Matrix Experiments Using Orthogonal Arrays

Comments on HW#2 and Quiz #1

Questions on the Reading

Quiz

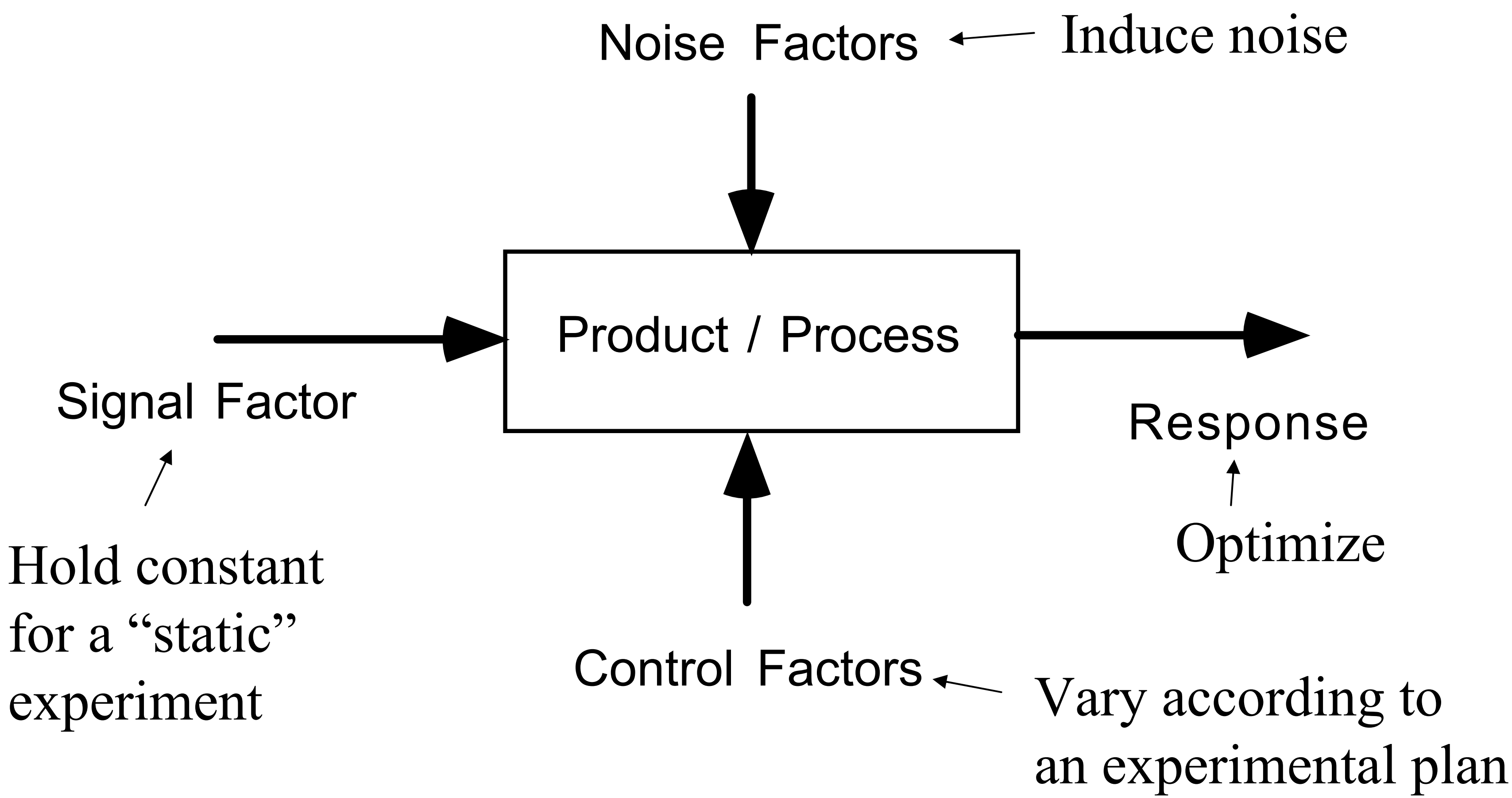
Brief Lecture

Paper Helicopter Experiment

Learning Objectives

- Introduce the concept of matrix experiments
- Define the balancing property and orthogonality
- Explain how to analyze data from matrix experiments
- Get some practice conducting a matrix experiment

Static Parameter Design and the P-Diagram



Parameter Design Problem

- Define a set of control factors (A,B,C…)
- Each factor has a set of discrete levels
- Some desired response η (A,B,C…) is to be maximized

Full Factorial Approach

- Try all combinations of all levels of the factors ($A_1B_1C_1$, $A_1B_1C_2$,...)
- If no experimental error, it is guaranteed to find maximum
- If there is experimental error, replications will allow increased certainty
- BUT ... #experiments = $\#levels^{\#control\ factors}$

Additive Model

- Assume each parameter affects the response independently of the others

$$\eta(A_i, B_j, C_k, D_i) = \mu + a_i + b_j + c_k + d_i + e$$

- This is similar to a Taylor series expansion

$$f(x,y) = f(x_o, y_o) + \left.\frac{\partial f}{\partial x}\right|_{x=x_o} \cdot (x - x_o) + \left.\frac{\partial f}{\partial y}\right|_{y=y_o} \cdot (y - y_o) + \text{h.o.t}$$

One Factor at a Time

	Control Factors				
Expt. No.	A	B	C	D	
1	2	2	2	2	η_1
2	1	2	2	2	η_2
3	3	2	2	2	η_3
4	2	1	2	2	η_4
5	2	3	2	2	η_5
6	2	2	1	2	η_6
7	2	2	3	2	η_7
8	2	2	2	1	η_8
9	2	2	2	3	η_9

Orthogonal Array

	Control Factors				
Expt. No.	A	B	C	D	
1	1	1	1	1	η_1
2	1	2	2	2	η_2
3	1	3	3	3	η_3
4	2	1	2	3	η_4
5	2	2	3	1	η_5
6	2	3	1	2	η_6
7	3	1	3	2	η_7
8	3	2	1	3	η_8
9	3	3	2	1	η_9

Notation for Matrix Experiments

$$L_9\,(3^4)$$

Number of experiments

Number of levels

Number of factors

$$9=(3-1)\times 4+1$$

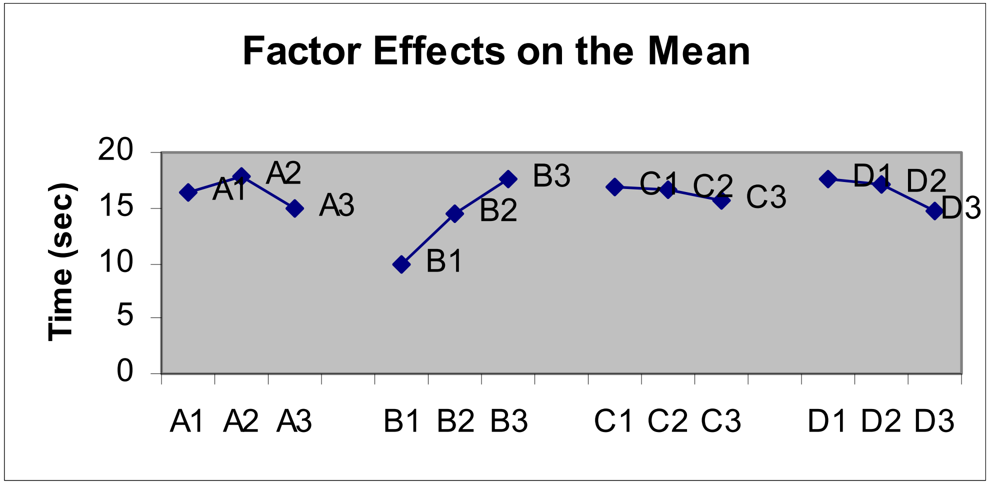
Why is this efficient?

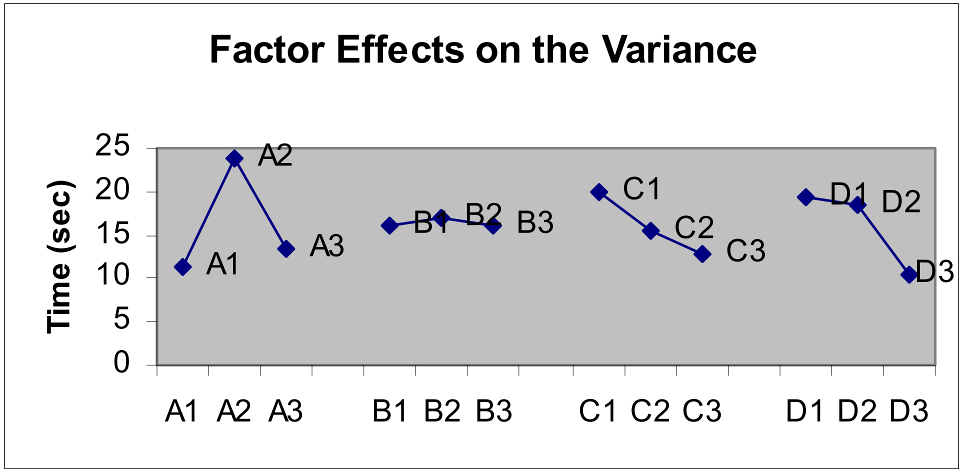
- One factor at a time
  - Estimated response at A_3 is $\eta_3 = \mu + a_3 + e_3$
- Orthogonal array
  - Estimated response at A_3 is

    $\eta_3 = \mu + a_3 + 1/3(e_7 + e_7 + e_7)$
  - Variance sums for independent errors
  - Error variance ~ 1/replication number

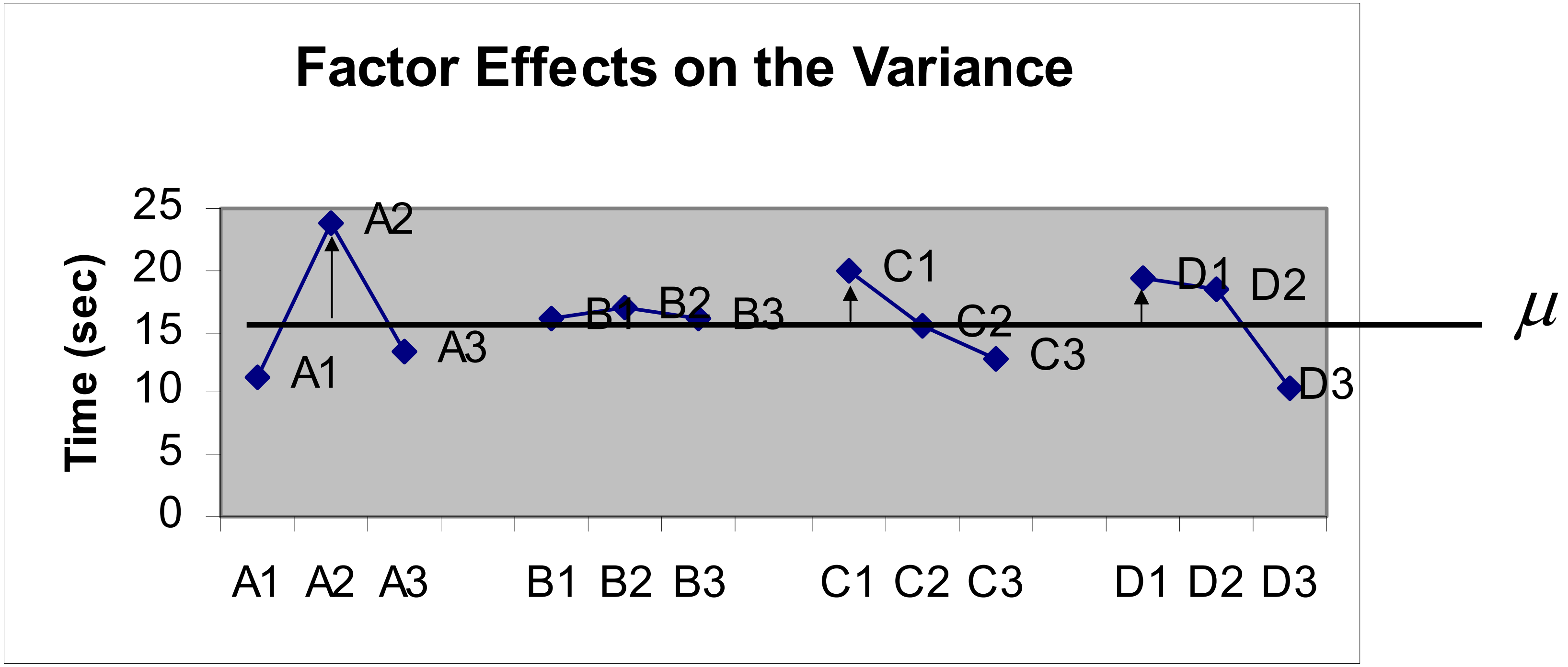
Factor Effect Plots

- Which CF levels will you choose?
- What is your scaling factor?

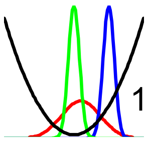




Prediction Equation



$$\eta(A_i, B_j, C_k, D_i) = \mu + a_i + b_j + c_k + d_i + e$$



Inducing Noise

	Control Factors				
Expt. No.	A	B	C	D	
1	1	1	1	1	η_1
2	1	2	2	2	η_2
3	1	3	3	3	η_3
4	2	1	2	2	η_4
5	2	2	3	1	η_5
6	2	3	1	2	η_6
7	3	1	3	3	η_7
8	3	2	1	3	η_8
9	3	1	2	1	η_9

{

	Noise Factor
Expt. No.	N
1	1
2	2

Analysis of Variance (ANOVA)

- ANOVA helps to resolve the relative magnitude of the factor effects compared to the error variance
- Are the factor effects real or just noise?
- I will cover it in Lecture 7
- You may want to try the Mathcad “resource center” under the help menu