

Solution to Quiz #3 Steps in Robust Design (Data Analysis)

The factor effect plots below represent the results of an orthogonal array based matrix experiment.

- 1) How many total experiments were required to gather the data represented in the figures assuming one noise factor with two levels in the outer array and no replicates?

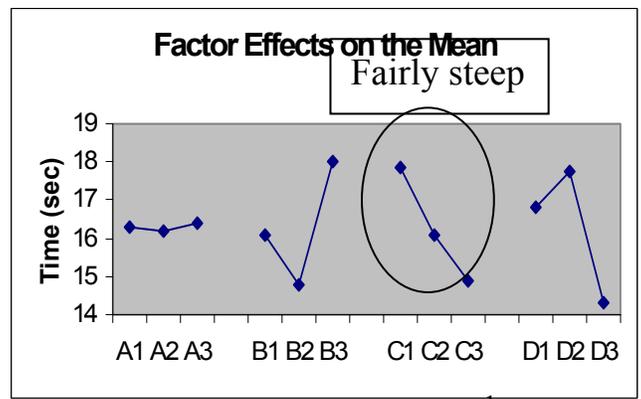
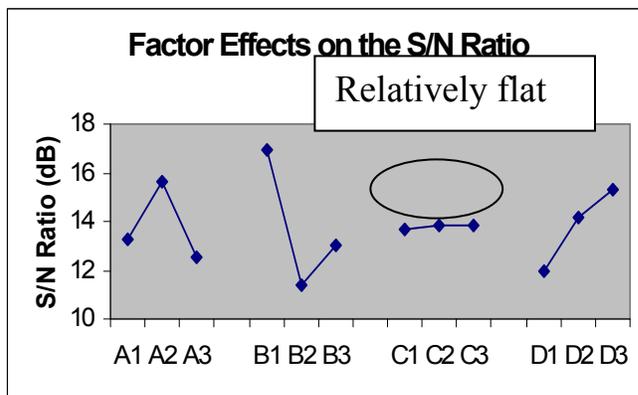
The data represents 4 control factors with 3 levels each. The required degrees of freedom are therefore

$$1+(3-1)*4=9\text{DOF}$$

This experiment happens to fit the $L_9 (3^4)$ exactly. A noise strategy including one noise factor in an outer array with no replicates will require just two experiments per row of the L_9 for a **total of 18 experiments**.

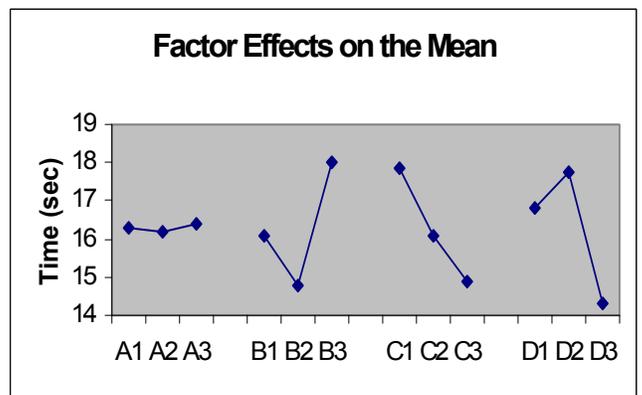
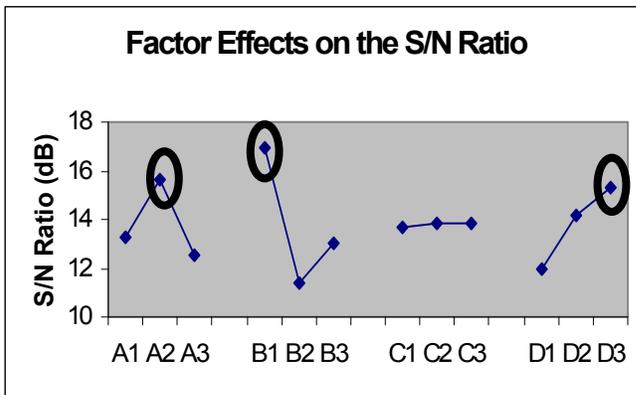
- 2) Which control factor should be used as the scaling factor assuming the additive model holds?

Factor C is the best choice for scaling factor. A good scaling factor should have minimal effect on the S/N ratio. So you're looking for a flat line on the control factor plot for S/N ratio. A good scaling factor should also have a strong effect on the mean so that it will allow us to adjust back onto target even when the mean is substantially off target. So, you're looking for a good spread between the highest and lowest mean response on the control factor plot for means. Factor C has both of these desired properties.



3) What settings of the other control factors will maximize the S/N ratio assuming the additive model holds?

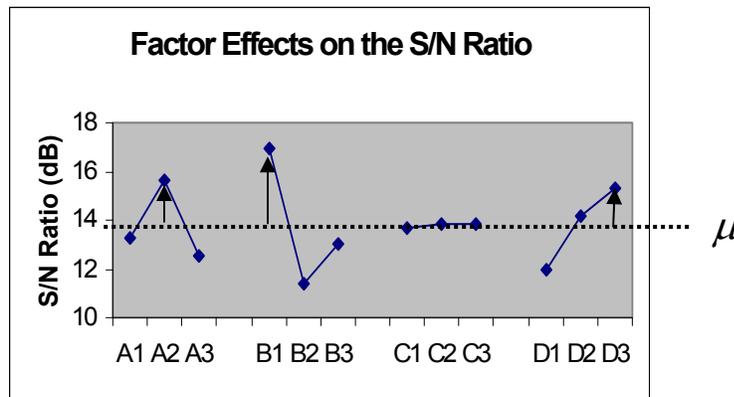
A2, B1, D3 are the control factor levels that will provide the highest predicted S/N ratio. Just take all the control factors aside from the scaling factor and pick the levels with the highest mean value of signal to noise ratio (the highest point on the factor effect plot for S/N).



4) What will be the predicted S/N ratio at the optimal settings selected in problem 3 above? (Estimate your answer from the graphs).

Remember, the average of the control factor level means (e.g., m_{A1} , m_{A2} , m_{A3}) for *any* control factor is always equal to the grand mean (μ) for the experiment. In other words, the control factor effect plot for any control factor is always centered on the grand mean. Looking at the factor effect plot for S/N ratio, the grand mean appears to be about 14 dB.

According to the additive model, the predicted S/N ratio is the grand mean plus the sum of the factor effects. From the factor effects plot, you can estimate the S/N ratio as $14\text{dB}+2\text{dB}+3\text{dB}+1\text{dB}=\mathbf{20\text{dB}}$



5) After optimizing the S/N ratio, what level should be selected for the scaling factor in order to place the mean response of the system at 16 seconds? (Assume an additive model holds for the mean response of the system)

Again, the average of the control factor level means (e.g., m_{A1} , m_{A2} , m_{A3}) for *any* control factor is always equal to the grand mean (μ) for the experiment. This applies equally well to the means as it does to the S/N ratio if an additive model is assumed to hold. Looking at the factor effect plot for mean, the grand mean appears to be about 16.2 seconds.

You've already decided to set the factor levels **A2**, **B1**, **D3** in order to increase the S/N ratio. Given these changes in control factor settings, the predicted mean is the grand mean plus the sum of the factor effects. From the factor effects plot, you can estimate the S/N ratio as $16.2\text{sec} - 1.2\text{sec} - 1.5\text{sec} = 13.5\text{sec}$.

In order to get back on target at 16 seconds, we should use the scaling factor to raise the mean time by 2.5 seconds. **Setting C1** should get us most of the way back on target, but we may want to go to a slightly lower level of control factor C beyond the level we tested to get the product back on target.

