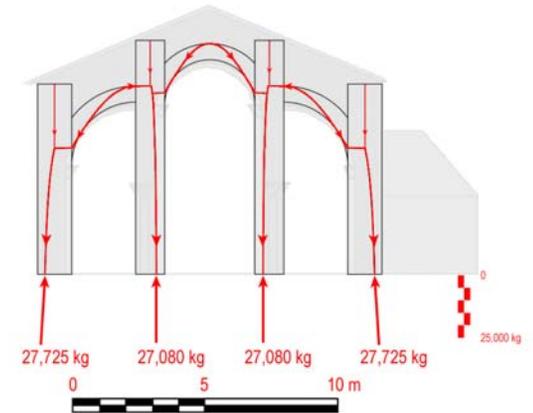
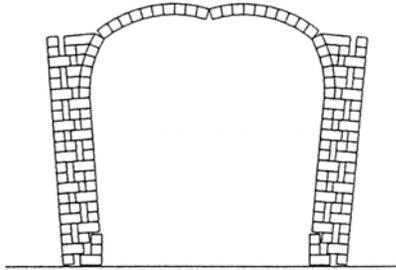


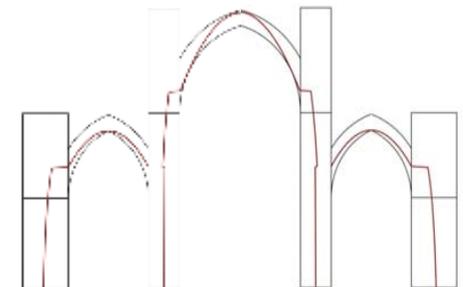
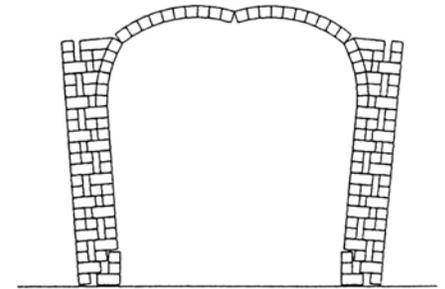
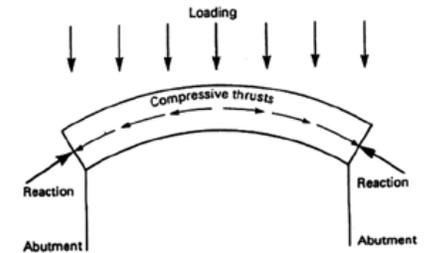
# Analysis of Historic Structures



## Lecture 2: Intro to Masonry Structure

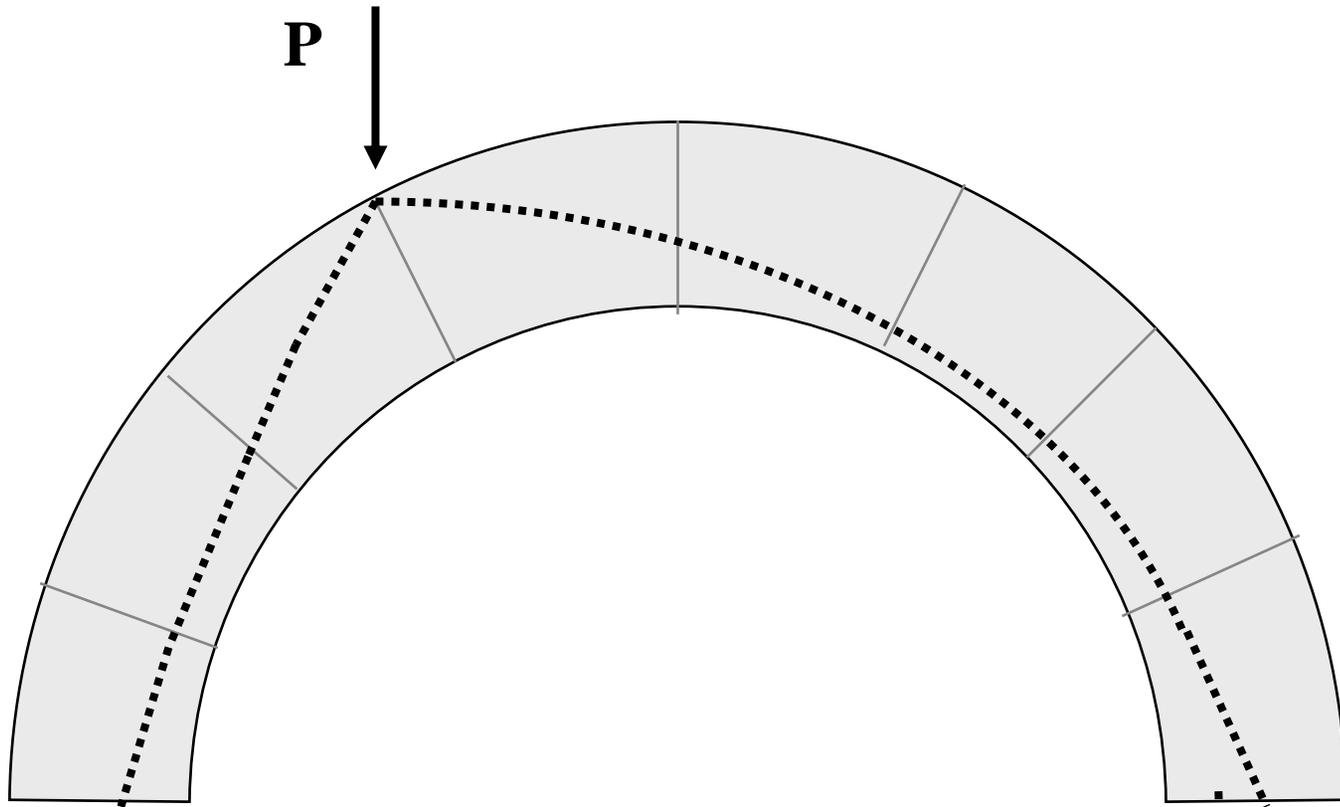
# Review of Last Meeting

- Principles of historic structure
  - EQUILIBRIUM
- Lower bound and upper bound theorems
- Possible research topics

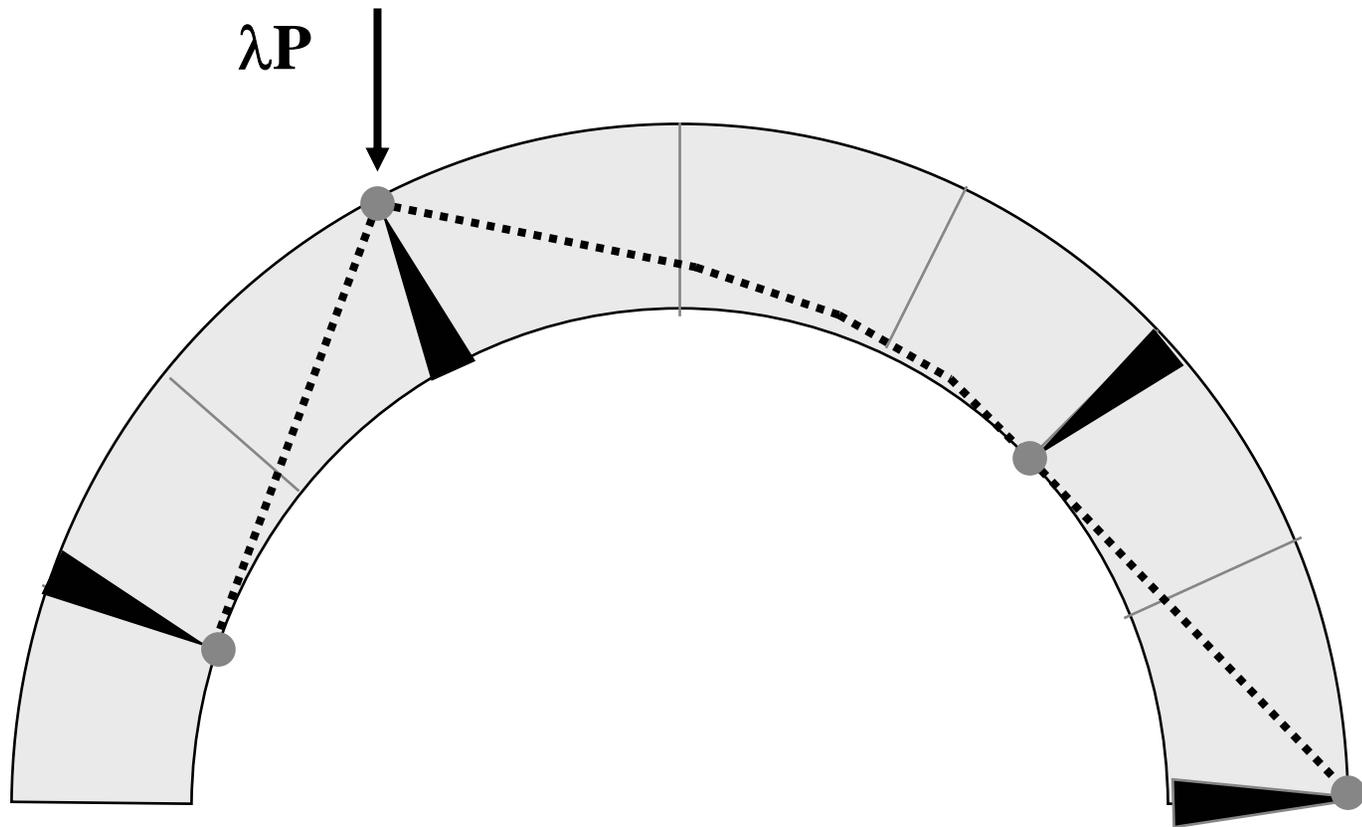


# Lower Bound Theorem

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# Upper Bound Theorem



# Theorems of Limit Analysis

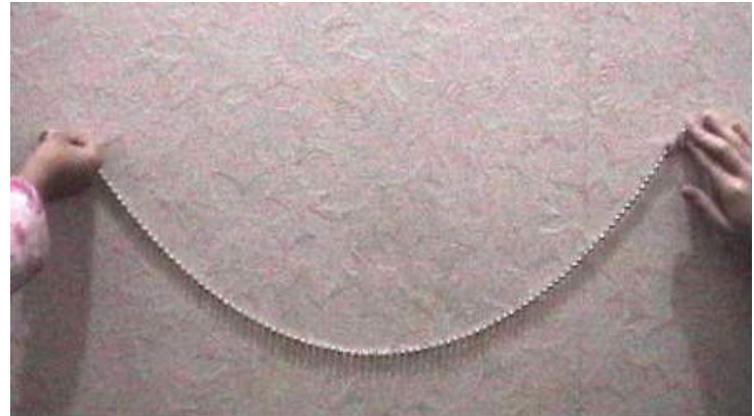
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- 1. Lower Bound:** If you can demonstrate at least one possible equilibrium state, then the structure can also find at least one possible stable state
- 2. Upper Bound:** When the load path can no longer be contained within the structure, and it is the unique and largest possible load, then it is the collapse load

# Hooke's "2<sup>nd</sup>" Law (1675)

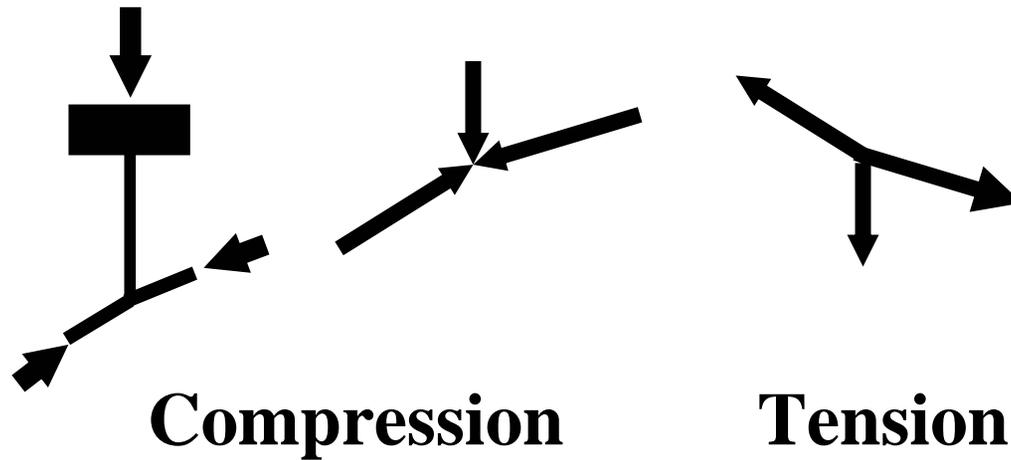
*“ut pendet continuum  
flexile, sic stabit  
contiguum rigidum  
inversum”*

**As hangs the flexible  
line, so but inverted  
will stand the rigid  
arch.**



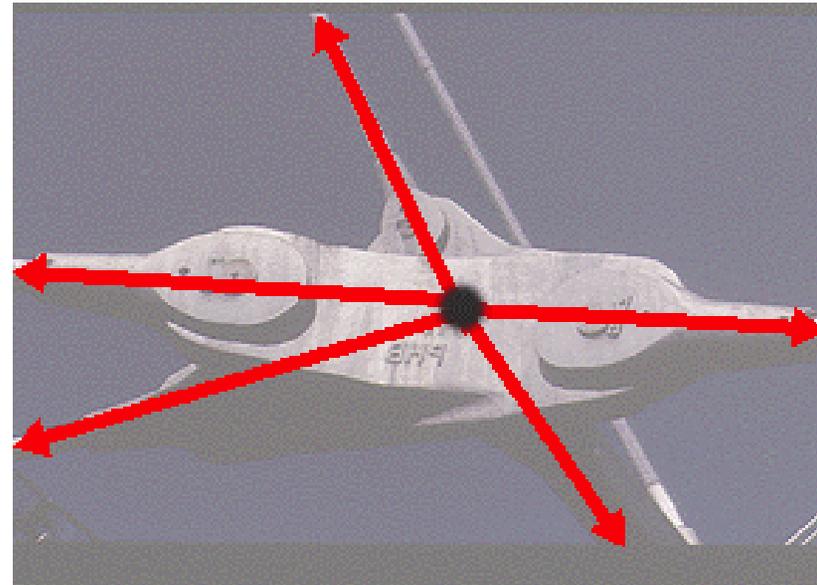
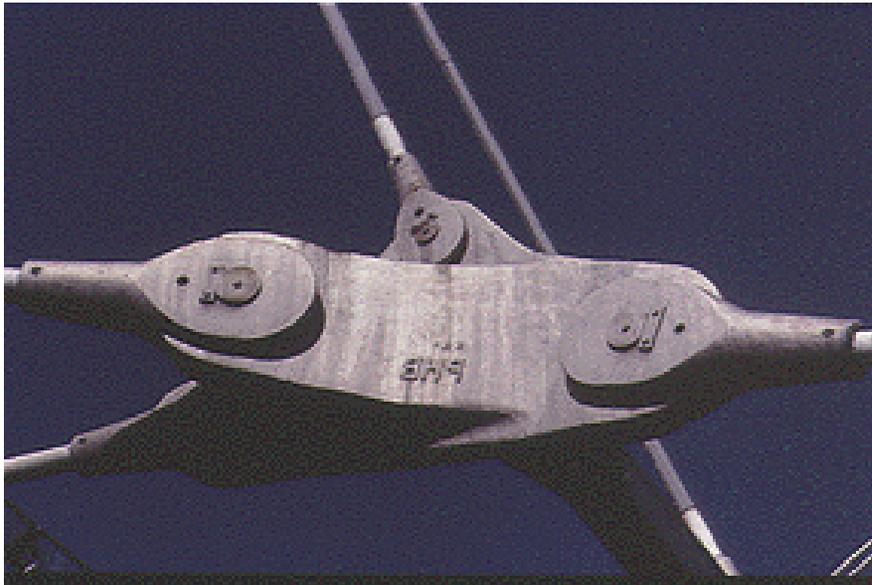
# Compression vs. Tension

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# Equilibrium at a Point

$\Sigma F = 0$  (sum of forces is zero)



# Structural Equations

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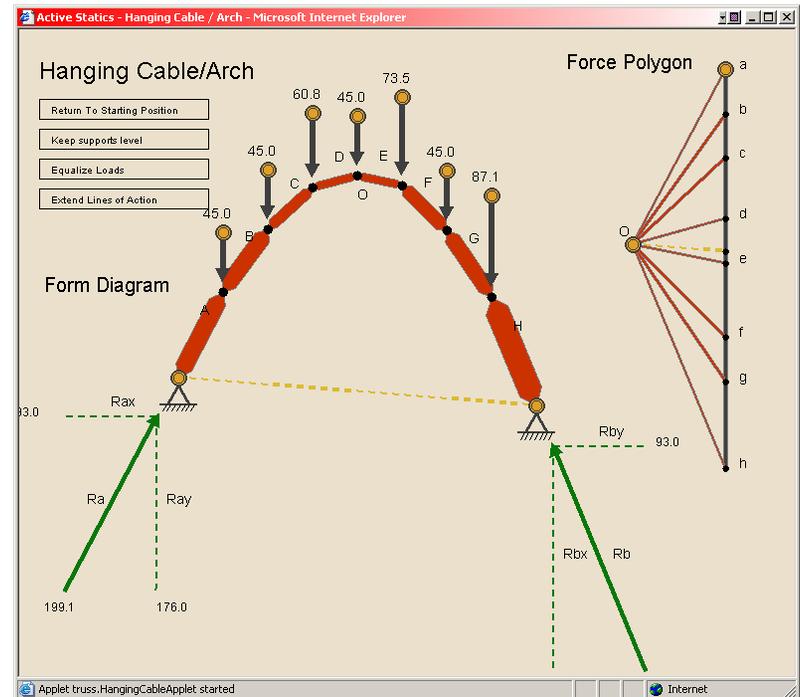
**Only three types of equations:**

- 1) Equilibrium**
- 2) Material properties (elasticity, etc)**
- 3) Compatibility (geometry)**

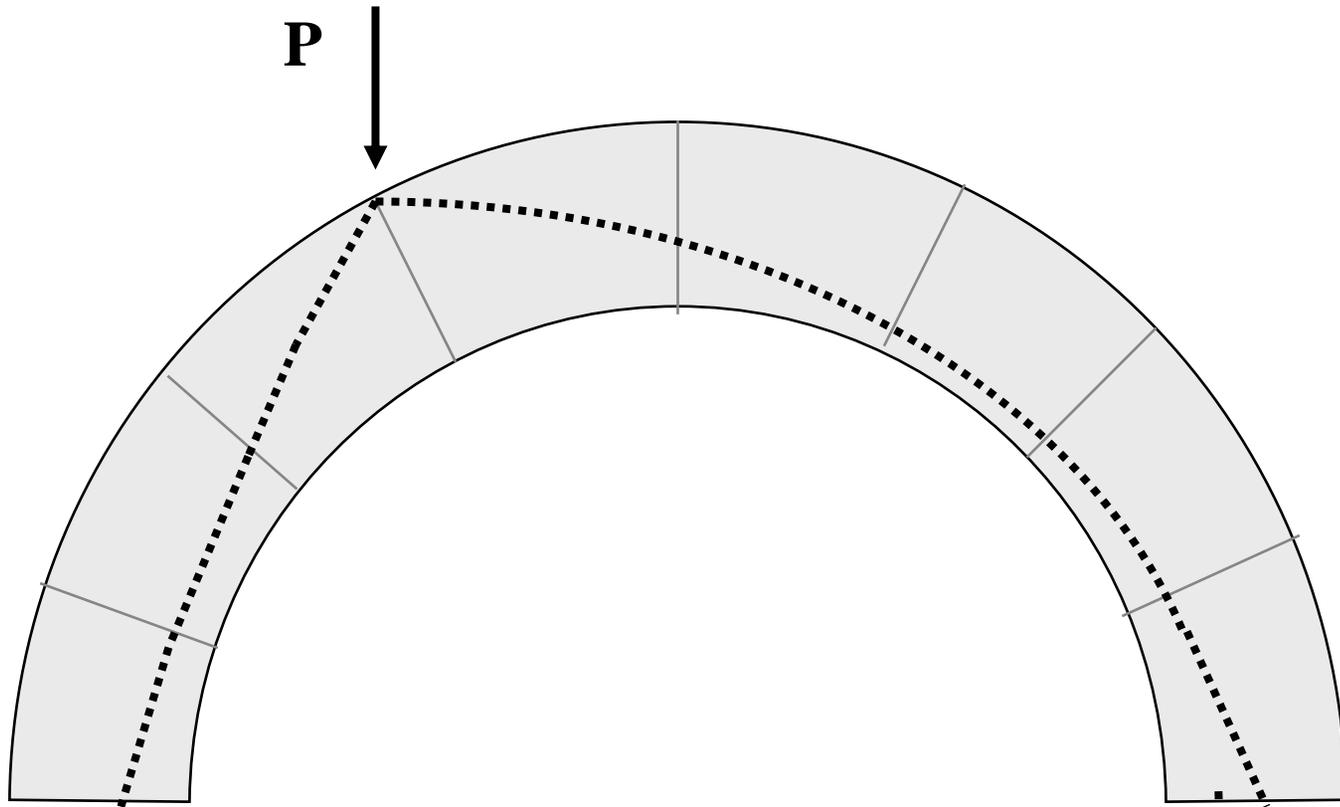
**We will focus on equilibrium equations because they are the most important.**

# Graphic Statics

Applet by Simon Greenwold

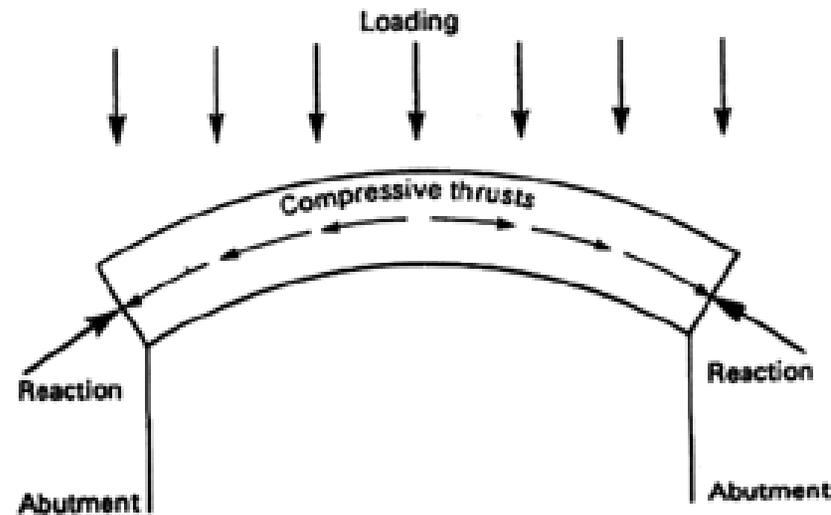


# Hooke's Hanging Chain



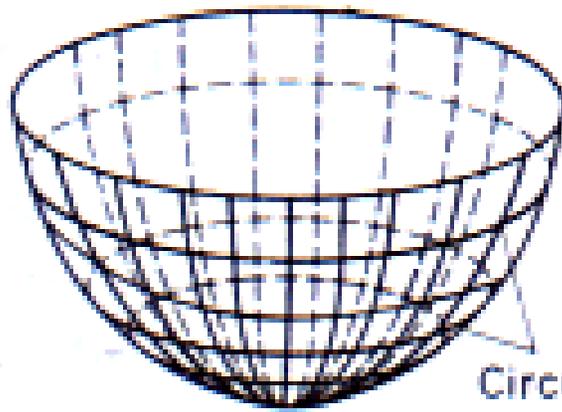
# Design of Masonry

- **Main principle: must be kept in compression**

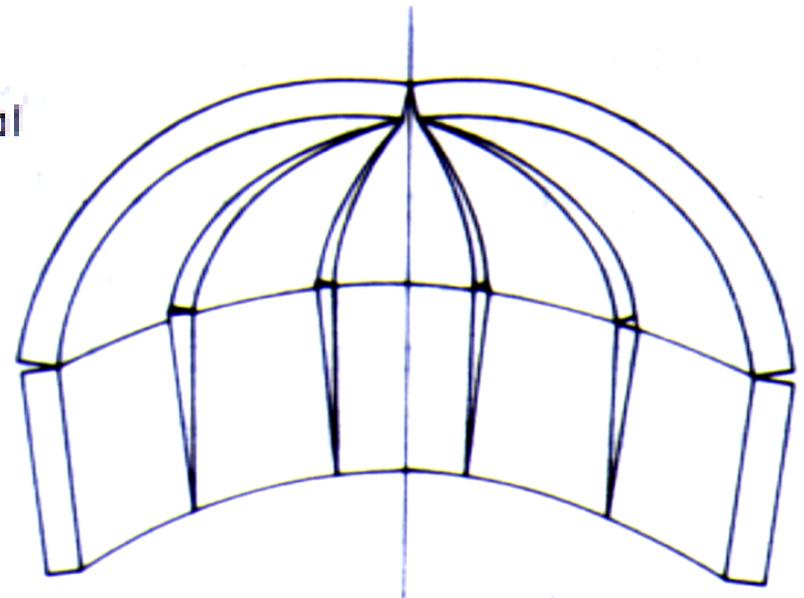


- **Also applies to cast iron, unreinforced concrete, and other “brittle” materials**

# 3D Vaults: “Slice” into arches



Circumferential cables

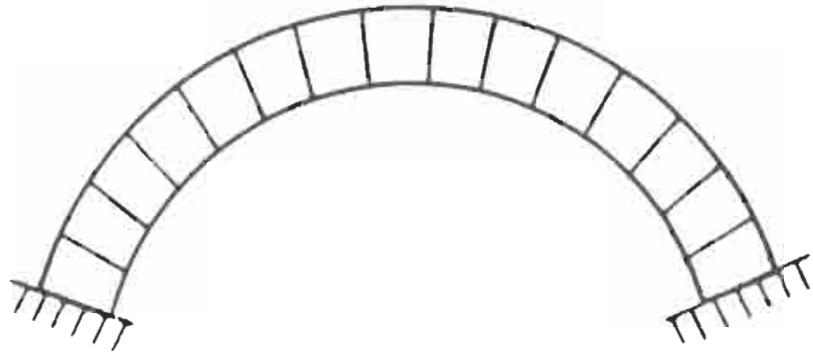


(a) Collapse of dome and drum in “orange - slice” segments

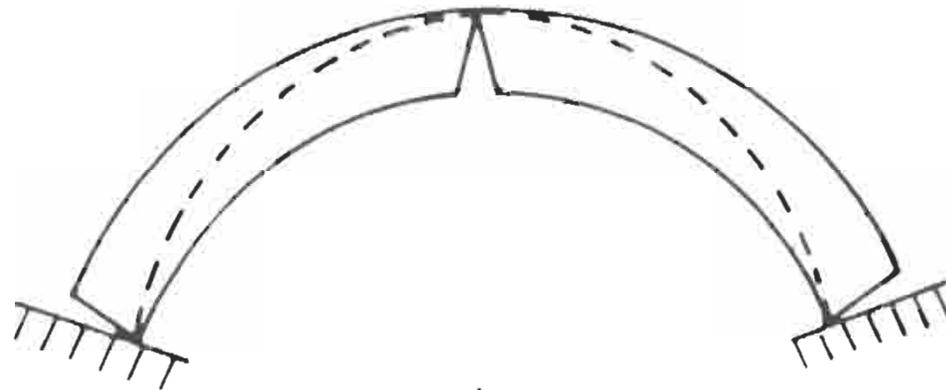
# Structural Analysis of Masonry

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- **The Stone Skeleton**  
by Jacques Heyman
- **Three main assumptions:**
  - No tensile strength
  - Infinite compressive strength (rigid)
  - Sliding does not occur

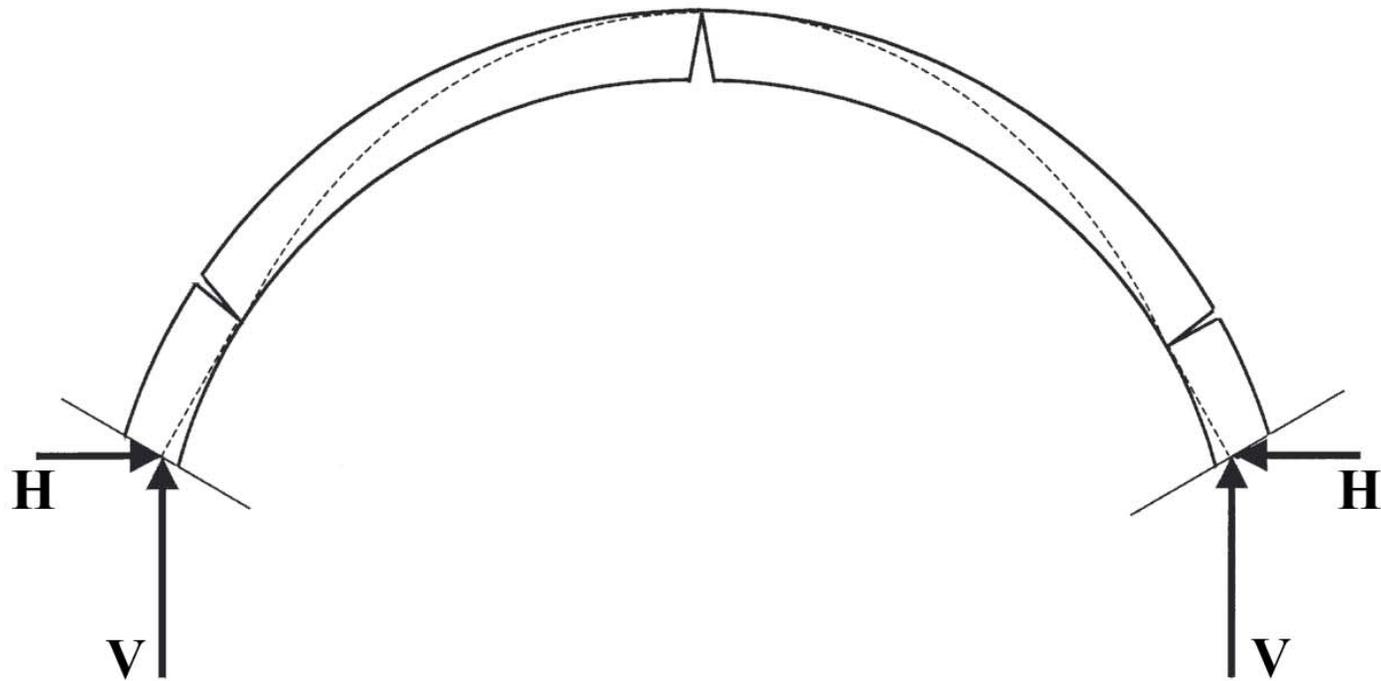


(a)

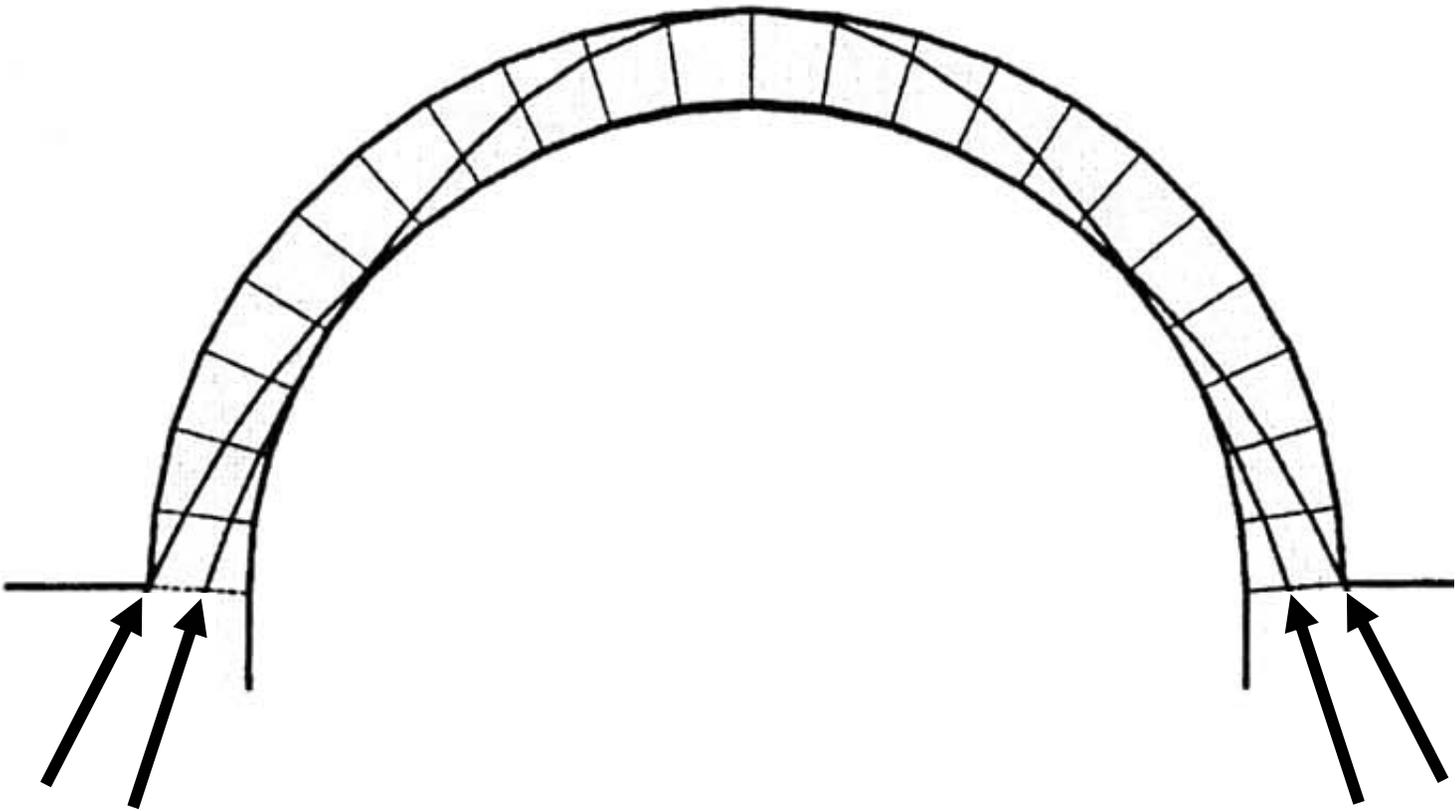


(b)

# Arch on Spreading Supports

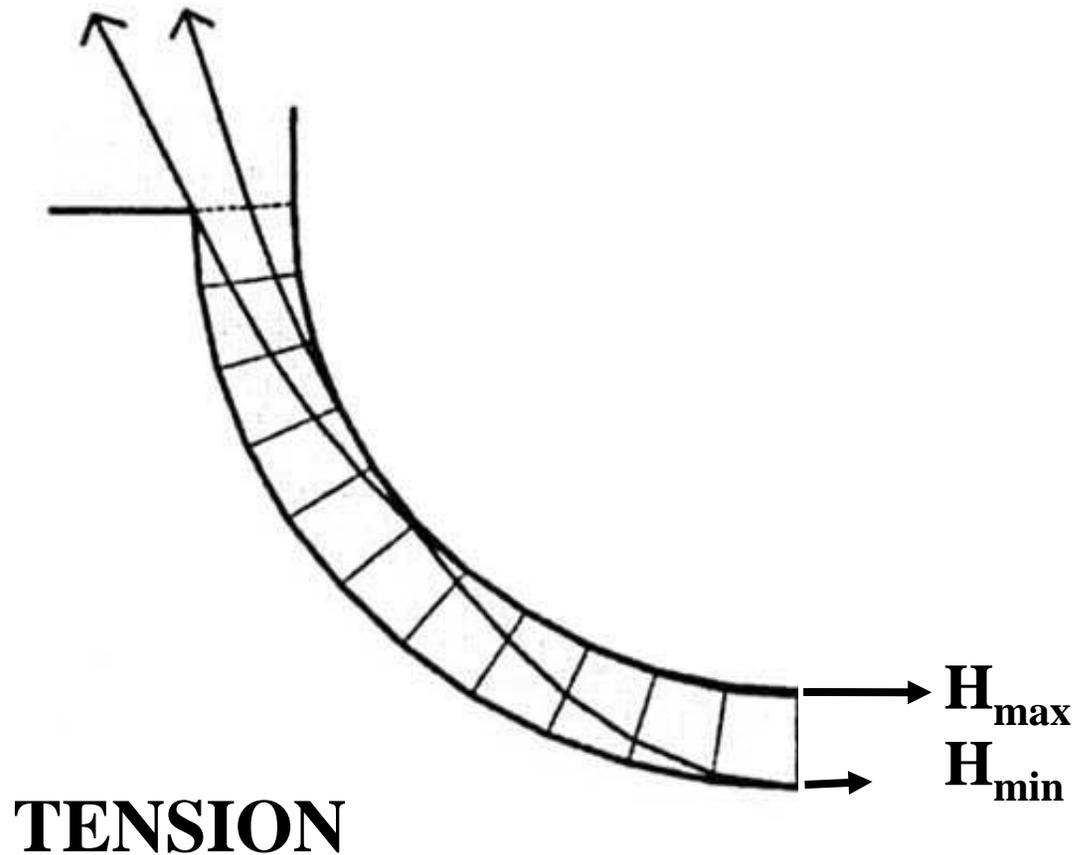


# Range of Arch Thrust

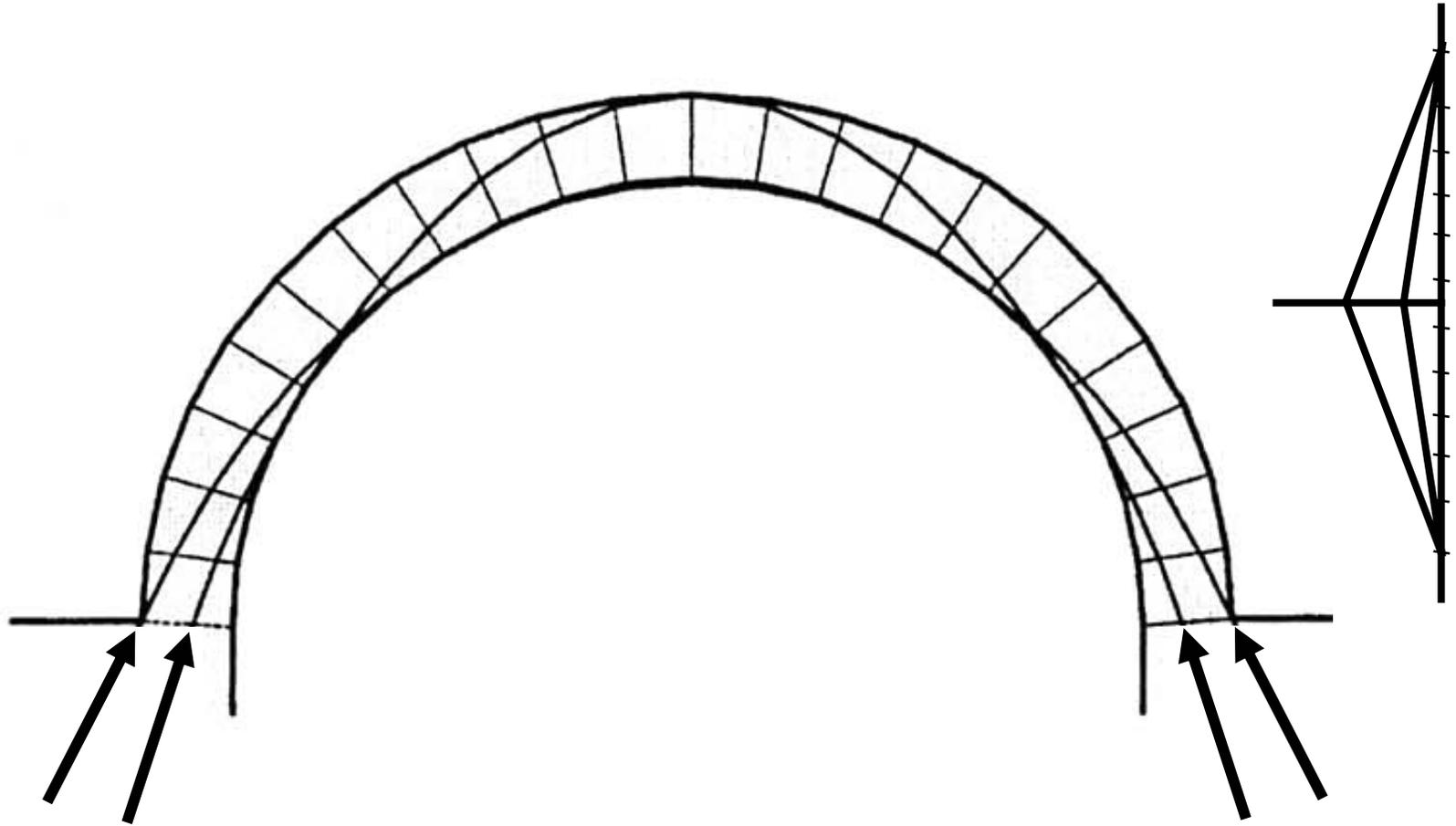


**Internal thrust lines due to self weight of arch**

# Range of Arch Thrust

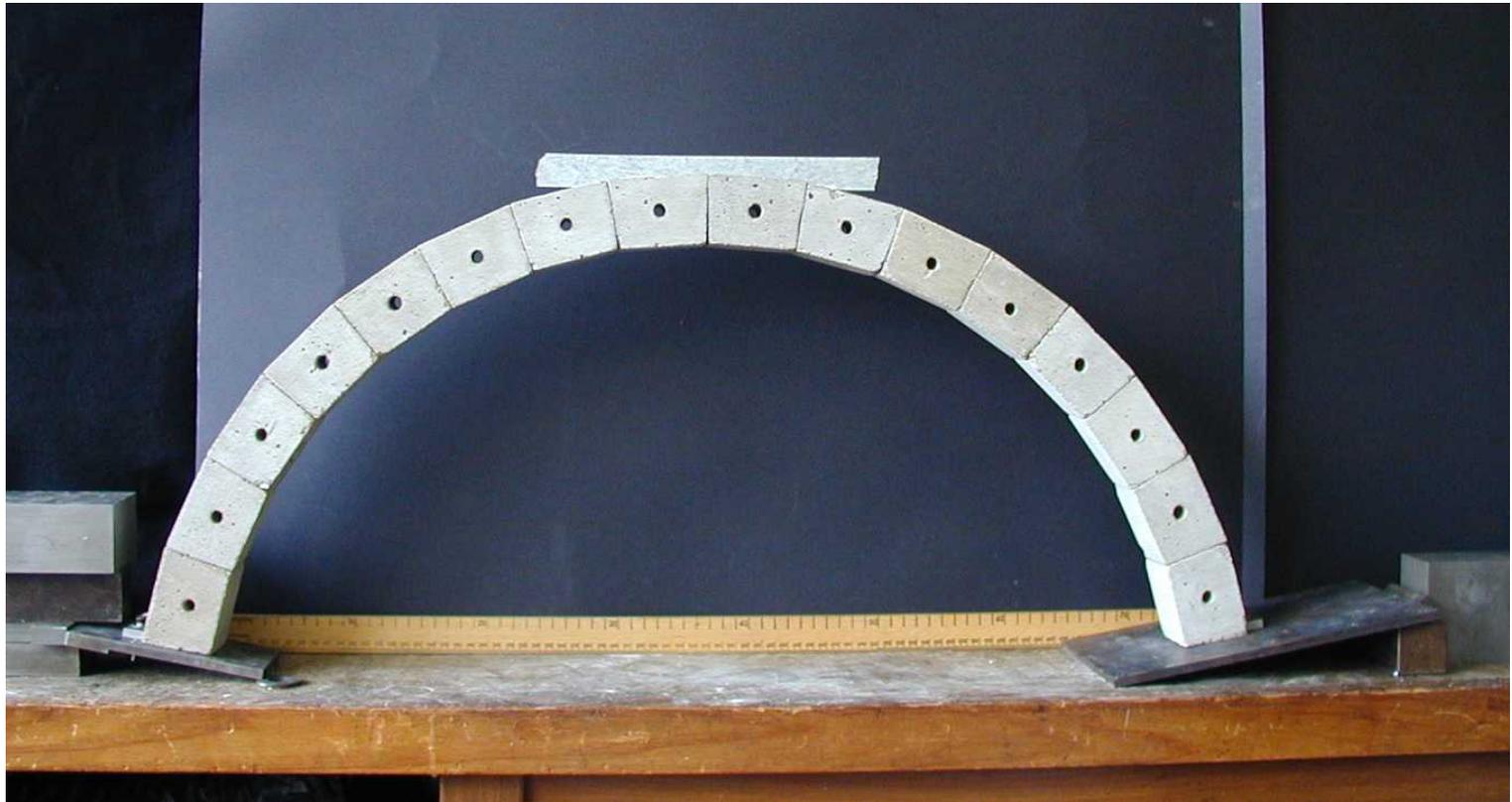


# Range of Arch Thrust

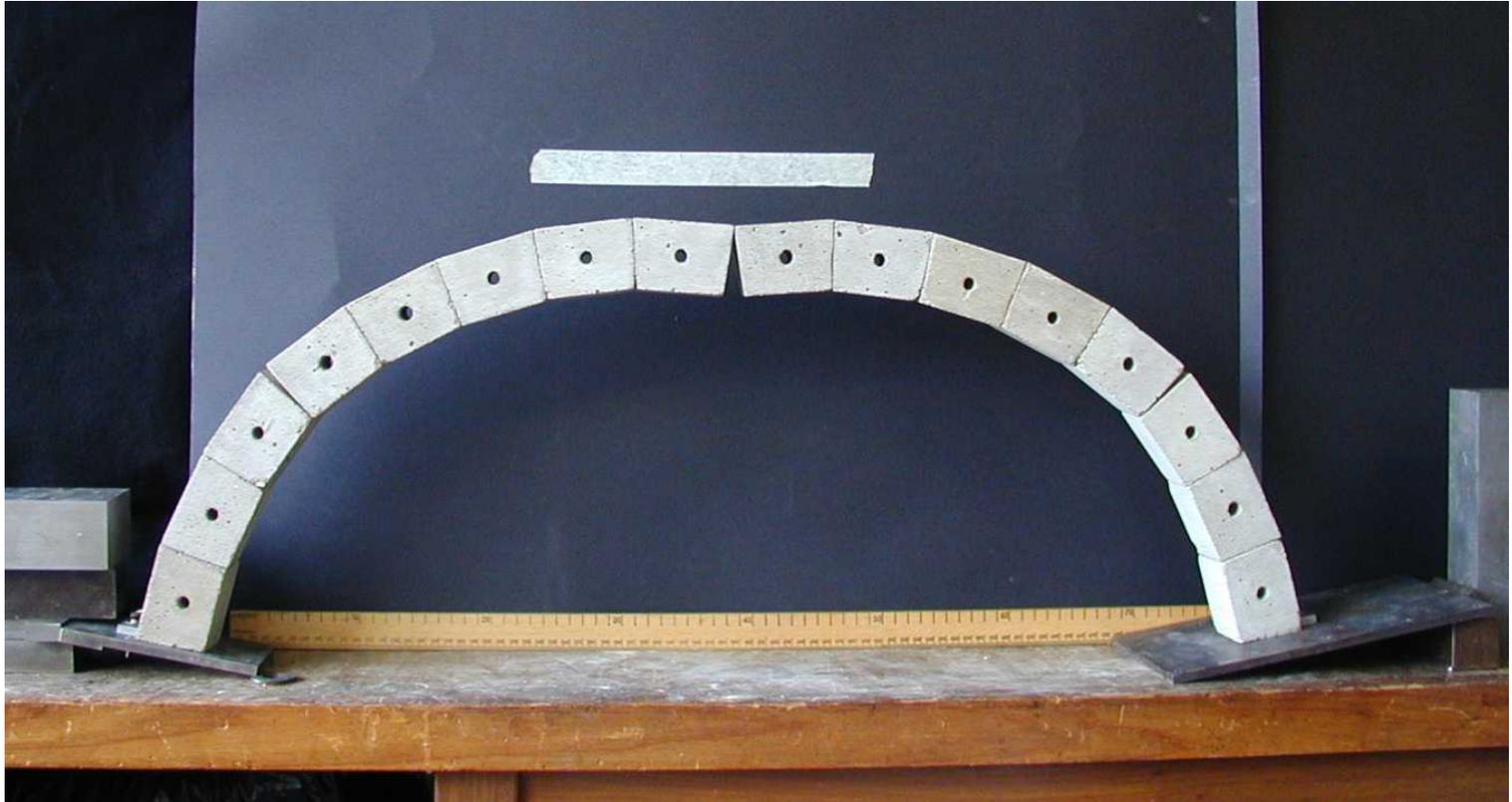


**COMPRESSION**

# Model Arch Experiment



# Model Arch at Collapse State



# **Understanding cracks in masonry**

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- 1. Why do cracks occur?**
- 2. What do they tell us?**
- 3. Are they a cause for concern?**

# Understanding cracks in masonry

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- 1. Why do cracks occur?**
  - Small movements of supports*
- 2. What do they tell us?**
  - Where forces are NOT acting*
- 3. Are they a cause for concern?**
  - Usually not, but they can be*

# Understanding of Collapse

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## Causes of collapse:

### 1. Displacements

-Foundation movements, mortar “creep” over time

### 2. Overloading (truck on a bridge)

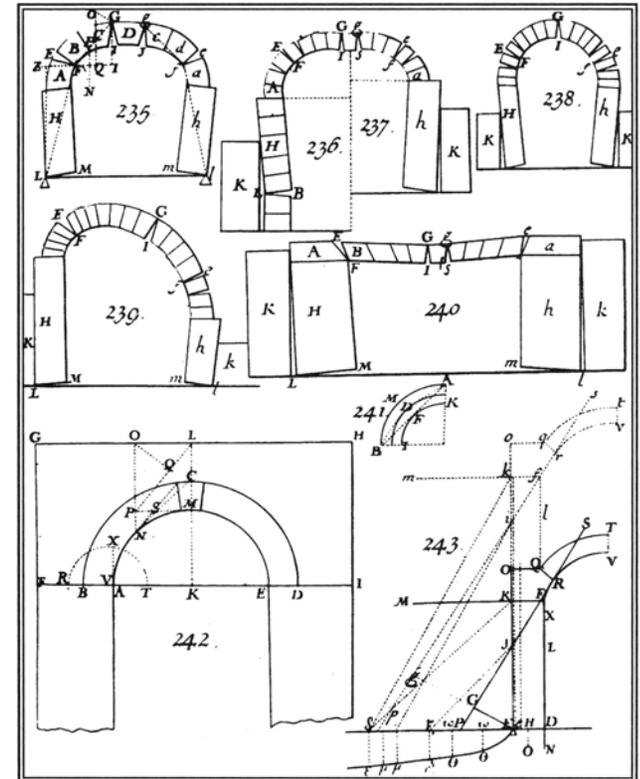
-Water on vaults, collapsing roof on vault

### 3. Accelerations

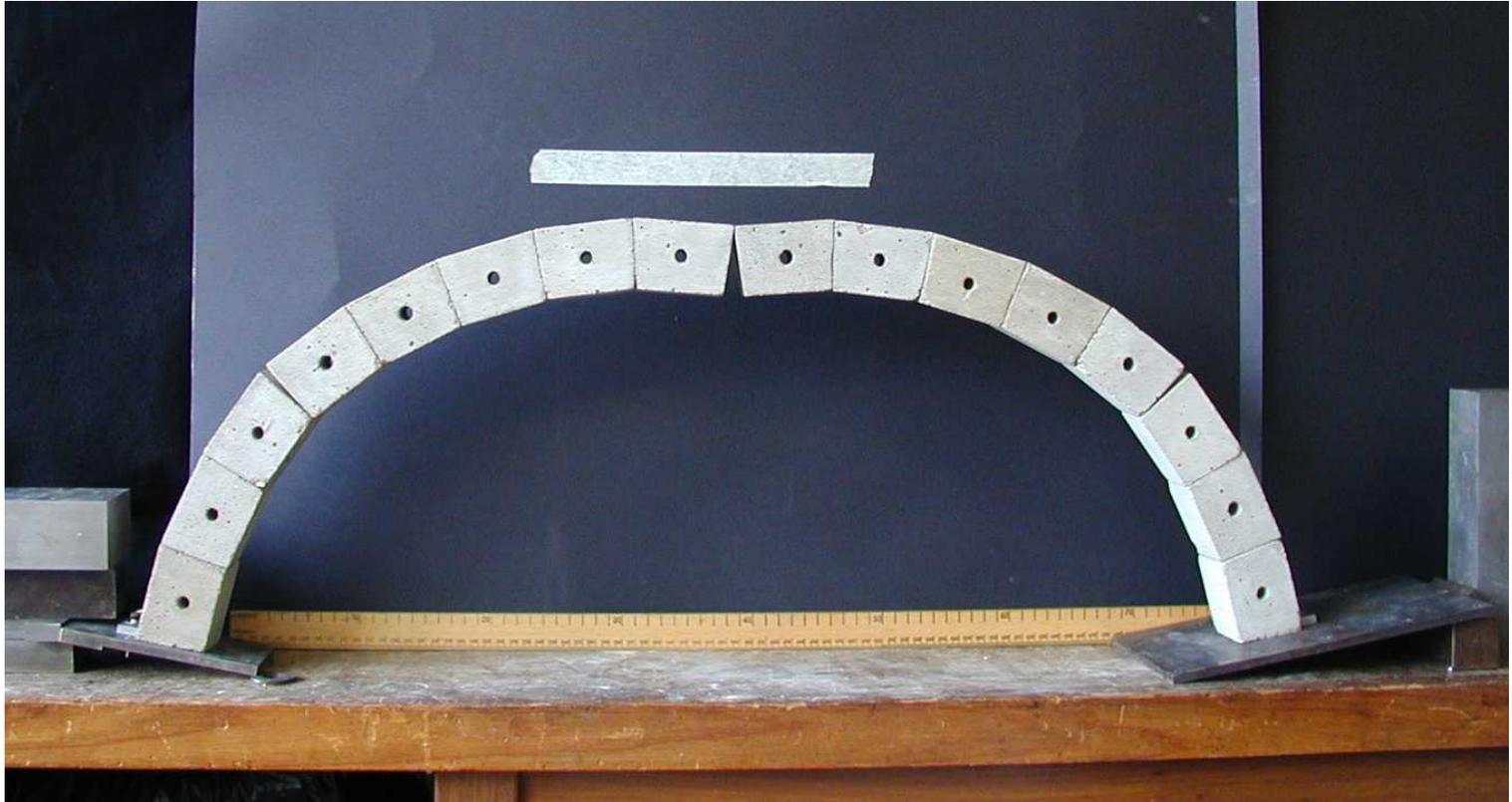
-Vibrations, earthquakes

# Design and Analysis of Unreinforced Masonry

- **Stability rather than failure of the material is the dominant concern**
- **Collapse occurs when the load path can no longer be contained within the masonry**



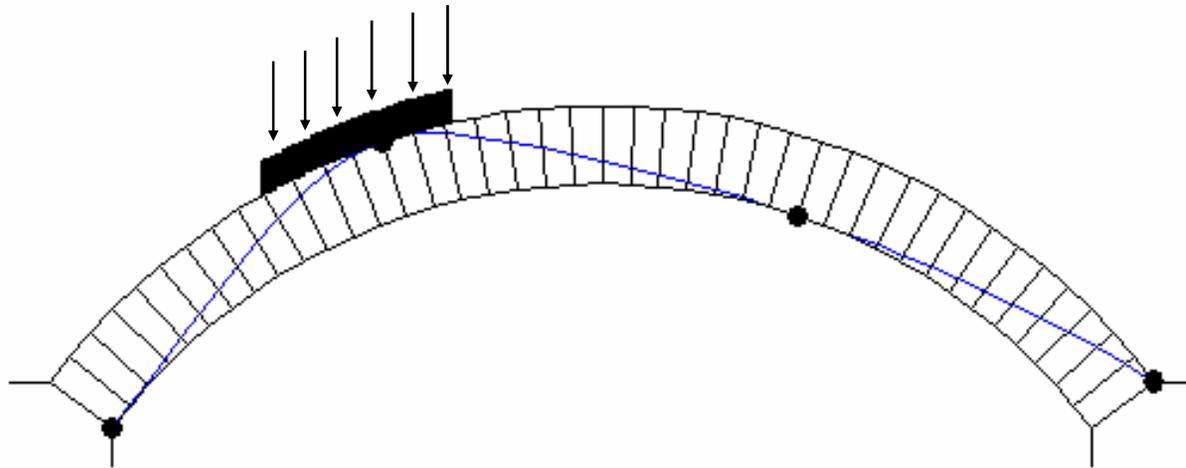
# Model Arch at Collapse State



# Single Span Stone Arch

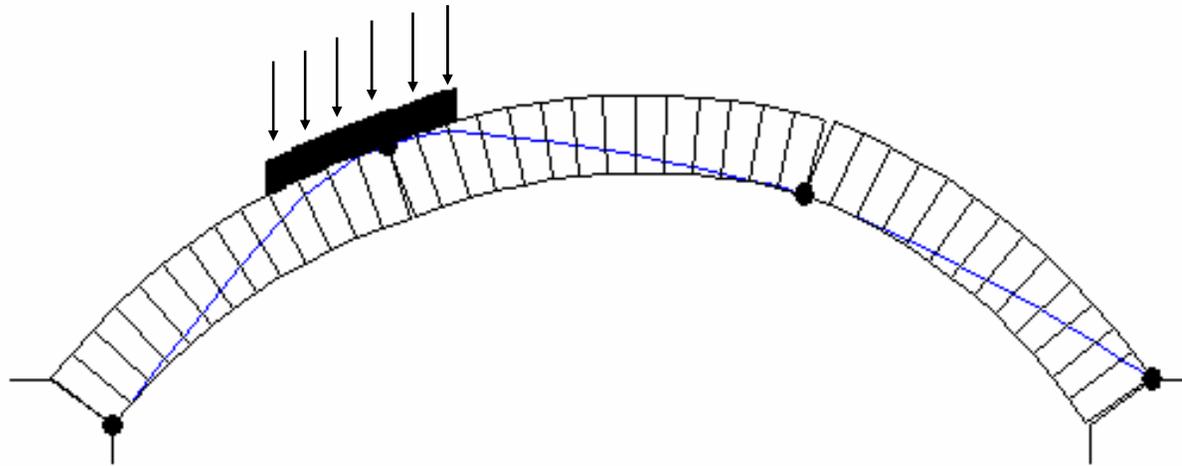


# Single Span Arch

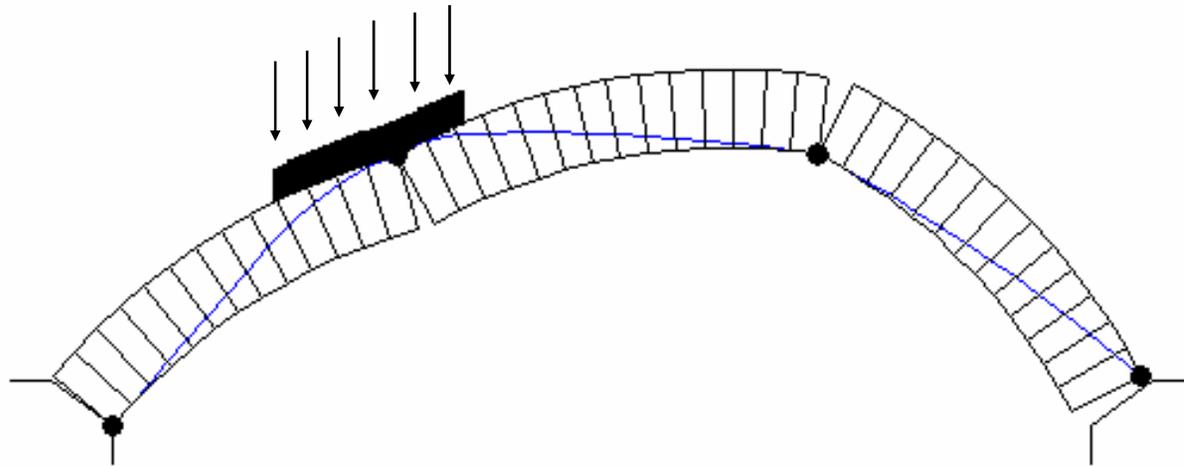


# Single Span Arch

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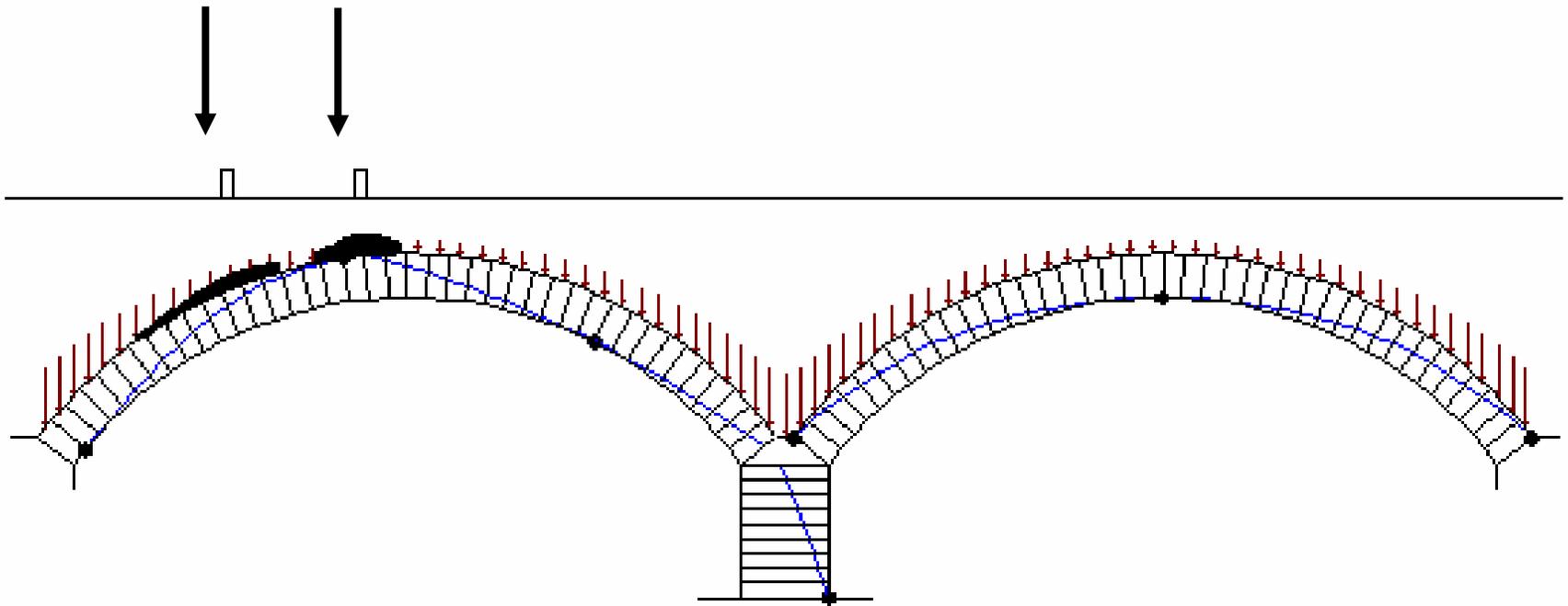
# Single Span Arch



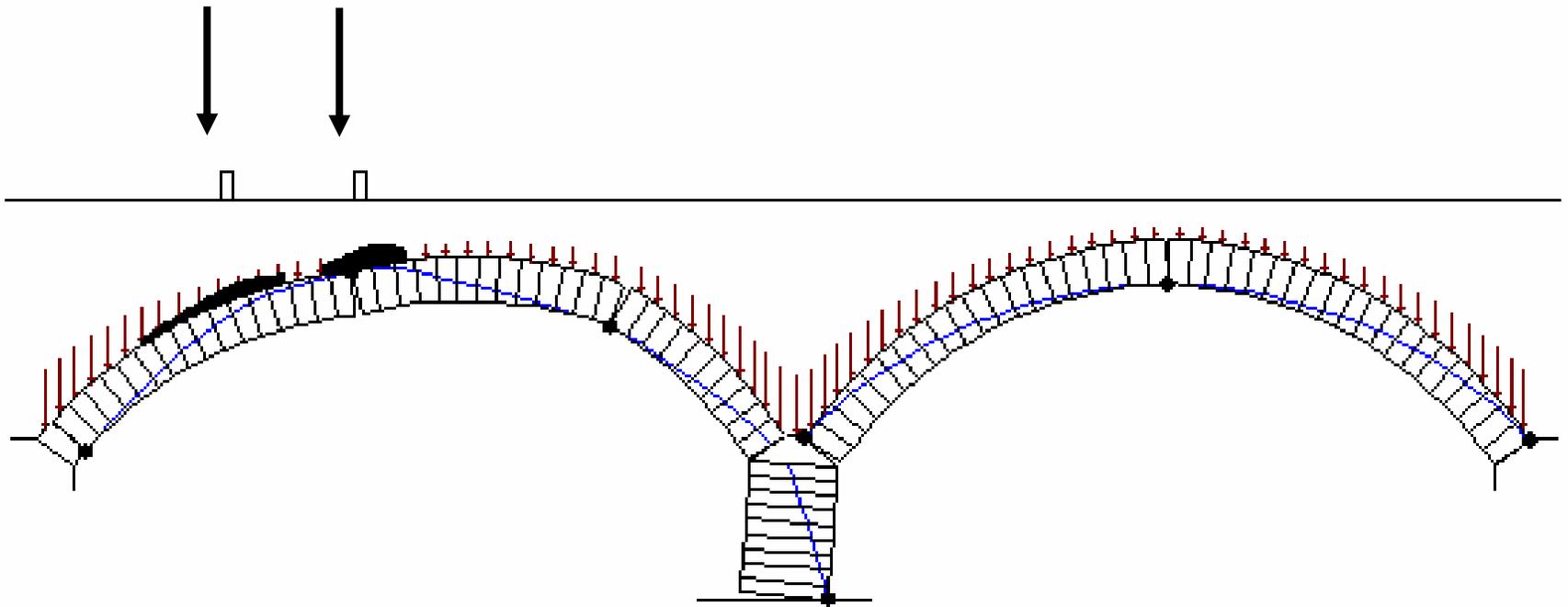
# Double Span Stone Arch



# Double Span Arch with Fill

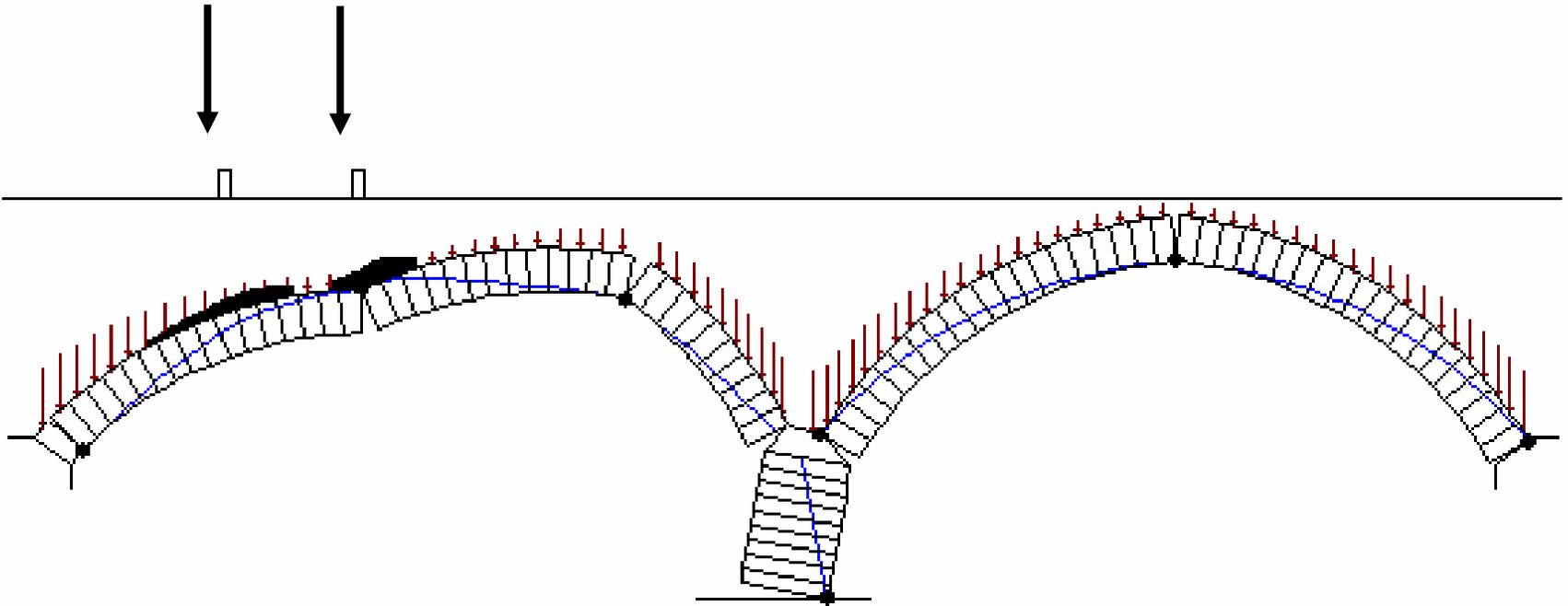


# Double Span Arch with Fill





# Double Span Arch with Fill



- This makes sense for bridges, but buildings don't usually have trucks driving on top of the vaults
- Deformation over time can cause collapse in buildings

# Conclusions

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- **Unreinforced masonry structures have very low stress levels: stability, not strength, governs the safety**
- **Determine collapse states based on thrust line analysis using graphic statics**
- **Equilibrium equations are most important when analyzing historical structures**