

Solution to Quiz #6

Constructing Orthogonal Arrays

NOTE: You may wish to refer to the table of orthogonal arrays on the last page.

The first four problems relate to the catapult experiment. The control factors and their levels are:

Cup position – High, Medium, Low

Post Pin – Up, Middle, Down

Stop pin – Fore, Aft

Draw angle -- 10, 20, and 30 degrees

- 1) If you are interested in main effects only, how many degrees of freedom does the system have?

1 for the grand mean

$3 \times (3-1)$ for three 3 level factors

$1 \times (2-1)$ for one 2 level factor

8 DOF Total

- 2) What experiment would you conduct to resolve these main effects? Explain your choice and name any techniques you applied.

The L8 will not work without modification since it has no three level factors.

The L9 is probably the best choice. Apply the dummy level technique to turn the two level factor, Stop Pin, into a three level factor.

An L18 is the smallest matrix that will accommodate this problem without modification. However, this is so much more experimentation than the L9 that, unless interactions are suspected, it is probably not the best choice.

- 3) If you are interested in post position / draw angle interactions in addition to main effects, how many degrees of freedom does the system have?

$(3-1) \times (3-1) = 4$ Additional DOF

Therefore, the entire system has $8+4=$ **12 DOF**

4) What experiment would you conduct? How many columns of what type (i.e., two factor, three factor, etc) would you leave unassigned?

The L12 will not work since it has no three level columns.

The L16 will not work for the same reason.

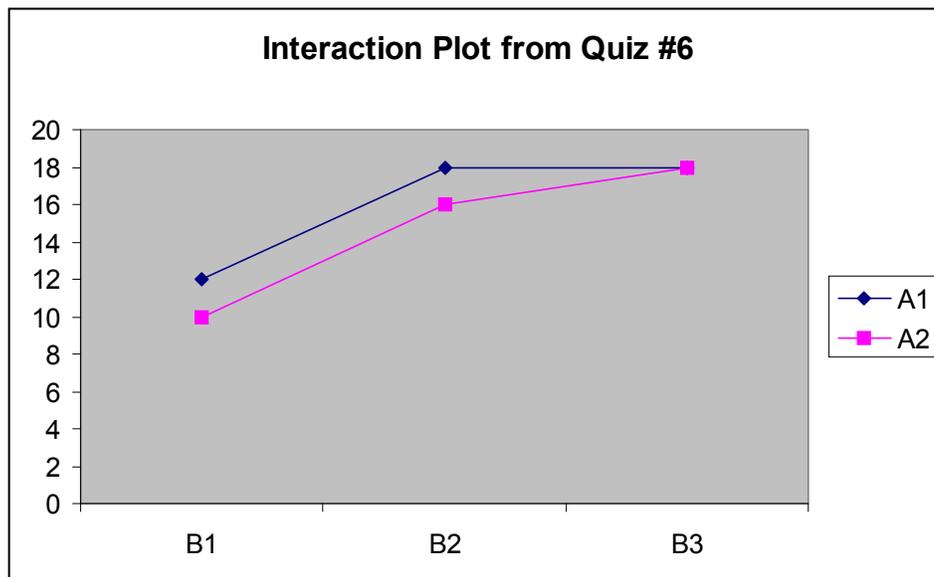
The L16' (4^5) was chosen by many students. You can indeed use dummy levels to assign all four control factors to the columns of this array. You will still have one column to leave unassigned. However, that column will not allow you to resolve interactions between the two three level factors. The interaction between two three level factors requires $(3-1)*(3-1)=4$ DOF. The unassigned column has only $(4-1)=3$ DOF. IN fact, since you used dummy levels to make the three level factors into four level factors, you'd need $(4-1)*(4-1)=9$ DOF or three 4 level columns $3*(3-1)=9$ DOF to study the interactions. The L16 just won't work for this system if you want to study that interaction.

The L18 was chosen by most students. Since it has seven three level columns, you can leave two 3 level columns empty giving you the required 4DOF you need. In fact, you'd have four columns left, so you could study two more 3 level factors. HOWEVER, if you study the linear graph of the L18, you'll see that it is not possible to study any three level by three level interactions with the L18. Since you didn't have the linear graph for reference, you got full credit for answering L18.

But, in fact, **you'd need to use the L27 to study all the factors and interactions specified in this problem.** One choice is to assign Draw angle and Post pin to columns 1 and 2. Leave columns 3 and 4 empty to allow the study of interactions between columns 1 and 2. Finally, use a dummy level to assign the two level factor to one of the remaining three level columns. This seems inefficient, but it seems to be the best solution based on the standard arrays.

5) Given the data from the L18 below, sketch the interaction plot for factors 1 and 2.
 Note: The L18 has the special property that interaction between columns 1 and 2 is orthogonal to the other columns. Hence, the interactions between 1 and 2 can be resolved even when no columns are left unassigned.

Exp. No.	Columns								η (dB)
	1	2	3	4	5	6	7	8	
1	1	1	1	1	1	1	1	1	12
2	1	1	2	2	2	2	2	2	13
3	1	1	3	3	3	3	3	3	11
4	1	2	1	1	2	2	3	3	18
5	1	2	2	2	3	3	1	1	17
6	1	2	3	3	1	1	2	2	19
7	1	3	1	2	1	3	2	3	18
8	1	3	2	3	2	1	3	1	20
9	1	3	3	1	3	2	1	2	16
10	2	1	1	3	3	2	2	1	10
11	2	1	2	1	1	3	3	2	9
12	2	1	3	2	2	1	1	3	11
13	2	2	1	2	3	1	3	2	17
14	2	2	2	3	1	2	1	3	16
15	2	2	3	1	2	3	2	1	15
16	2	3	1	3	2	3	1	2	19
17	2	3	2	1	3	1	2	3	18
18	2	3	3	2	1	2	3	1	17



Standard Orthogonal Arrays

Orthogonal Array	Number of Rows	Maximum Number of Factors	Maximum Number of Columns at These Levels			
			2	3	4	5
L_4	4	3	3	-	-	-
L_8	8	7	7	-	-	-
L_9	9	4	-	4	-	-
L_{12}	12	11	11	-	-	-
L_{16}	16	15	15	-	-	-
L'_{16}	16	5	-	-	5	-
L_{18}	18	8	1	7	-	-
L_{25}	25	6	-	-	-	6
L_{27}	27	13	1	13	-	-
L_{32}	32	31	31	-	-	-
L'_{32}	32	10	1	-	9	-
L_{36}	36	23	11	12	-	-
L'_{36}	36	16	3	13	-	-
L_{50}	50	12	1	-	-	11
L_{54}	54	26	1	25	-	-
L_{64}	64	63	63	-	-	-
L'_{64}	64	21	-	-	21	-
L_{81}	81	40	-	40	-	-