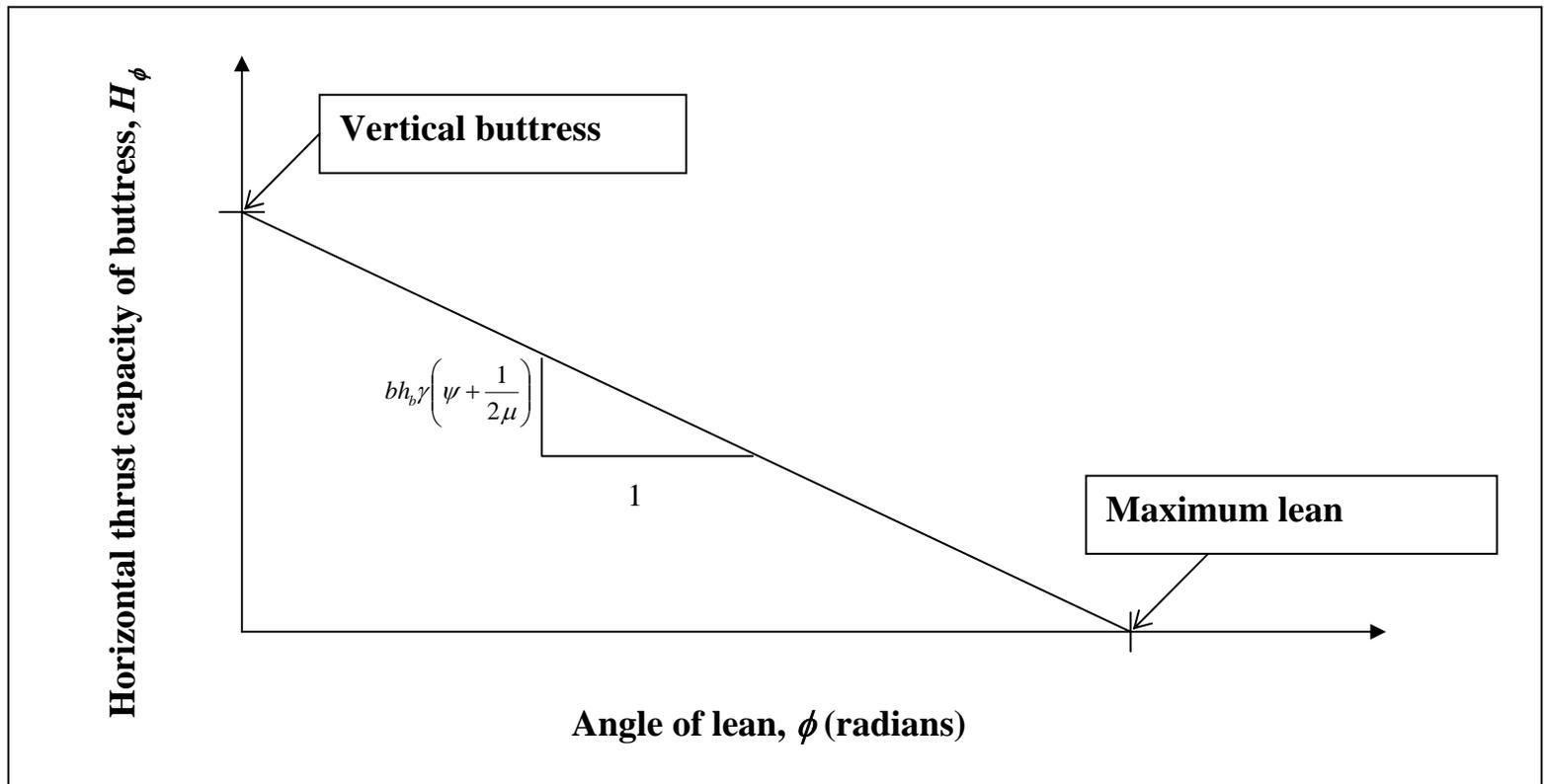
# Gothic Buttress Failure State



# Leaning Buttress

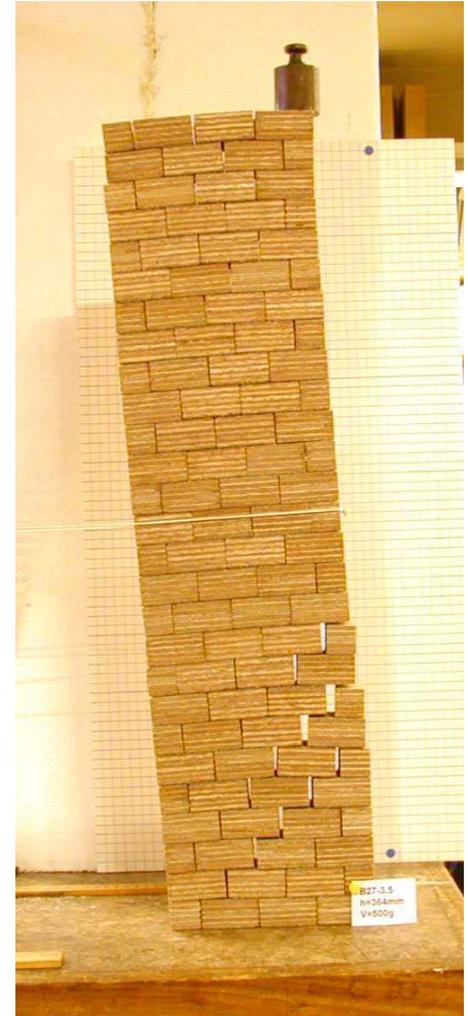


# Leaning Buttress Capacity

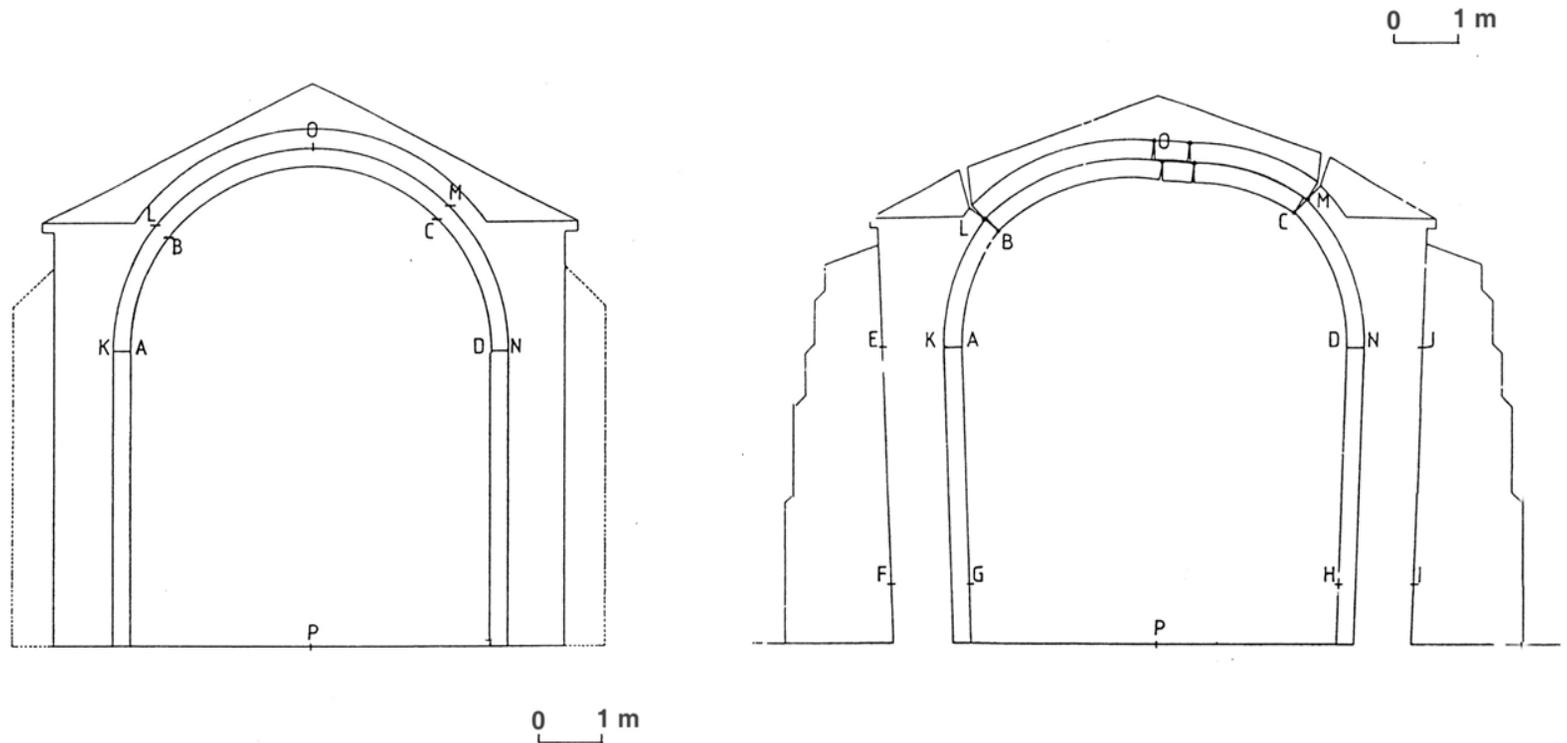


# Load Capacity of a Buttress

- A masonry buttress will fracture at collapse, reducing its load capacity.
- A leaning buttress has a linear reduction in capacity, based on a small angle approximation as the centroid shifts horizontally.

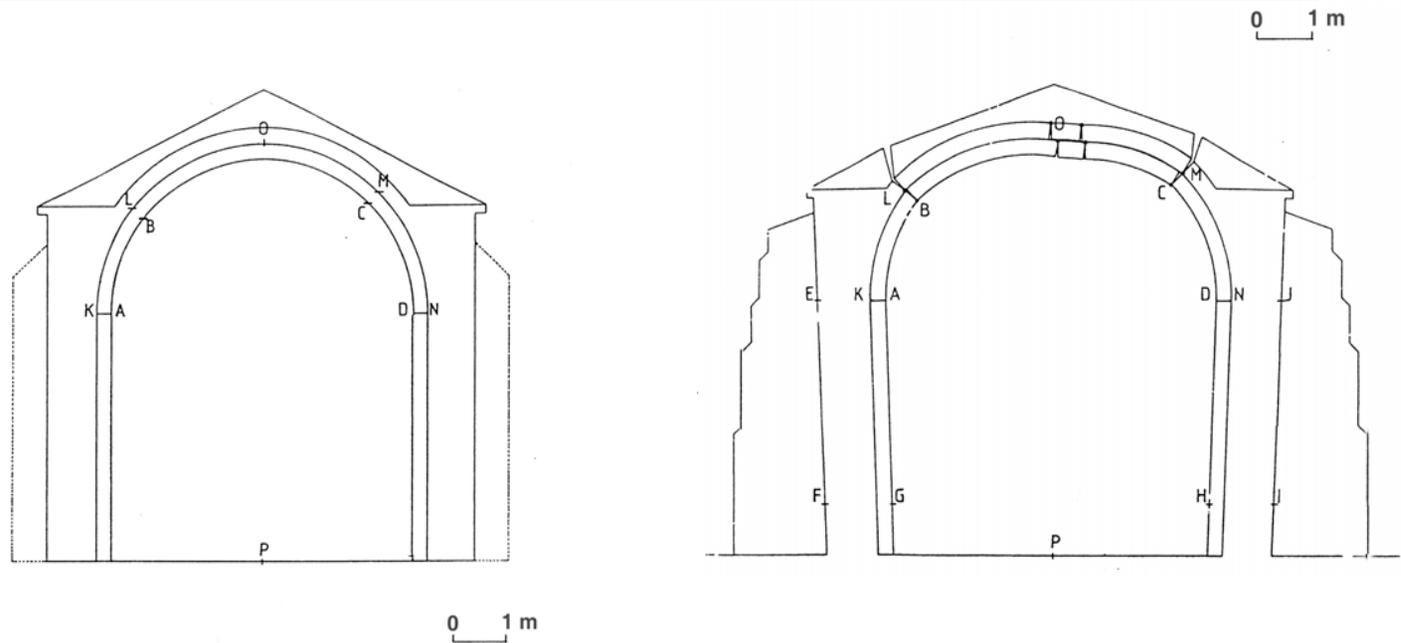


# Church in Guimarei, Spain



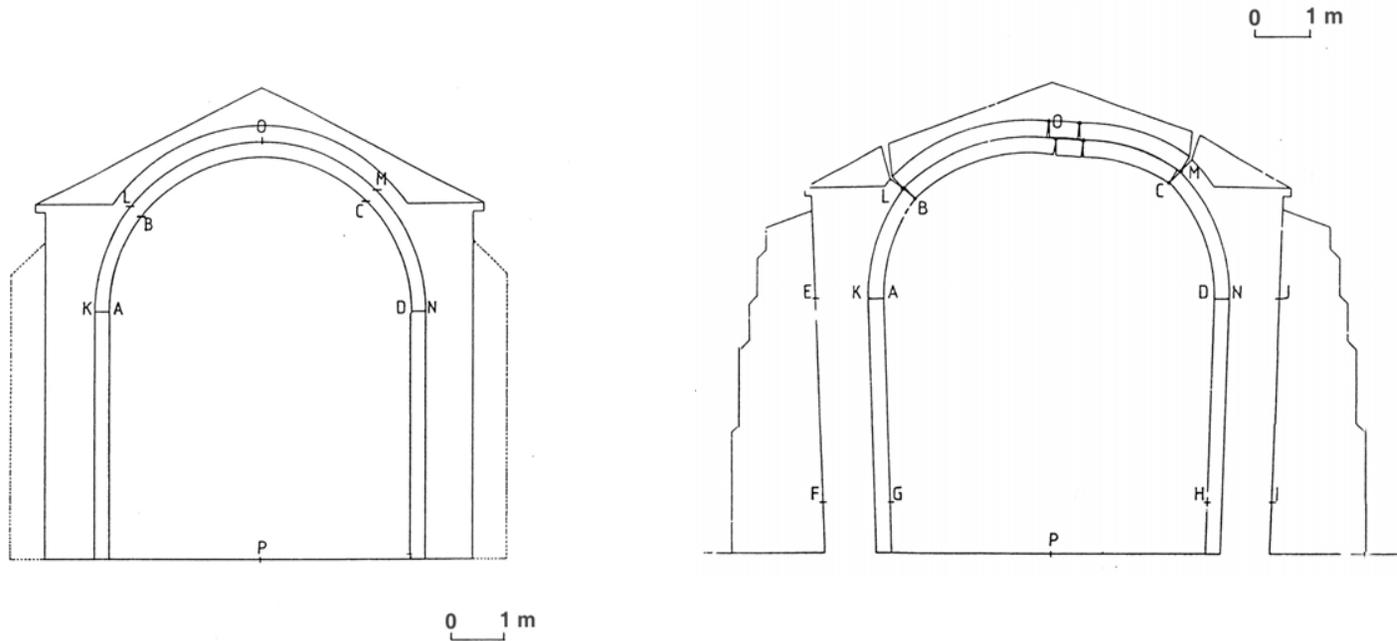
**Geometry changes may threaten stability of the structure**

# Church in Guimarei, Spain



- 1. Buttress leans outward (e.g. foundation deforms)**
- 2. Arch deforms and thrust increases**
- 3. Buttress leans further and thrust increases further.**

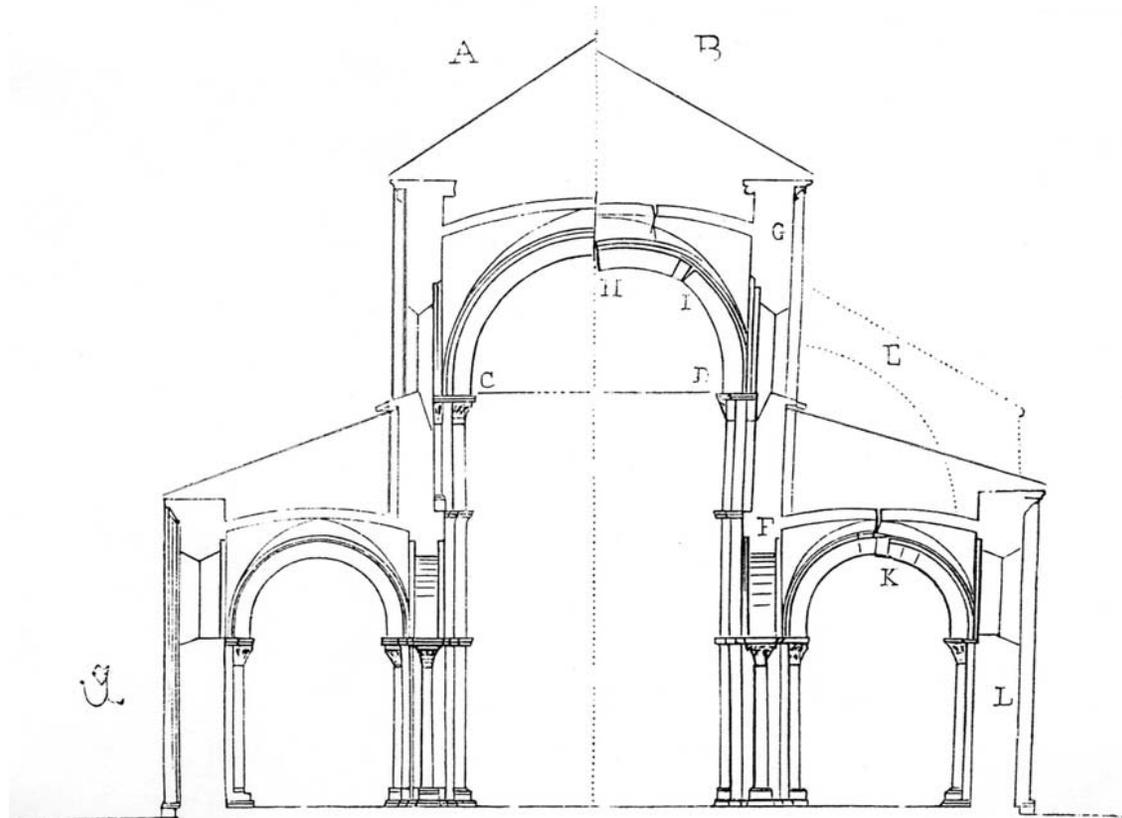
# Church in Guimarei, Spain



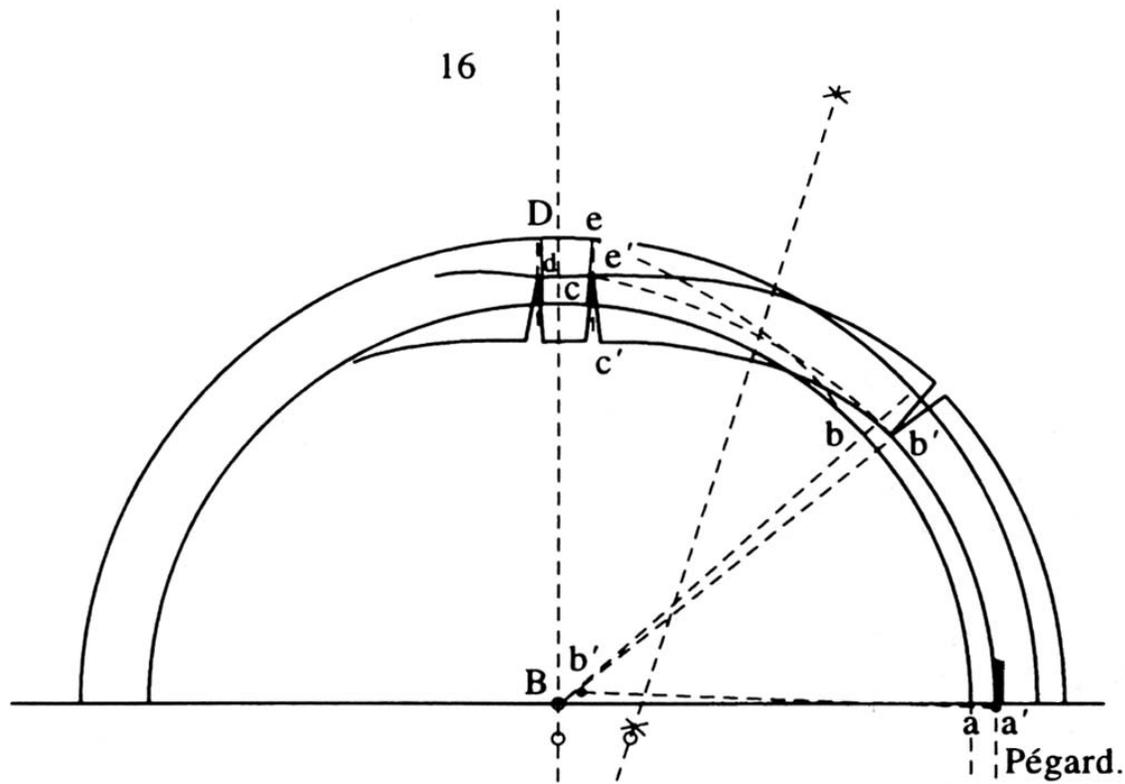
**Must solve three problems:**

- 1. Load capacity of buttress (and influence of lean)**
- 2. Collapse state of arch on spreading supports**
- 3. Analysis of arch supported on leaning buttresses**

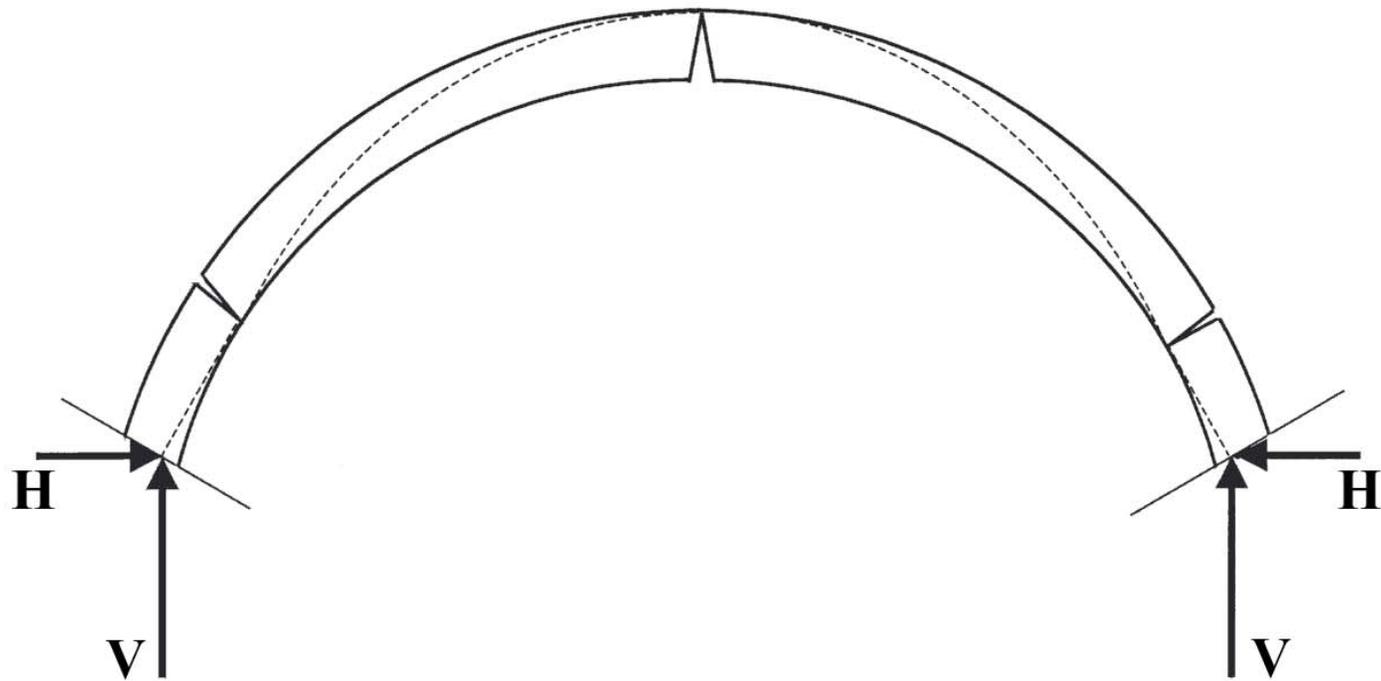
# Spreading Arches: Viollet's study of Vezelay (1854)



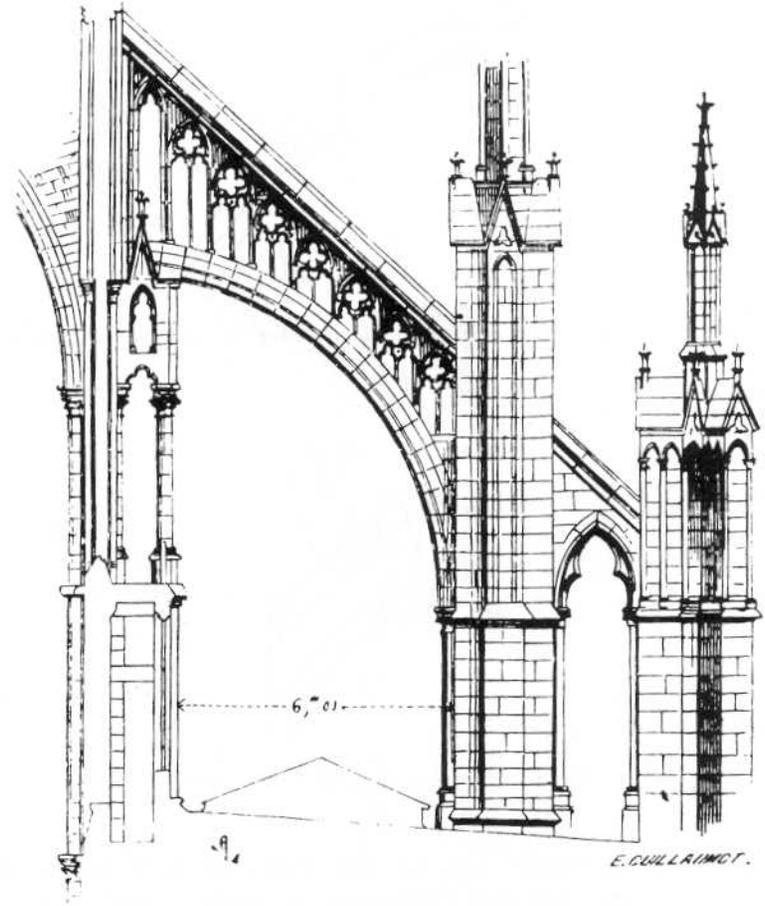
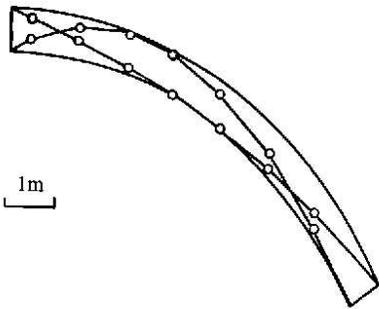
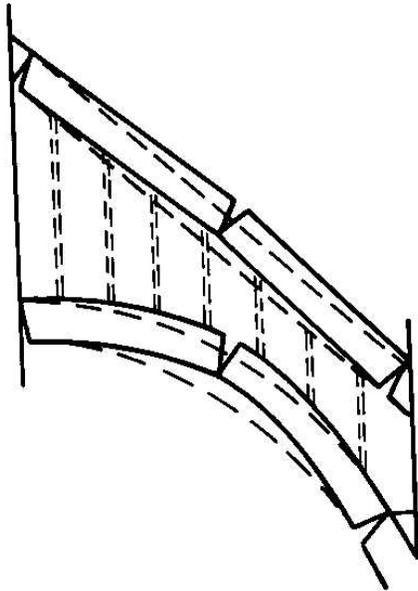
# Viollet-le-Duc on Spreading Arches



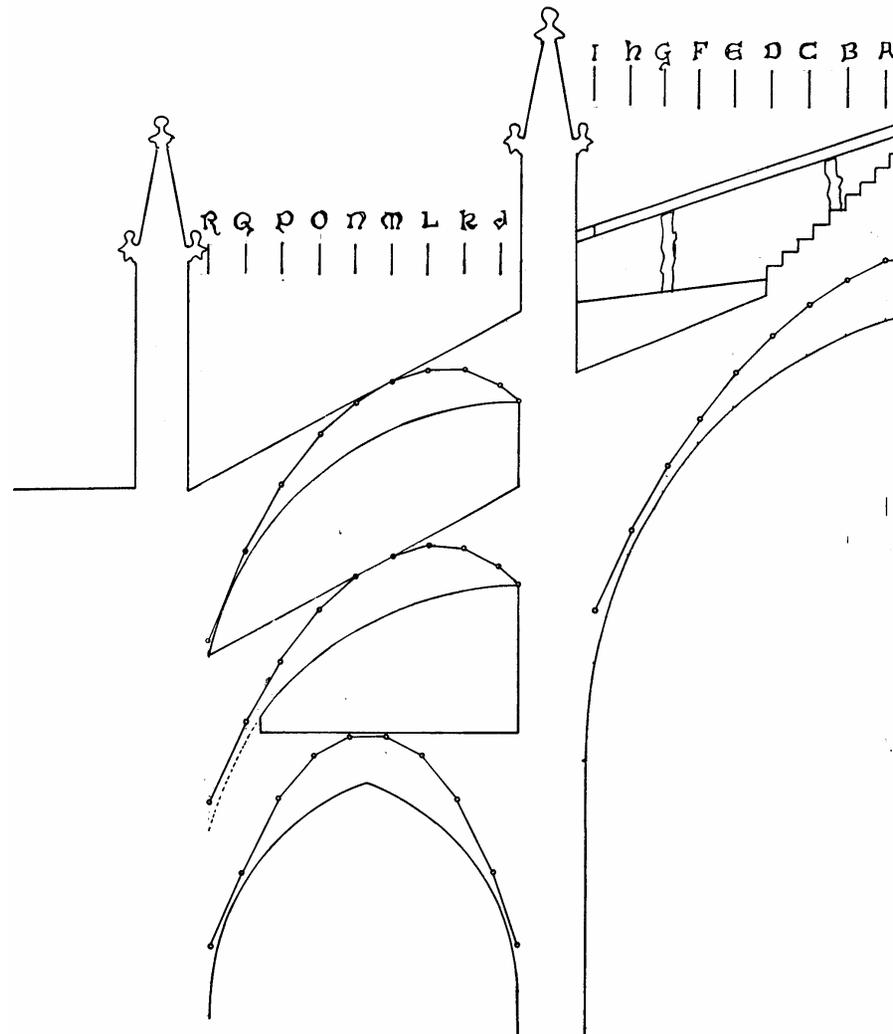
# Arch on Spreading Supports



# Amiens Possible Collapse Mode



# Flying Buttresses at Palma de Mallorca



# Conclusions

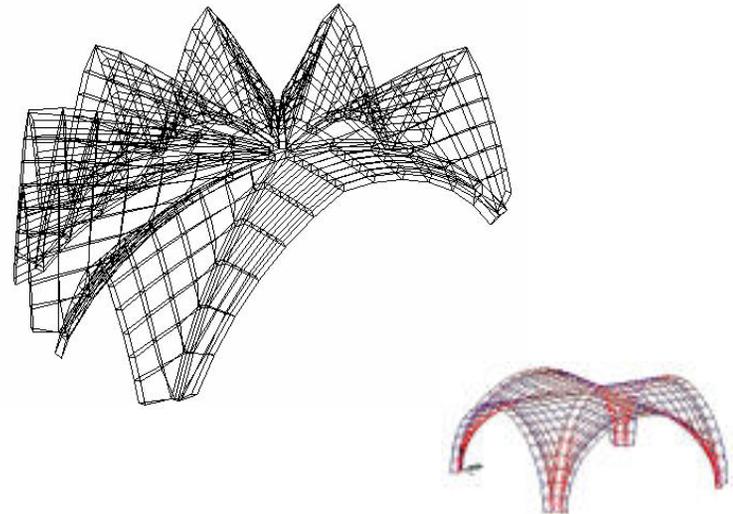
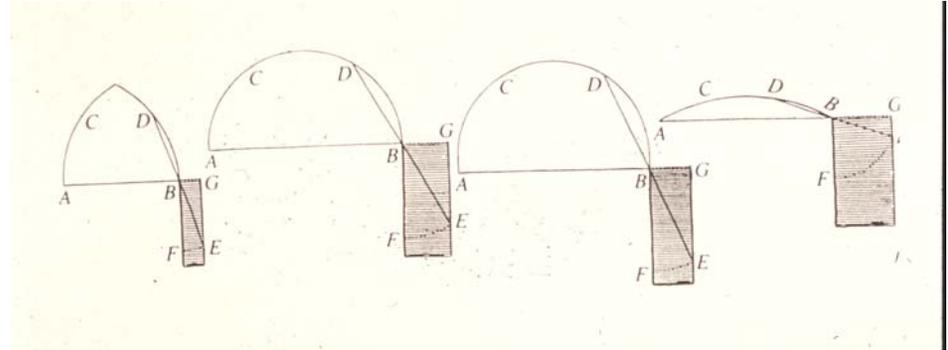
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- **Unreinforced masonry structures have very low stress levels: stability, not strength, governs the safety**
- **Limit analysis can be used to determine collapse states based on thrust line analysis**
- **Capacity for displacements may be more important than load capacity (particularly for historic buildings)**
- **For high vaulted buildings, the arch will collapse and the buttress will remain standing in most cases.**

# Research Papers on Masonry

- **Comparative studies**

- Arches
- Vaulting
- Buttresses
- Individual structures



# Research Papers on Masonry

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- Tile vaulting  
(Guastavino)
- Gothic
- Romanesque
- Mamluk
- Maya/Aztec
- Mycenaean tholos  
tombs
- Individual structures