

Solution to Quiz #5 Analysis of Variance

- 1) In Taguchi methods, the F value for a factor effect is defined as
- The probability that we reject the hypothesis that the factor effect equals zero.
 - The sum of squares due to a factor divided by the error variance.
 - The mean square due to a factor divided by the error variance.
 - The sum of squares due to a factor divided by the DOF for the factor.

C is the correct answer. It is very simply the definition of the F value in Taguchi methods. Note that in traditional one-way ANOVA, the F test statistic is defined differently

$$F := \frac{\left(\frac{SSB}{dfB}\right)}{\left(\frac{SSW}{dfW}\right)}$$

Where SSB is the sum of squares within treatments and SSW is the sum of squares within treatments.

A is incorrect. To compute a probability from an F test statistic, you need to integrate the F distribution over the critical region. This can be done in Mathcad with the pF function or in excel with the FDIST function. F statistics are not normally used to compute probabilities in robust design.

B is incorrect. The sum of squares due to a factor divided by the DOF of the factor is the mean square due to a factor. This is the key difference. You should note the similarity in form between the F as defined in one way ANOVA and the Taguchi use of ANOVA. Both the numerator and denominator are divided by DOF.

D is incorrect. The error variance needs to be in the formula.

- 2) In Taguchi methods, “pooling” the variance of the factor effects with the lowest contributions to the total sum of squares:
- a) Provides an unbiased estimate of error variance.
 - b) Provides an approximate estimate of error variance without the need for replicates.
 - c) Reduces the error variance thereby increasing the efficiency of a matrix experiment.
 - d) Cannot be performed before the confirmation experiment is complete.

B is the correct answer. According to Phadke (pg. 57), “an approximate estimate of error variance can be obtained by pooling the sum of squares corresponding to the factors having the lowest mean square”. No replicates are required for the procedure since the sum square due to a factor measures variation between rows of a matrix experiment, not within rows.

A is incorrect. According to Phadke (pg. 57), “by the traditional statistical assumptions, pooling gives a biased estimate of error variance.” The procedure for making an unbiased estimate based on replicates of test conditions is outlined in Box, Hunter, and Hunter, “Statistics for Experimenters”, p. 319. It involves pooling error from replicates within all the treatments of an experiment.

C is incorrect. The only way to reduce error variance is to conduct experiments more carefully and make more careful measurements.

D is incorrect. The Taguchi style ANOVA can be completed before the confirmation experiment. In fact, it may be useful to do so because it may influence the choice of some of the control factor levels. If a control factor is judged to be insignificant in its effect on S/N, you may want to set it at the least expensive level.

The **data** below represent the results from an L9 with one noise factor in an outer array and three replicates. The **table** below the **data** represents the results of a one way (single factor) ANOVA ($\alpha=0.05$) with the first row representing the first sample and the second row representing the second sample. Note that this is not the usual use of ANOVA in Taguchi methods.

3) What does the F crit value represent?

The F crit value is the lowest value of F for which we will accept the null hypothesis (at $\alpha=0.05$) that the samples in the two rows were taken from populations with the same mean. The null hypothesis in this case is a test that the control factor settings tested in rows 1 and 2 did not have a significant effect on the mean response of the system under the noise conditions we induced.

4) What conclusion can you draw from this analysis?

We reject the null hypothesis that the two samples were drawn from populations with the same mean. In other words, based on this data, we are not 95% certain that the two treatments change the mean response of the system.

5) Explain a few differences between this analysis and a Taguchi style use of ANOVA.

This use of ANOVA assigns probabilities to hypotheses. Taguchi ANOVA does not.

This use of ANOVA assumes the samples come from a population that is normally distributed. Taguchi style ANOVA does not require this assumption because it makes no probability statements.

This use of ANOVA tested the differences between *rows* (different rows are different treatments) while Taguchi F values are related to differences between the effect of *control factors*.

Taguchi ANOVA would normally be applied to S/N ratios computed from the rows of data. It would not be applied directly to the data in the rows.

In one way ANOVA, the F crit value depends on the amount of data collected. No matter how much noise (experimental error) exists, with enough replicates, you can resolve small differences in the treatments. In Taguchi style ANOVA, they set the standards ($F > 4$ is substantial) without regard to the amount of data collected. They generally recommend you don't perform replicates.

Experiment #	N1 #1	N1 #2	N1 #3	N2 #1	N2 #2	N2 #3
1	19	19.5	20.2	14	14	14.6
2	53.5	54.5	54	38.2	39.3	40.3
3	102.5	104.5	105	78.5	79.5	81.1
4	28	30.3	34	19	21.6	22
5	59	60.5	60.6	38.3	38.5	38.2
6	72.7	75.7	76.5	57.9	58.7	61.9
7	22	23.7	26	12.1	12.4	12.6
8	52.7	54.1	54.7	37.8	38	38
9	61	64.5	64	42.7	42.8	43.1

Anova: Single
Factor

SUMMARY

Groups	Count	Sum	Average	Variance
Row 1	6	275.3	45.88333	76.26967
Row 2	6	318.1	53.01667	125.0777

ANOVA

Source of Variation	SS	df	MS	F	P-value	F crit
Between Groups	152.6533	1	152.6533	1.516318	0.24635064	4.964591
Within Groups	1006.737	10	100.6737			
Total	1159.39	11				