$\sigma := 1$

This is required for some plots below.

Supplement to 16.881 Homework#2

Exploration of the Quadratic Loss Function

Dan Frey, 6 JUN 98

The areas I expect you'll need to change to do homework#2 are highlighted.

$ORIGIN := 1$ Let 1 be the first index in any vector.

$A_o := 10$ Cost to scrap the resistor [cents]

$m := 100$ Nominal value of the resistance (in ohms)

$\Delta_o := 5\% \cdot m$ Allowable variation in the diameter is +/- 5%

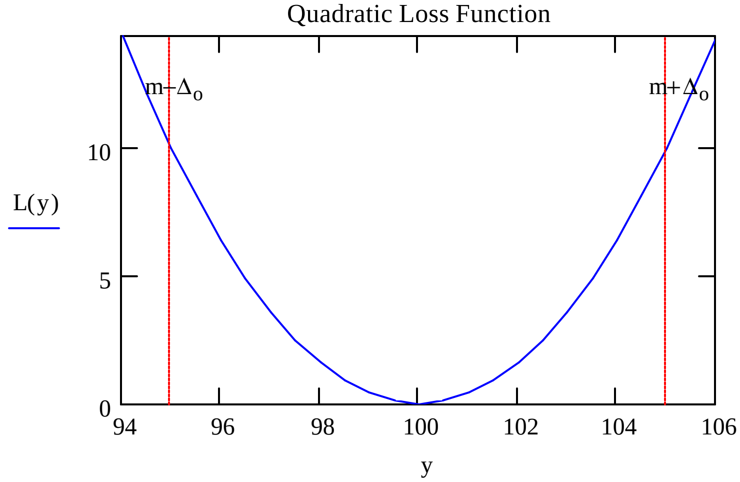
$k_1 := \frac{A_o}{\Delta_o^{\,2}}$ $k_2 := \frac{A_o}{\Delta_o^{\,2}}$

$$L(y) := \begin{cases} k_1 \cdot (y - m)^2 & \text{if } y > m \\ k_2 \cdot (y - m)^2 & \text{if } y \le m \end{cases}$$

Define the quadratic loss function

$y := m - 1.2 \cdot \Delta_o, m - 1.2 \cdot \Delta_o + \frac{\Delta_o}{10} .. m + 1.2 \cdot \Delta_o$ Define a range over y for the purpose of plotting

Quadratic Loss Function



Create a Monte Carlo simulation of the manufacture of the resistors.

$n := 1000$ Number of resistors to be manufactured

$\sigma := \frac{\Delta_o}{6}$

$\mu := m + 1.5\cdot\sigma$

$R := \mathrm{rnorm}(n, \mu, \sigma)$ Create a vector with all of the resistance values of the resistors we manufactured.

Set up the format for a histogram of the data.

$number\_of\_bins := 20$

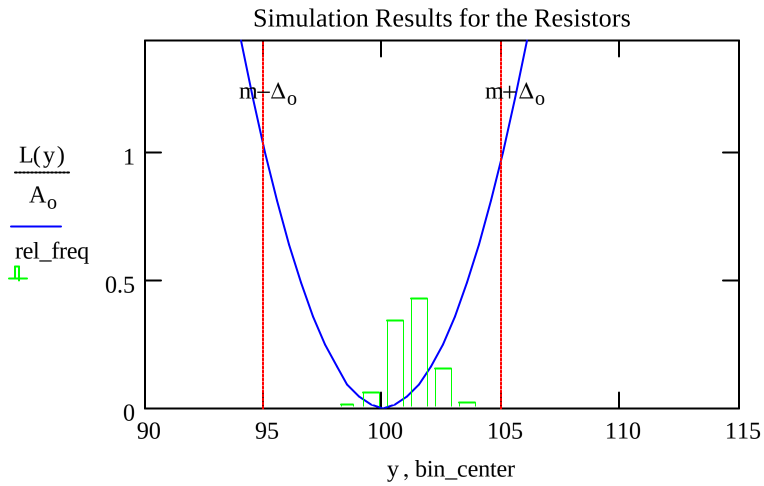
$width\_of\_bins := \frac{4\cdot\Delta_o}{number\_of\_bins}$

$j := 1 .. number\_of\_bins + 1$

$bin_j := m - 2\cdot\Delta_o + width\_of\_bins\cdot j$ Define a vector with the start and end points of the bins.

$rel\_freq := \frac{\mathrm{hist}(bin, R)}{n}$ Compute the relative frequency distribution over interval.

$bin\_center := bin + 0.5\cdot width\_of\_bins$



$$\text{Average\_quality\_loss} := \frac{1}{n} \cdot \sum_{i=1}^{n} L\left(R_i\right)$$

$$\text{Average\_quality\_loss} = 0.845$$ in cents

How does this compare to the theoretically derived figure?

$$\int_{m-2\cdot\Delta_o}^{m+2\cdot\Delta_o} k_1 \cdot (y-m)^2 \cdot \left[\frac{1}{\sigma \cdot \sqrt{2 \cdot \pi}} \cdot e^{\frac{-(y-\mu)^2}{2\cdot\sigma^2}}\right] dy = 0.903$$ in cents