Flexibility

1. Introduction and Definition
2. Taxonomy of flexibility
3. Real Options & Other Financial Tools

Introduction and Definition

We have introduced the notion of the tradespace and the use of it to find Pareto optimal solutions. The idea of uncertainty was introduced as a different way of organizing the tradespace. It has also been found necessary to expand the tradespace to include other attributes: for example, flexibility, robustness etc.

While many space systems have proven to be very successful, as measured by meeting technical requirements, it is often the case that they have outlived their nominal design lives. For example, the DMSP satellites have lived many more years than they were designed for. The GPS satellites have an average life of over ten years even though they were designed for seven years. Part of the reason for this is that if a satellite makes it past it's infant mortality stage, the redundancy that is part of the design process combined with the design and operational workarounds for the space environment tend to be successful. Thus some satellite systems have ended up being used well beyond their design lives and in particular, have been used for missions somewhat different from what they were envisioned for. A good example is the DSP satellites. They were designed for the strategic mission of warning of attack from intercontinental ballistic missiles. They have actually been used in several conflicts for the detection of tactical missiles. This use required a substantially different architecture for use of the DSP results from that envisioned when it was designed. As another example, many commercial communication satellites routinely live 15 or more years. Indeed the limiting factor tends to be fuel on board the system for precision station-keeping. Even when this runs out and the satellite starts to drift (at geosynchronous orbit), it can be still be used for a while as long one has a tracking antenna in the ground architecture. As another consideration, the rate of change of digital technology is much shorter than the lifetime of many modern satellite systems. Note that this was not true in the early days of spaceflight when satellite lifetimes was measured in weeks. Given this, the satellite has to be designed to be using parts which will be generations out of date at the end of its life. These examples suggest that for long lived systems, the considerations of flexibility should be included in the architectural design. That is, given that the space system architecture may end up being used in different ways than originally envisioned, can the flexibility to do this be embedded in the design.

We start with a formal definition of flexibility. Flexibility is defined as the property of a system to respond to changes in initial requirements and objectives, after it has been fielded, in a timely and cost effective way[i]. This definition places the emphasis on change after the system has been fielded. Presumably change before the system is fielded would be accomplished through design modifications. As pointed out in Lamasourre & Saleh[ii], flexibility is necessary as a response to uncertainty in use. If we knew exactly how a

system would be used for its design lifetime and beyond, then it could and would be designed into the system from the beginning. We should note that the term flexibility has a number of definitions all related to the common sense use of the word. We should note also that the ESD definitions paper has a slightly different definition of flexibility. In this document, the definition of flexibility is
"The property of a system that is capable of undergoing changes with relative ease. Such changes can occur in several ways: a system of roads is flexible if it permits a driver to reach from one point to another using several paths; flexibility may indicate the ease of programming the system to achieve a variety of functions; flexibility may indicate the ease of changing the system's requirements with a relatively small increase in complexity or rework". This definition is more related to the underlying notions of a network and relates flexibility to the number of possible interconnections between two points. A system is flexible if it has a large number of interconnections or can form them very easily. In this sense the brain is flexible, since human beings have shown the ability to learn many new tasks through the ability to easily form new pathways in the brain. Nilchiani reviews many of the definitions.

Flexibility in space architectures has been achieved in several ways. Since space architectures always have space-based pieces and ground based pieces as well as software pieces, we can consider these parts of the architecture separately. The huge difference between them is that the ground-based piece is physically accessible and thus can be physically changed. By contrast, the hardware space based piece for the vast majority of space systems is physically inaccessible (with the obvious exception of systems that can be reached by the Space Shuttle) and thus changes in the system can only be accomplished through software uploads. This has been the primary way that flexibility has been built into the space parts of space system architectures. A wonderful example is Iridium. The satellites in this (failed) constellation were launched with minimal software loads and then the final software loads where installed on orbit. Of course, while flexibility due to software changes can accomplish a lot, it cannot accomplish things that need fundamental physical changes. For example, if a battery fails, a software change may reduce the need for the stored power in the battery but it will not replace the battery. As another example, new software algorithms may reduce the need for a high gain antenna (Galileo) but no software processing will turn an IR sensor into a UV sensor if the UV data was not originally collected.

Taxonomy of Flexibility

More generally, flexibility can be induced in a space system architecture in the following ways that follow a structured analysis of an architecture

1) Software changes at the spacecraft level e.g Galileo
2) Software changes at the ground station level e.g GPS
3) Changes in the communications link structure e.g DSP
4) Changes in constellation configuration e.g planned changes for Iridium
5) Additions to the constellation e.g smallsats which operate in close proximity to a larger satellite to replace communication links or enhance memory or add a new sensor

6) Changes to the ground station hardware e.g SBIRS High
7) Changes to the space based hardware which fall in two classes. These are changes which extend the life of a satellite, for example refueling or replacing a solar array with one of the same design & power. Then there are changes which give rise to new functionality on the spacecraft. These would include a change to improve the chip in a digital signal processor or a change to add an atomic oxygen cleaner to a GEO satellite. From the point of view of the utility function, in the first case the utility function does not change, it is extended in time and in the second case it needs to be redefined.

Flexibility is often cited as a virtue of a system. One earlier way to try and understand it was outlined in Shaw who argued that flexibility was really the ease of moving around on the design space formed from the cost per function metric. This was based fundamentally on the idea of a design surface and the ability to get from one design to another being measured locally by a partial derivative.

As pointed out in the Nilchiani review, it has been best developed in the manufacturing literature by people who have looked at production lines under variable demand. Saleh draws from this manufacturing literature and carefully distinguishes between the use of the word flexibility and what are often taken as synonym's i.e. adaptability and robustness. He shows that flexibility properly defined focuses on changes after a system is fielded in its requirements and its environment. In a follow on paper (Saleh1,2002), he argues that flexibility is necessary as a response to long lived systems in an uncertain environment. Ironically, space systems are so relatively long lived since they tend to have large amounts of redundancy placed on them to mitigate risk. In a sense, risk mitigation exposes them to the uncertain environment. This was a lesson learned in the early days of the space program, namely that it was critical to get beyond the infant mortality stage of these complex systems. This is accomplished through double and triple redundancy. For example, most modern satellites have an "A" side processor and a "B" side processor which are cross strapped so that the "B" side can take over for the "A" side. As another example, there are usually two paths from the propulsion tanks to the thrusters and multiple cross connected station keeping thrusters. The price of all this redundancy is extra mass to orbit but the benefit is on the average long lived systems.

Since the need for flexibility has been argued above and formally defined, a critical question is how to value it. This is essential since in the absence of a specific value associated with it, it will either be designed in accidentally or deliberately taken out as a response to budget pressures in design. That is, in the absence of quantitative tools for evaluation it will be seen as a "nice to have" feature rather than a "necessary feature". An interesting example is given from the Galileo spacecraft (which was saved ultimately by software flexibility).

The high gain antenna on Galileo was of an umbrella design with ribs which would deploy and carry the antenna with them. In the original design, as a "nice to have" flexibility feature, the central jackscrew which was the core of the umbrella design was driven by a motor that could be reversed so that the antenna could be opened and closed. However, in the critical design phase just before starting to build the system, the reversing motor was replaced by one that could only turn in one direction. This was done to save money and in part because no one could see the value of the reversing motor. Galileo was then launched with the single direction motor. When the command was given to deploy the antenna, this motor went to its maximum torque but was unable to deploy the antenna. It is believed that some of the ribs had vacuum welded to their housings and the motor could not exert enough torque to break the bonds. At this point, the value of the reversing motor became clear since with it, the controllers could have periodically stressed the bonds (by repeatedly reversing the motor). It is believed that this would have broken the vacuum bonds and freed the high gain antenna. However, it never was deployed and the ultimate "fix" to Galileo came from coming up with new data compression tools and using the backup omni-directional low gain antenna (put in for reasons of redundancy). It is clear that the decision on Galileo would have been different if it was possible to quantify the benefit of flexibility as well as the cost. When only the cost is quantified and the benefit is not then the tendency is to remove flexibility from the system as budgets get tighter.

The question of how to value flexibility has been answered in part by the development of the theory & language of real options to which we now turn. We should note that there may be several ways to value flexibility. The great advantage of the real options approach is that it allows the valuation of flexibility in the same context and manner as the net present value approach. Since this is well accepted, it allows the computation of value on a basis that can be broadly accepted. However, it is based fundamentally on

Real Options & Financial Valuation Tools

This section (taken from the SM of thesis of McVey[iii]) introduces the concepts behind real options, considers the benefits and downfalls of other financial valuation tools, investigates different scenarios that yield themselves to being valued using real options, and illustrates how real options can be used to evaluate projects in the aerospace industry. This section also includes an example valuation, comparing net present value, decision tree analysis, and real options.

Some of the tools shown below, namely net present value/discounted cash flow and real options are used throughout the remainder of this section to value the satellite servicing market. The net present value/discounted cash flow approach is used in the satellite servicing analysis in the McVey thesis to capture the value of each case before accounting for flexibility. The real options approach is utilized to take into account the inherent flexibility in satellite servicing. The background for their use is presented here.

What is the Real Options Approach?

The real options approach is a financial valuation technique that uses the concepts behind financial option pricing theory (OPT) to value "real" (non-financial) assets. It is a tool that can be used to value projects that have "risky" or contingent future cash flows, as well as long-term projects; projects that are typically undervalued by standard valuation tools.

An option is defined as the ability, but not the obligation, to exploit a future profitable opportunity.

Most projects have options embedded in them. These options give managers the chance to adapt and revise decisions based upon new information or developments. For example, if a project is determined to be an unprofitable venture for a company, the project can be abandoned. The option to abandon a project has value, especially when future investments are necessary to continue the project. The real options approach captures this value, along with the value of uncertainty in a project. Real options and option pricing theory will be used interchangeably throughout the remainder of the section.

How Does Real Options Compare to Standard Valuation Techniques?

Traditional Net Present Value (NPV)

NPV is a standard financial tool that compares the positive and negative cash flows for a project by using a discount rate to adjust future dollars to "current" dollars. The following equation can be used to calculate NPV.

$$NPV = \sum_{i=1}^{N} \frac{C_i}{(1+r)^i}$$

where r is the discount rate, C_i is the cash flow in period i, and N is the total number of periods. The discount rate is determined by the expected rate of return in the capital markets and accounts for the "riskiness" of the project.

Two major deficiencies exist in this method. Managerial flexibility is ignored, and the choice of discount rates is very subjective. Managers often use inappropriately high discount rates to value projects (Dixit[iv], 1994). In addition, NPV does not take into account the flexibility and influence of future actions inherent in most projects. Both using a high discount rate and ignoring the flexibility of using future "options" to make strategic decisions tend to lead to the under valuation of projects. However, one of the primary benefits of the NPV approach is that it is simple and understood by many people.

Discounted Cash Flow (DCF)

DCF is simply the sum of the present values of future cash flows. It has the same drawbacks as listed for NPV. It inherently assumes that an investor is passive. This means that once a project is started it will be completed without future strategic decisions based upon future information or outcomes. Thus, it typically leads to undervalued projects because it does not take into account the value of the options for future action. As with NPV, one its main benefits is its universal use. It is also adaptable to many types of projects.

Decision Tree Analysis

Decision analysis is a straightforward method of laying out future decisions and sources of uncertainty. It uses probability estimates of outcomes to determine the value of a project. By doing this, it is one of the few methods that takes into account managerial flexibility. The major downfall to this approach is that probability estimates are generally very subjective and as such are hard to form with much precision. The equations for this method are presented below in the example calculation.

Simulation Analysis

Simulation analysis lays out many possible paths for the uncertain variables in a project. Unfortunately, it is difficult to model decisions that occur before the final decision date using simulation analysis. This and the use of a subjective discount rate are the major drawbacks of this method of valuation.

Where can the Real Options Approach be Utilized?

The real options approach is a suitable method for valuing projects that:

- include contingent investment decisions
- have a large enough uncertainty that it is sensible to wait for more information before making a particular decision
- have a large enough uncertainty to make flexibility a significant source of value
- have future growth options
- have project updates and mid-course strategy corrections

As can be seen above, a real options analysis is not needed for all cases. Traditional methods of valuation correctly value businesses that consistently produce the same or slightly declining cash flow each year without further investment or follow-on opportunities. Real options are not necessary for projects with negligible levels of uncertainty. (Amram, 1999)

Where can Real Options be Utilized in the Aerospace Industry?

The following are hypothetical examples used to illustrate the value of real options.

Waiting-To-Invest Options

BizJet, a company that produces business jets, is considering becoming the first to enter the supersonic business jet market. It has the option to start development today or to wait until the market outlook changes. Real options can capture the value of delaying this decision until the market uncertainty is resolved.

Growth Options

CallSat, a company that offers satellite cellular phone service, is considering entering the market in the populated areas of South America. This would require a significant investment. If this investment is made it would leave the option open to increase service in the future to the less populated areas of South America if the market proved to be worthwhile. A real options analysis of this project would include the value of the future option to increase service area.

Flexibility Options

Entertainment Sat is considering developing a constellation of satellites that provides either standard satellite television service or a new pay-per-view downloadable movie service. Instrument A is needed on the satellite to provide television service and Instrument B is needed to provide downloadable movie service. Instrument C is more expensive than both A and B but it allows the satellite to provide either television or movie service. Real options can be used to value the flexibility of Instrument C, taking into account the fact that if one of the two markets proves to be less profitable than expected, or the opposite occurs, Instrument C has the ability to capture the most profitable market at any given time.

Exit Options

Sky ISP, a proposed satellite internet service provider, is interested in providing very fast internet connections throughout the US, using a constellation of satellites. Their fear is that the market is not large enough to support the substantial investment necessary to fund the development of satellites. Market forecasts look good today but what will they look like in a year when the satellites will be launched, requiring additional funding? Real options recognizes that the project can be abandoned if the market forecasts deteriorate. This option to abandon has value in that it limits the downside potential of a project.

Learning Option

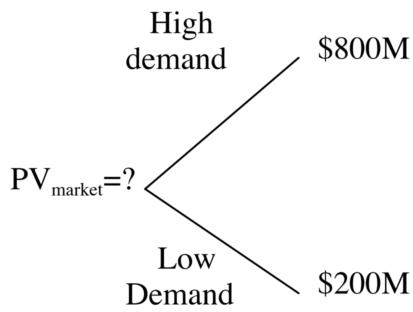
StarSat is doing research on a new tracking instrument that will help satellites point more accurately towards their target. There are several different levels of accuracy foreseen as feasible, each requiring an additional investment. StartSat has the ability to stage its investments in order to capitalize on learning effects. If through developing the first tracker they gain knowledge about how to develop the next tracker, the future investment can be altered. The real options approach values the contingent decisions based upon the learning curve that StarSat faces.

Valuations: Using the Binomial Real Options Approach

This section will walk the reader through a simple example of valuation to illustrate the differences between net present value, decision tree analysis, and real options. The reader should take note of a few key points throughout the example. First, the NPV approach does not correctly value options because it assumes that once a project is started, it will be completed regardless of the outcome. Second, DTA and OPT valuations both take into account managerial flexibility, but do not result in the same answers. This is due to the way the two methods discount the value of options. DTA uses the same discount rate to discount the underlying project as well as the options. Since an option is always more risky than its underlying asset (Brealey and Myers, 1996), OPT valuations discount the option at a higher rate. This is more consistent with the theory that riskier cash flows should be discounted at a higher rate.

The valuation will be based upon the following scenario:

> Sky ISP, as introduced previously, faces the following scenario. The market outlook for one year from now will either have high or low demand. If the demand for Internet connections is high, the market will be worth $800M and if the demand is low the market will be worth $200M. The satellites will be launched in one year for a cost of $300M. An initial investment of $250M must be made today in order to continue building the satellites needed to complete the system.
>
> In financial market terms, the launch scenario corresponds to owning a call option on a stock with a price equal to the value (see calculation below) of the market and an exercise price of $300M (the cost of launching the satellites).



The market outcomes are illustrated in Figure 0-1.

Figure 0-1 Predicted Market Outcomes for Year 1

The information needed in the analysis to follow is summarized here ($M):

• Initial investment today to continue building satellites, I:	$250
• Present value of market without option to launch (calculation below), S:	$509
• Future value of market with high demand, uS:	$800
• Future value of market with low demand, dS:	$200
• Probability of high demand, p:	60%
• Probability of low demand, 1-p:	40%
• Discount rate, r:	10%
• Exercise price, E:	$300
• Maturity, t:	1 year
• Risk-free interest rate, r_f:	5%

In this example, a distinction will be made between the market and the project. The value of the market is defined as the amount of money a business would make if entering the market had zero costs associated with it. The value of the business/project is defined as the value of the market minus the cost of entering the market.(i.e. the exercise price of the option). In this case, the cost of entering the market is the cost of launch. The present value of the market is:

$$PV = \frac{p(uS) + (1-p)(dS)}{(1+r)} = \frac{(0.6)(\$800) + (0.4)(\$200)}{1.1} = \$509$$

Net present value calculation

The NPV of the business is found using the following formula.

$$NPV = p\frac{uS - E}{1+r} + (1-p)\frac{dS - E}{1+r} - I$$

This formula simply takes the value of the project in year 1 and discounts it back to year 0. Using the assumptions above the net present value of the project is:

$$NPV = 0.6 \times \frac{\$800 - \$300}{1.1} + 0.4 \times \frac{\$200 - \$300}{1.1} \text{-\$250=-\$14}$$

The NPV valuation assumes that the option is exercised regardless of the market outcome. This is obviously flawed because a rational manager would not choose to launch the satellites if the demand were lower than the cost of launch. This leads to a negative NPV valuation.

Decision tree analysis calculation

Traditionally, this project would either not be undertaken because of its negative valuation, or a manager would go with his/her "gut" feeling that Sky ISP is a worthwhile project. Although this project is worthwhile as long as one considers the options (a.k.a. managerial flexibility) involved, it would be helpful to be able to quantify the manager's "gut" feeling. One method of remedying this is to use decision tree analysis. In finance terms, this method recognizes the manager's ability to not exercise the call option (i.e. launch the satellites) if the demand is low. This is illustrated below, where circles represent event nodes and squares represent decision nodes. The bold lettering indicates what decision a rational manager would make in the given situation.

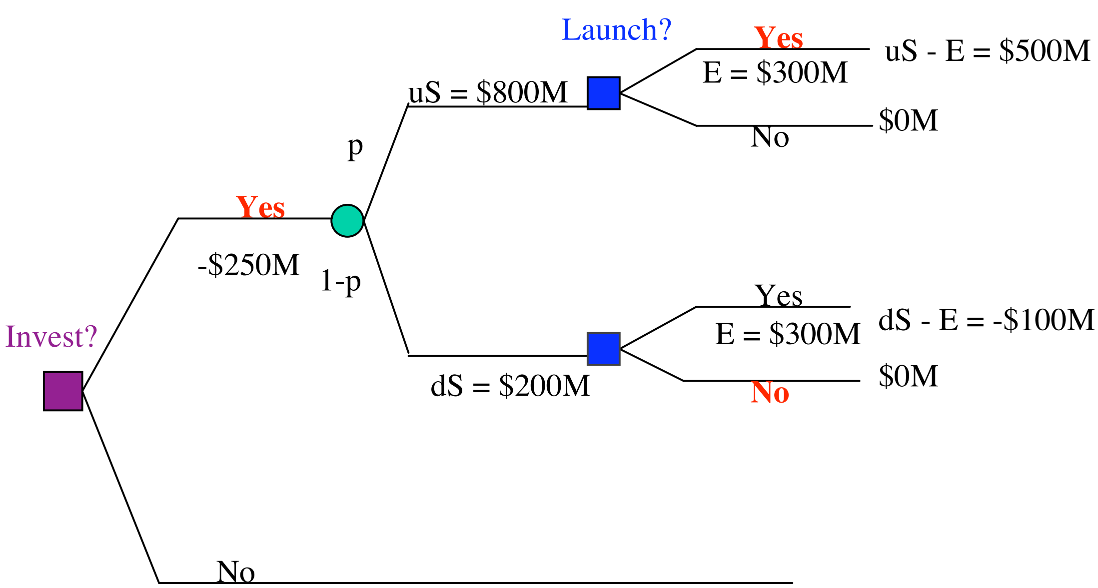


Figure 0-2: Decision Tree used for Decision Analysis and Real Options Valuation

The value of this project, according to decision analysis, is calculated using the following formula.

$$V_{DTA} = \frac{p \times \max(uS - E, 0) + (1 - p) \times \max(dS - E, 0)}{(1 + r)^t} - I$$

In this case, the decision tree analysis method gives:

$$V_{DTA} = \frac{(0.6)\max(\$800 - \$300, 0) + (0.4) \times \max(\$200 - \$300, 0)}{1.1} - \$250$$

$$= \frac{0.6(\$500) + 0.4(0)}{1.1} - \$250 = \$23$$

This valuation is significantly higher than the NPV approach because it assumes that the project would be abandoned if the launch costs exceeded the size of the market.

Real options calculation

The final approach covered here is real options. Using the binomial method (Brealey and Myers, 1996), there are two ways to approach this valuation. The one that will be used here is the risk-neutral approach.

The risk-neutral approach is based on the surprising fact that the value of an option is independent of investors' preferences towards risk. Therefore, the value of the option in a risk-neutral world, where investors are indifferent to risk, equals the option's value in the real world.

If Sky ISP were indifferent to risk, the manager would be content if the business offered the risk-free rate of return of 5%. The value of the market is either going to increase to \$800, a rise of 57%, or decrease to \$200, a fall of 61%.

$$\begin{matrix} Expected \\ Return \end{matrix} = \begin{pmatrix} Probability \\ of \quad rise \end{pmatrix} \times 57\% + \begin{pmatrix} 1 - \begin{matrix} Probability \\ of \quad rise \end{matrix} \end{pmatrix} \times (-61\%) = 5\%$$

This yields a probability of rise in the risk-neutral world of 56%. The *true* probability of the market rising is 60%. However, options are always riskier than their underlying asset (i.e. the project itself), which leads to the use of different probabilities for valuation. The use of risk-neutral probabilities effectively increases the discount rate used to value the option.

If there is low demand in the market, the market with the option to launch will be worth nothing. On the other hand, if the demand in the market is high, the manager will choose to launch and make 800-300 = 500, or \$500M. Therefore, the expected future value of the market with the option is

$$\begin{pmatrix} Probability \\ of \quad rise \end{pmatrix} \times 500 + \begin{pmatrix} 1 - \begin{matrix} Probability \\ of \quad rise \end{matrix} \end{pmatrix} \times 0 = (.56 \times 500) + (.44 \times 0) = \$280$$

Still assuming a risk-neutral world, the future value is discounted at the risk-free rate to find the current value of the project with the option to launch as

$$\frac{Expected \quad future \quad value}{1 + r_f} - I = \frac{280}{1.05} - 250 = \$16$$

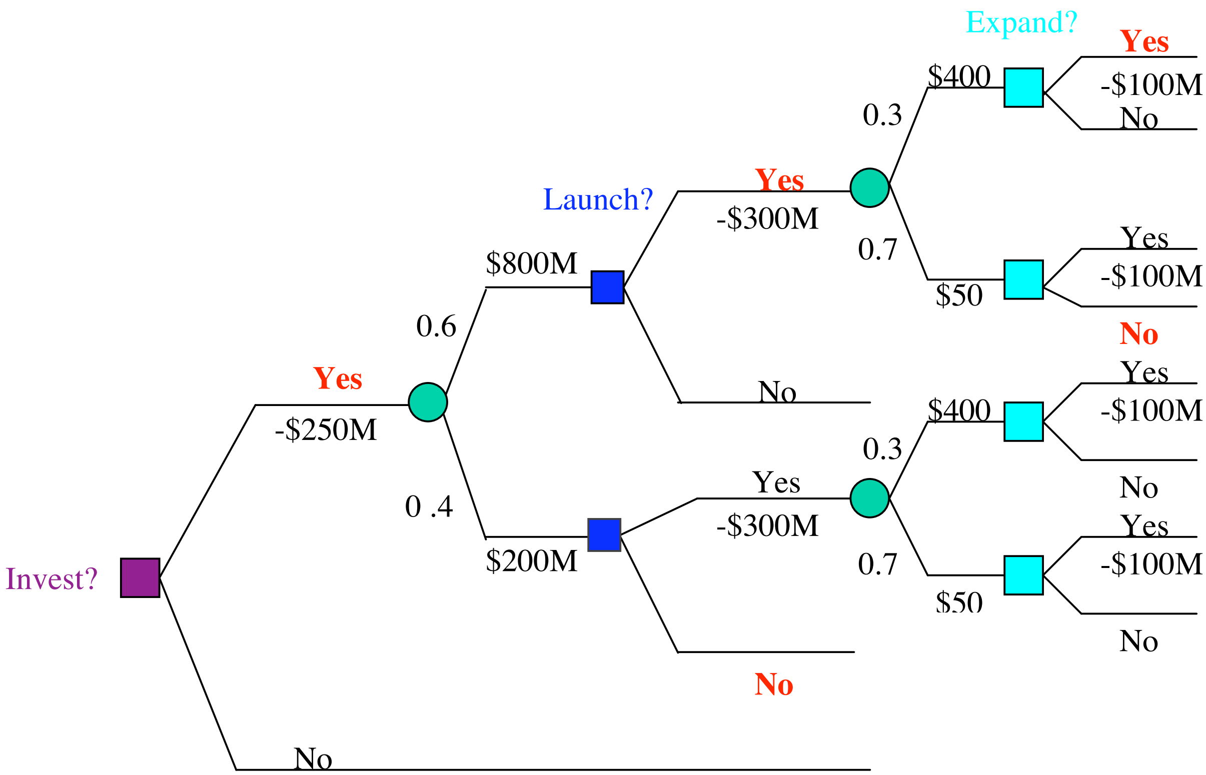
The value of the option to launch is the difference between the value of the business with the option (the OPT valuation) and the value of the business without the option (i.e. the NPV).

$$\begin{matrix} Value \quad of \quad business \\ with \quad launch \\ option \end{matrix} - \begin{matrix} Value \quad of \quad business \\ without \quad launch \\ option \end{matrix} = \begin{matrix} Value \\ of \\ option \end{matrix}$$

$$= \$16M - (-\$14M) = \$30M$$

Multiple option example

Although single options are often very important to analyze, as they may be all that a business faces, multiple or compound options are generally more interesting. Adding another option to the one discussed above produces interesting results. Assume that one-year after the launch decision is made, a new satellite data transfer market emerges. This market has a 30% chance of being worth \$400M and a 70% of being worth \$50. The operations and marketing costs of entering this new market amount to $100M. The situation is illustrated graphically in the figure below, where the probabilities shown are the probabilities used in the NPV and decision analysis valuations. The risk-neutral



probabilities, used in OPT, are not shown.

Figure 0-3 Example of Multiple Options

Although the calculations will not be covered in detail here, the NPV, DTA, and OPT valuations are listed below for various cases. In the analysis the following are used:

- uS_1: Future value of original market with high demand
- dS_1: Future value of original market with low demand
- E_1: Exercise price of option to launch (cost of launch)
- uS_2: Future value of new data transfer market with high demand
- dS_2: Future value of new data transfer market with low demand
- E_2: Exercise price of option to expand (cost of expansion)
- r_1: Discount rate for launch option = 10%
- r_2: Discount rate for option to expand = 15%
- r_r: Risk-free interest rate = 5%

All numbers below are in $M.

Table 0-1 Various Cases of Two Option Valuation

Case number	uS_1	dS_1	E_1	uS_2	dS_2	E_2	NPV	DTA	OPT	Option to launch	Option to expand
1	**800**	**200**	**300**	**400**	**50**	**100**	**30**	**65**	**52**	**22**	**75**
2	800	200	300	**600**	50	100	77	105	89	11	127
3	800	**50**	300	400	50	100	-25	65	59	84	75
4	800	200	**400**	400	50	100	-61	11	0	61	75
5	800	200	300	400	50	**200**	-14	51	40	54	50

Case 1 is the baseline case for the rest of the analysis. It illustrates how the addition of the option to expand significantly increases the value of the project.

Case 2 illustrates the effect of increasing the upside potential of the option to expand. As can be seen above, it significantly increases the value of the project. It also makes the option to launch worth less because even if the initial demand were low, the option to expand makes it worthwhile to launch the satellites.

Case 3 illustrates the effect of decreasing the future value of the original market with low demand. In this case, the value of the option to launch increases because one can choose not to launch if the demand were low. As expected, the DTA value of the project does not change because the manager would only launch if the demand were high.

Case 4 illustrates the effect of increasing the exercise price of the option to launch (i.e. increasing the launch costs). The NPV valuation becomes negative, while the OPT valuation goes to zero. The reason that the DTA valuation remains positive is due to the way in which discounting takes place.

Case 5 illustrates the effect of increasing the exercise price of the option to expand (i.e. increasing the cost of entering the new data transfer market). The NPV is much more negative than the DTA or OPT valuations because it does not correctly value options. In addition, this case is a good example of the DTA valuation being greater than the OPT valuation. This is due to the different ways that DTA and OPT treat discount rates.

Extension to the Black-Scholes Formula

Thus far, all quantitative discussion of real options in this thesis is based upon the binomial method. This is a simplified version of option pricing theory that assumes that there are only two possible outcomes for a project. Although this method can be used to value options over short time periods or in very special cases where only two outcomes are possible, it is often unrealistic.

One means of solving this issue is to break the total time period into smaller intervals. For an example of this refer to Brealey and Myers, 1996. As the time interval period used for each option shortens, the valuation becomes more realistic because more outcomes are possible. Ideally one would keep shortening the interval periods until eventually the stock price (or project value) varies continuously. This leads to a continuum of possible outcomes. Fortunately, this is exactly what the Black-Scholes formula, which the authors were awarded the 1997 Nobel Prize in Economics for, does. The formula is

$$V_{OPTION} = P \times N(d_1) - PV(E) \times N(d_2)$$

where

$$\begin{cases} N(x) = \int \frac{1}{\sqrt{2\pi}} e^{-y^2} dy \\ d_1 = \frac{\left[\ln(P/E) + \left(r + \sigma^2/2\right) \times t\right]}{\sigma\sqrt{t}} \\ d_2 = d_1 - \sigma\sqrt{t} \end{cases} \qquad N : cumulative \quad normal \quad distribution \quad function$$

and

P = share price (value of project)

r = risk-free interest rate

PV(E) = present value of exercise price of option (discounted using risk-free rate)

t = number of periods to exercise date

_ = volatility of the share price per period of rate return (continuously compounded) on stock

In addition to accounting for the fact that projects generally have a continuum of possible outcomes, the Black-Scholes formula does not require an arbitrary discount rate. Although the binomial method does not technically use a discount rate, using a discount rate is almost always inevitable to determine the present value of the price of the stock (i.e. project).

---

[i] J. Saleh, E. Lamassoure and D. Hastings, " Space Systems Flexibility provided by On-Orbit Servicing I", Journal of Spacecraft and Rockets, Volume 39, Number 4, p551-560, (2002)

[ii] E. Lamassoure, J. Saleh and D. Hastings, " Space Systems Flexibility provided by On-Orbit Servicing II", Journal of Spacecraft and Rockets, Volume 39, Number 4, p561-570, (2002)

[iii] McVey, M, “Valuation Techniques for Complex Space Systems: An Analysis of a Potential Servicing Market”, SM thesis, Dept of Aero and Astro, MIT, 2002

[iv] Dixit, A & Pindyck, R “Investment under Uncertainty” Princeton University Press, Princeton, 1994