Airline Fleet Assignment

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16.75 Airline Management

Outline:

- Problem Definition and Objective
- Fleet Assignment Network Representation
- Fleet Assignment Models and Algorithms
- Extension of Fleet Assignment to Schedule Design
- Conclusions

Airline Schedule Planning

Schedule Design

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Fleet Assignment

↓

Aircraft Routing

↓

Crew Scheduling

Select optimal set of *flight legs* in a schedule

A flight specifies origin, destination, and departure time

Contribution = Revenue - Costs

Assign crew (pilots and/or flight attendants) to flight legs

Fleet Assignment

<u>**Problem Definition**</u>

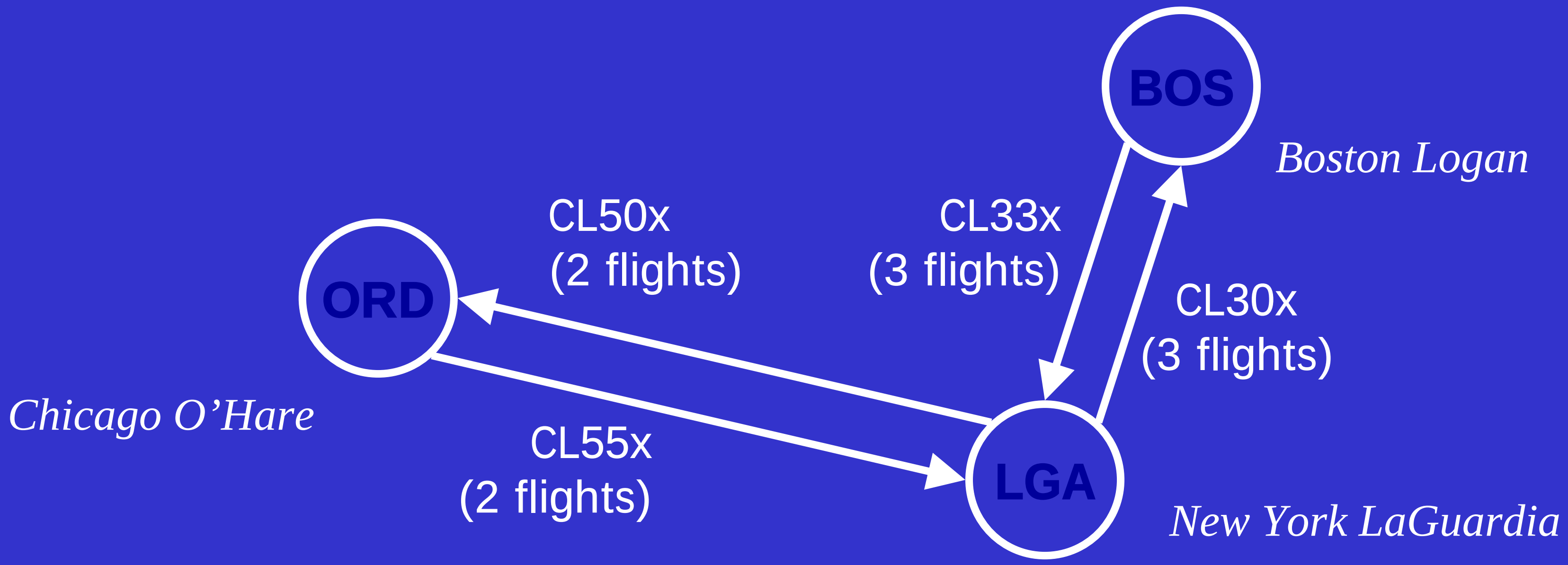
- Given:
  - Flight Schedule: a set of (daily) flight legs;
  - Aircraft fleet: consisting of different fleet types;
  - Passenger demand pattern;
  - Revenue and operating cost data;
- Find:

  A feasible fleet assignment, i.e. a mapping from flight legs to fleet types that maximizes

  profit = total revenue – total operating costs.

Fleet Assignment

- **<u>Class Exercise:</u>** Flight Network



Fleet Assignment

- **<u>Class Exercise:</u>** Flight Schedule, Fares, & Demand

Flight #	From	To	Dept Time (EST)	Arr Time (EST)	Fare [$]	Demand [passengers]
CL301	LGA	BOS	1000	1100	150	250
CL302	LGA	BOS	1100	1200	150	250
CL303	LGA	BOS	1800	1900	150	100
CL331	BOS	LGA	0700	0800	150	150
CL332	BOS	LGA	1030	1130	150	300
CL333	BOS	LGA	1800	1900	150	150
CL501	LGA	ORD	1100	1400	400	150
CL502	LGA	ORD	1500	1800	400	200
CL551	ORD	LGA	0700	1000	400	200
CL552	ORD	LGA	0830	1130	400	150

Fleet Assignment

- **Class Exercise:** Fleet Information

Fleet type	Number of aircraft owned	Capacity [seats]	Per flight operating cost [$000]	
			LGA - BOS	LGA – ORD
DC-9	1	120	10	15
B737	2	150	12	17
A300	2	250	15	20

Fleet Assignment

- **Class Exercise:**

Find:

An assignment of fleet types to the flights in this network that maximizes net profit.

Fleet Assignment

- Evaluating assignment profits:

$$c_{l,f} := fare_l \times \min(D_l, Cap_f) - OpCost_{l,f}$$

where:

$c_{l,f}$: profitability of assigning fleet type f to flight leg l;

$fare_l$: fare of flight leg l;

D_l : demand of flight leg l;

Cap_f : capacity of fleet type f;

$OpCost_{l,f}$: operating cost of assigning fleet type f to flight leg l.

Fleet Assignment

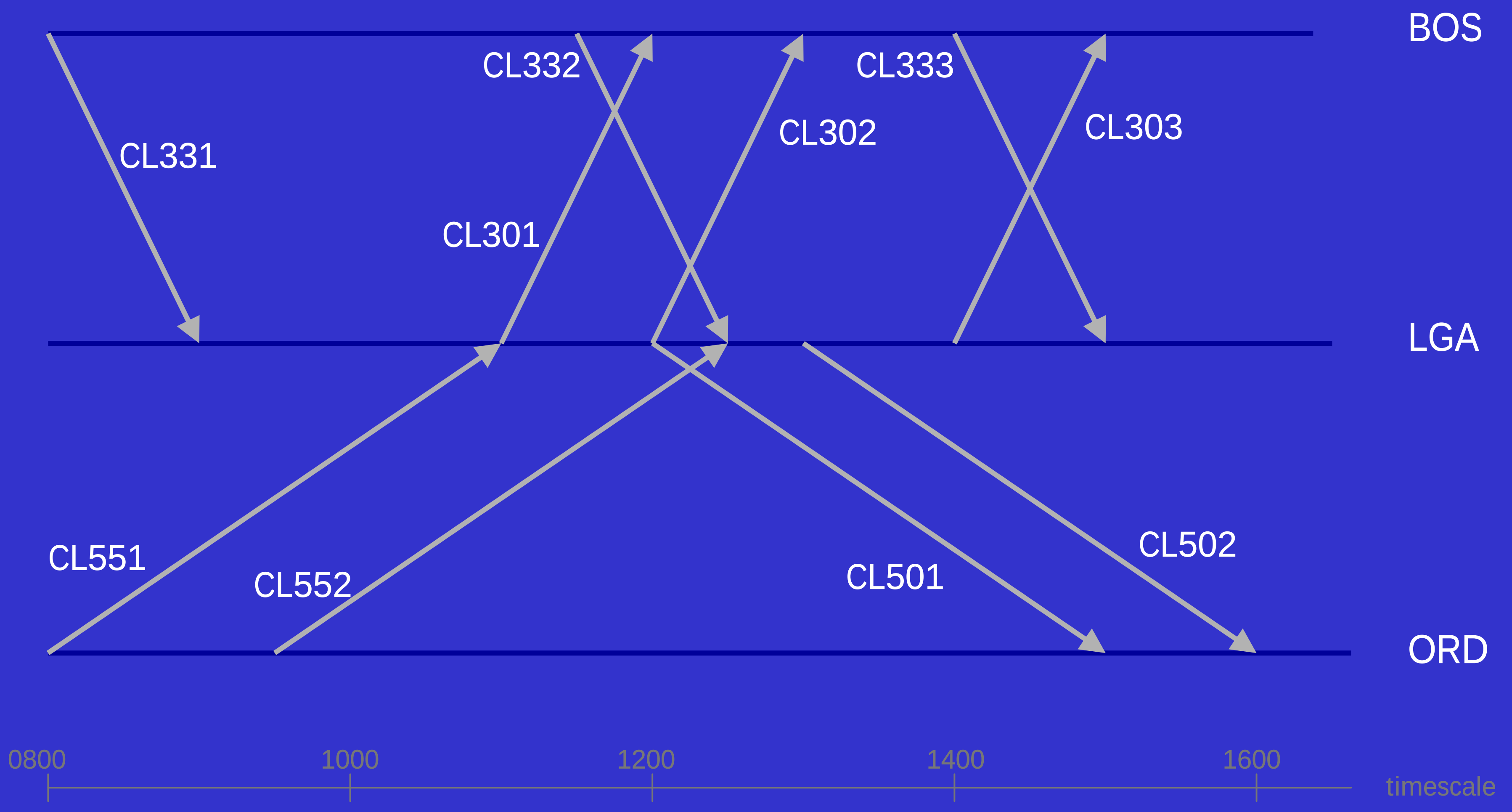
- **Class Exercise:** Evaluating assignment profitabilities…

Profitability [$000 per day]

Flight #	DC-9	B737	A300
CL301	8	10.5	22.5
CL302	8	10.5	22.5
CL303	5	3	0
CL331	8	10.5	7.5
CL332	8	10.5	22.5
CL333	8	10.5	7.5
CL501	33	43	40
CL502	33	43	60
CL551	33	43	60
CL552	33	43	40

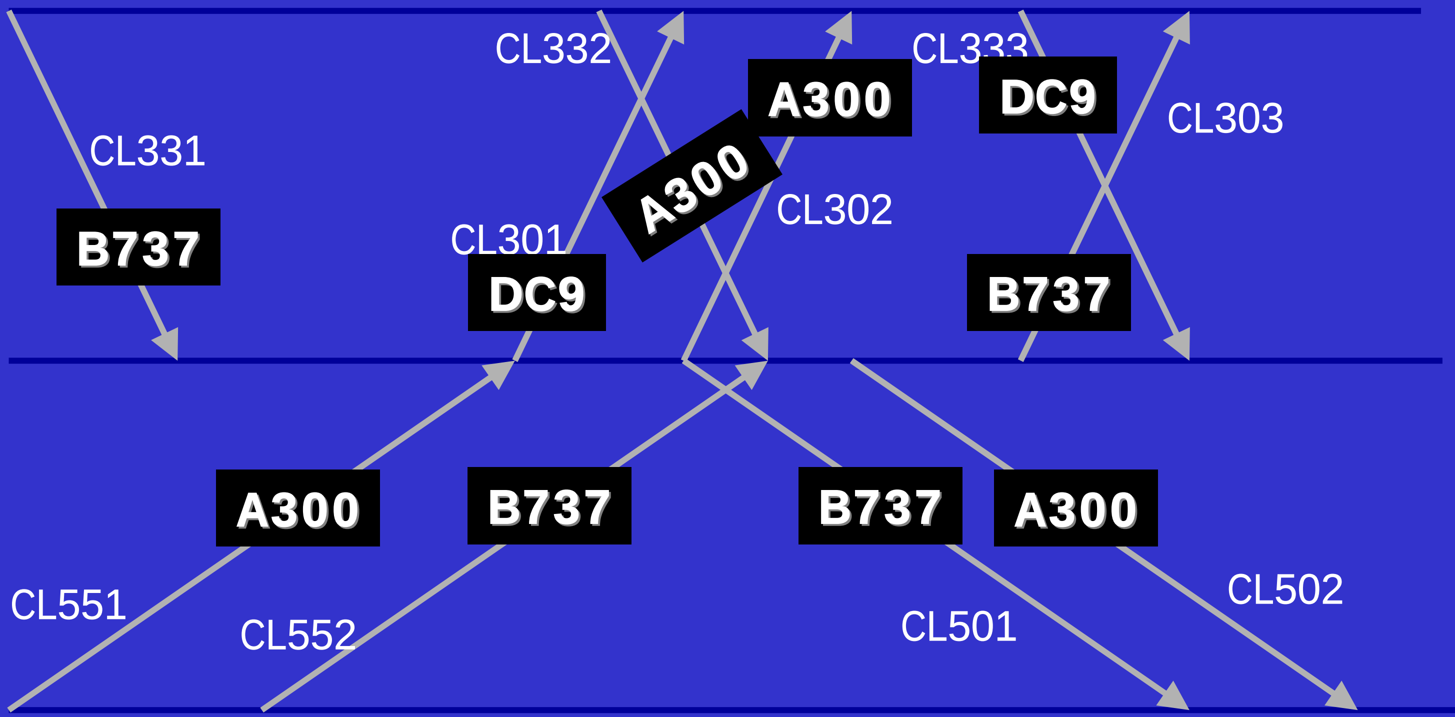
Fleet Assignment

- Time-Line Network:



Fleet Assignment

- Optimal solution:



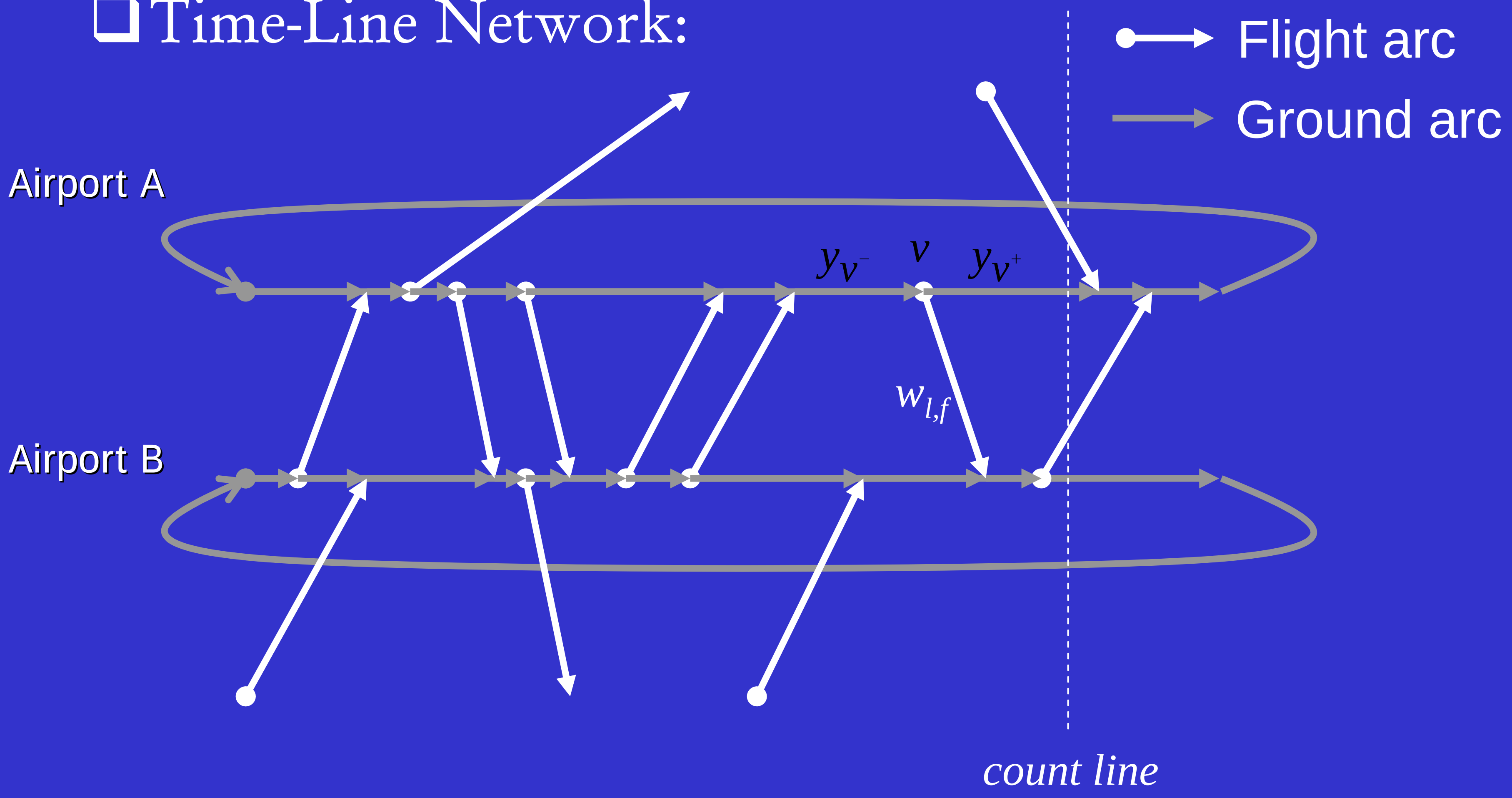
Leg	Δcapacity
CL301	-130
CL302	0
CL303	+50
CL331	0
CL332	-50
CL333	-30
CL501	0
CL502	+50
CL551	+50
CL552	0

Revenue = $428,500
Cost = $148,000

Profit = $280,500

Fleet Assignment

- Time-Line Network:



Notations

- Decision Variables
  - $f_{k,i}$ equals 1 if fleet type k is assigned to flight leg i, and 0 otherwise
  - $y_{k,o,t}$ is the number of aircraft of fleet type k, on the ground at station o, and time t
- Parameters
  - $C_{k,i}$ is the cost of assigning fleet k to flight leg i
  - N_k is the number of available aircraft of fleet type k
  - t_n is the "count time"
- Sets
  - L is the set of all flight legs i
  - K is the set of all fleet types k
  - O is the set of all stations o
  - $CL(k)$ is the set of all flight arcs for fleet type k crossing the count time

Fleet Assignment Model (FAM)

$$Min \sum_{k \in K} \sum_{i \in L} c_{k,i} f_{k,i}$$

Subject to:

$$\sum_{k \in K} f_{k,i} = 1 \qquad \forall i \in L$$

$$y_{k,o,t^-} + \sum_{i \in I(k,o,t)} f_{k,i} - y_{k,o,t^+} - \sum_{i \in O(k,o,t)} f_{k,i} = 0 \qquad \forall k,o,t$$

$$\sum_{o \in O} y_{k,o,t_n} + \sum_{i \in CL(k)} f_{k,i} \le N_k \qquad \forall k \in K$$

$$f_{k,i} \in \{0,1\} \qquad y_{k,o,t} \ge 0$$

Hane et al. (1995), Abara (1989), and Jacobs, Smith and Johnson (2000)

Constraints

- Cover Constraints
  - Each flight must be assigned to exactly one fleet
- Balance Constraints
  - Number of aircraft of a fleet type arriving at a station must equal the number of aircraft of that fleet type departing
- Aircraft Count Constraints
  - Number of aircraft of a fleet type used cannot exceed the number available

Objective Function

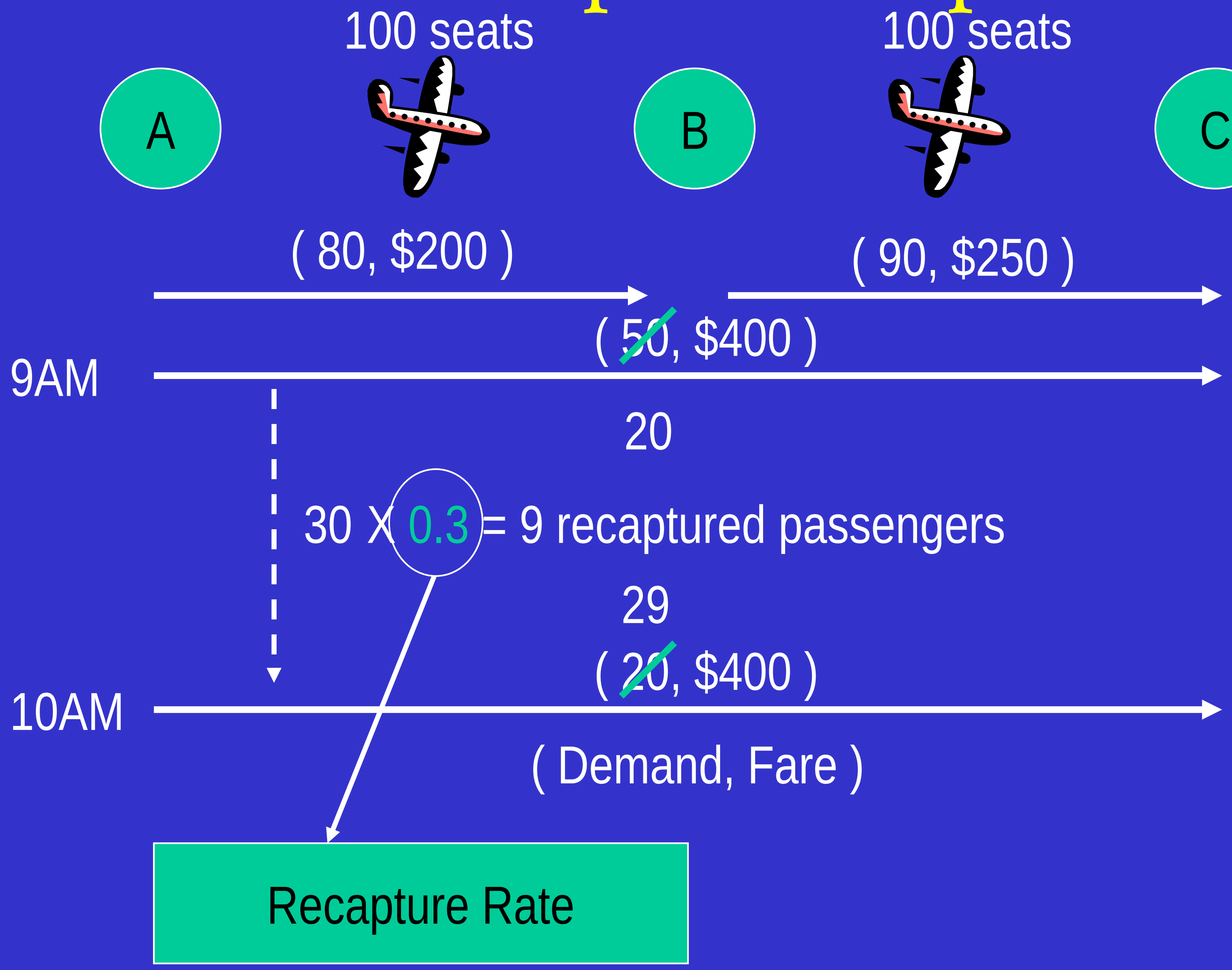
- For each fleet - flight combination: Cost ≡ Operating cost + Spill cost –Recaptured revenue
- Operating cost associated with assigning a fleet type *k* to a flight leg *j* is relatively straightforward to compute
  - Can capture range restrictions, noise restrictions, water restrictions, etc. by assigning "infinite" costs
- Spill cost for flight leg *j* and fleet assignment *k* = average revenue per passenger on *j* * MAX(0, unconstrained demand for *j* – number of seats on *k*)
  - Unclear how to compute revenue for flight legs, given revenue is associated with itineraries
- Recaptured revenue
  - Revenue from passengers that are recaptured back to the airline after being spilled from another flight leg

FAM Example: Spill

Demand = 100
Fare = $100

Fleet Type	Capacity	Spill Cost	Op. Cost	Assignment Cost
i	80	$2,000	$5,000	$7,000
ii	100	$0	$6,000	$6,000
iii	120	$0	$7,000	$7,000
iv	150	$0	$8,000	$8,000

FAM Example: Recapture



Fleet Assignment

- A few observations on FAM:
  - Nodes can be consolidated to reduce model size;
  - Fleet-specific time-line networks are possible;
  - Fleet assignment not aircraft assignment!
  - Note that feasibility of FAM implies that aircraft rotations exist (takes only a little bit of thinking);
  - However, these rotations might not be maintenance feasible…

Fleet Assignment

- Solvability:
  - FAM can be solved using standard branch-and-bound software;
  - Solution times are FAST, thanks to FAM's small LP gaps...

Fleet Assignment

Computational Sample: 2,044 flight legs, 9 fleet types

Problem size	
# of columns	18,487
# of rows	7,827
# of non-zero entries	50,034
Strength of formulation	
Root node LP solution	21,401,658
Best IP solution	21,401,622
Difference	36
Solution time [sec]	974

Fleet Assignment

- FAM suffers from a significant drawback in its modeling of the revenue side…
- Passengers book itineraries not flight legs…
- Capacity decisions on one leg will affect passenger spill on other legs…
- This phenomenon is known as network effects.

Fleet Assignment Model (FAM)

$$Min \sum_{k\in K}\sum_{i\in L} c_{k,i} f_{k,i}$$

Subject to:

$$\sum_{k\in K} f_{k,i} = 1 \qquad \forall i \in L$$

$$y_{k,o,t^-} + \sum_{i\in I(k,o,t)} f_{k,i} - y_{k,o,t^+} - \sum_{i\in O(k,o,t)} f_{k,i} = 0 \qquad \forall k,o,t$$

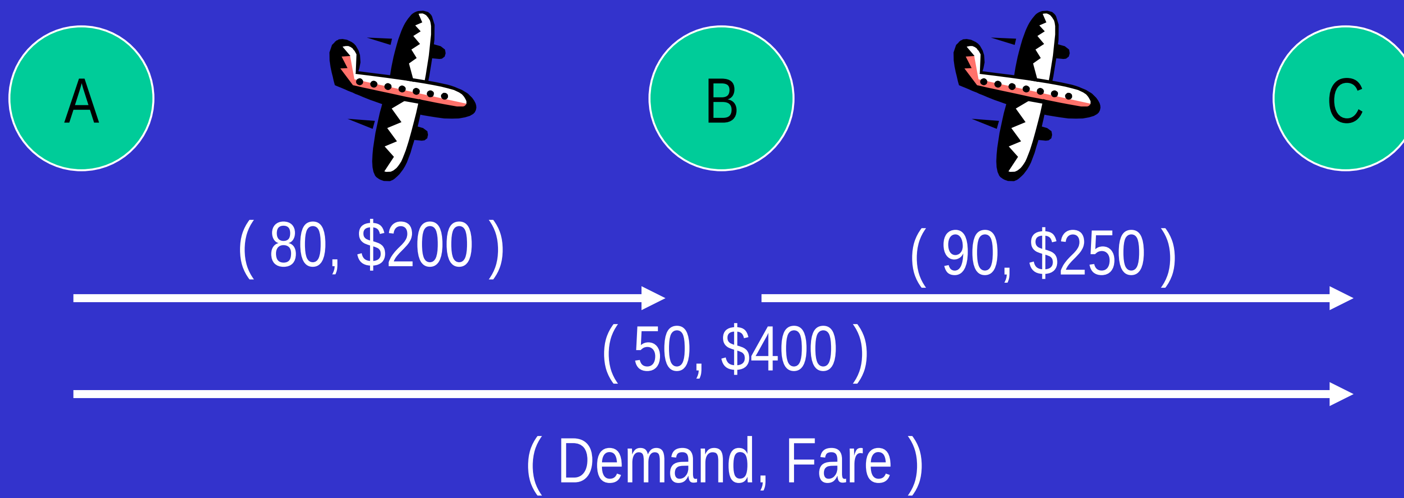
$$\sum_{o\in O} y_{k,o,t_n} + \sum_{i\in CL(k)} f_{k,i} \le N_k \qquad \forall k \in K$$

$$f_{k,i} \in \{0,1\} \qquad y_{k,o,t} \ge 0$$

Major Shortcoming:
FAM assumes leg independence

Hane et al. (1995), Abara (1989), and Jacobs, Smith and Johnson (2000)

FAM Example: Network Effects



Fleet Type	Capacity	Spill Cost
i	80	?
ii	100	?
iii	120	?
iv	150	$0

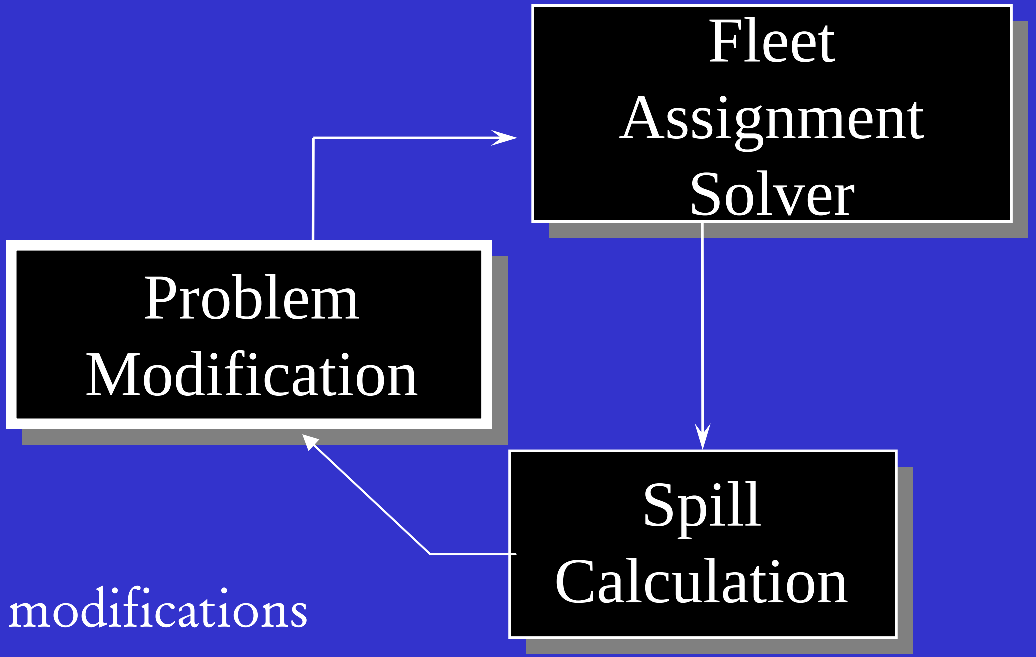
Leg Interdependence

Network Effects

Spill Cost Computation and Underlying Assumption

- Given:
  - Spill cost for flight leg *j* and fleet assignment *k* = average revenue per passenger on *j* * MAX(0, unconstrained demand for *j* – number of seats on *k*)
- Implication:
  - A passenger might be spilled from some, but not all, of the flight legs in his/ her itinerary

An Iterative Approach



- FAM Solver
  - Basic Fam
    - Possibly with minor modifications
- Spill Calculation:
  - Simulations
  - Passenger mix model
- Problem Modification:
  - Objective cost coefficient update

Passenger Mix

- Passenger Mix Model (PMIX)
  - Kniker (1998)
  - Given a fixed, fleeted schedule, unconstrained passenger demands by itinerary (requests), and recapture rates find maximum revenue for passengers on each flight leg

Network Effects and Recapture

Problem Modification

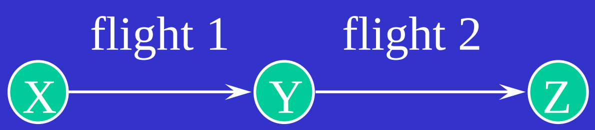
- Based on differences in expected spill from FAM and the Spill Calculator, we modify the FAM problem
  - Update objective cost coefficients
- Cost coefficient update, many heuristics possible

FAM Spill Calculation Heuristics

- Fare Allocation
  - Full fare - the full fare is assigned to each leg of the itinerary
  - Partial fare - the fare divided by the number of legs is assigned to each leg of the itinerary
  - Shared fare - the fare divided by the number of *capacitated* legs is assigned to each *capacitated* leg in the itinerary
- Spill Cost for each variable
  - Representative Fare
    - A "spill fare" is calculated; each passenger spilled results in a loss of revenue equal to the spill fare
  - Integration
    - Sort each itinerary by fare, spill costs are sum of x lowest fare passengers, where $x = max\{0, demand - capacity\}$

An Illustrative Example

Fleet Type	Seats
A	100
B	200



Market	Itinerary	Average Fare	No. of Pax
X-Y	1	$200	75
Y-Z	2	$225	150
X-Z	1-2	$300	75

Fleet Assign. Fl. 1- Fl. 2	Partial Alloc. Spill	Full Alloc. Spill	Actual Opt. Spill	Actual Opt. Spilled Pax
A-A	$30,000	$38,125	31,875	50 X-Z, 75 Y-Z
A-B	$11,250	$15,625	12,500	25 X-Z, 25 X-Y
B-A	$22,500	$28,125	28,125	125 Y-Z
B-B	$3,750	$5,625	5,625	25 Y-Z

Spill Calculation: Results

- For a 3 fleet, 226 flights problem:
  - The best representative fare solution results in a gap with the optimal solution of $2,600/day
  - Using a shared fare scheme and integration approach, we found a solution with an $8/day gap.
- By simply modifying the basic spill model, significant gains can be achieved

Itinerary-Based Fleet Assignment

- Impossible to estimate airline profit exactly using link-based costs
- Enhance basic fleet assignment model to include passenger flow decision variables
  - Associate operating costs with fleet assignment variables
  - Associate revenues with passenger flow variables

Itinerary-based Fleet Assignment Definition

- Given
  - a fixed schedule,
  - number of available aircraft of different types,
  - unconstrained passenger demands by itinerary, and
  - recapture rates,

Find maximum contribution

Network effects

Itinerary-Based FAM (IFAM)

$t_p^r \geq 0 \qquad f_{k,i} \in \{0,1\} \qquad y_{k,o,t} \geq 0$

Kniker (1998)

Itinerary-Based FAM (IFAM)

$$Min \sum_{k \in K} \sum_{i \in L} \text{‰}_{k,i} f_{k,i} + \sum_{p \in P} \sum_{r \in P} (fare_p - b_p^r fare_r)\, t_p^r$$

3 t_p^r

Subject to:

$$\sum_{k \in K} f_{k,i} = 1 \qquad \forall i \in L$$

$$y_{k,o,t^-} + \sum_{i \in I(k,o,t)} f_{k,i} - y_{k,o,t^+} - \sum_{i \in O(k,o,t)} f_{k,i} = 0 \qquad \forall k,o,t$$

$$\sum_{o \in O} y_{k,o,t_n} + \sum_{i \in CL(k)} f_{k,i} \le N_k \qquad \forall k \in K$$

1

$$\sum_{k} f_{k,i} SEATS_k + \sum_{r \in P} \sum_{p \in P} \delta_i^p t_p^r - \sum_{r \in P} \sum_{p \in P} \delta_i^p b_p^r t_p^r \ge Q_i \qquad \forall i \in L$$

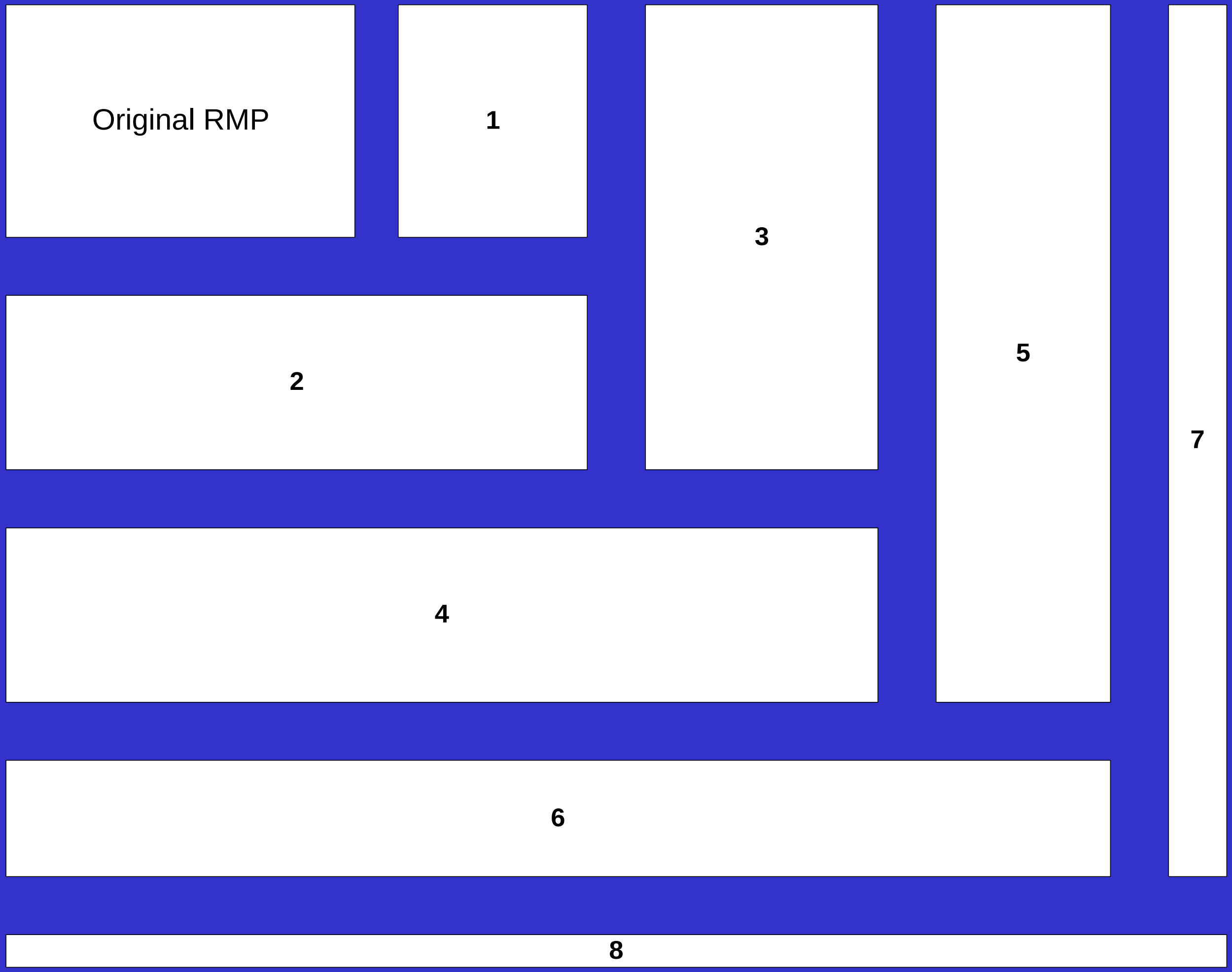
2

$$\sum_{r \in P} t_p^r \le D_p \qquad \forall p \in P$$

$$t_p^r \ge 0 \qquad f_{k,i} \in \{0,1\} \qquad y_{k,o,t} \ge 0$$

Kniker (1998)

Column and Constraint Generation



Implementation Details

- Computer
  - Workstation class computer
  - 2 GB RAM
  - CPLEX 6.5
- Full size schedule
  - ~2,000 legs
  - ~76,000 itineraries
  - ~21,000 markets
  - 9 fleet types
- RMP constraint matrix size
  - ~77,000 columns
  - ~11,000 rows
- Final size
  - ~86,000 columns
  - ~19,800 rows
- Solution time
  - LP: > 1.5 hours
  - IP: > 4 hours

88% Saving from Row Generation
> 95% Saving from Column Generation

Fleet Assignment

Computational Sample: 2,044 flight legs, 9 fleet types

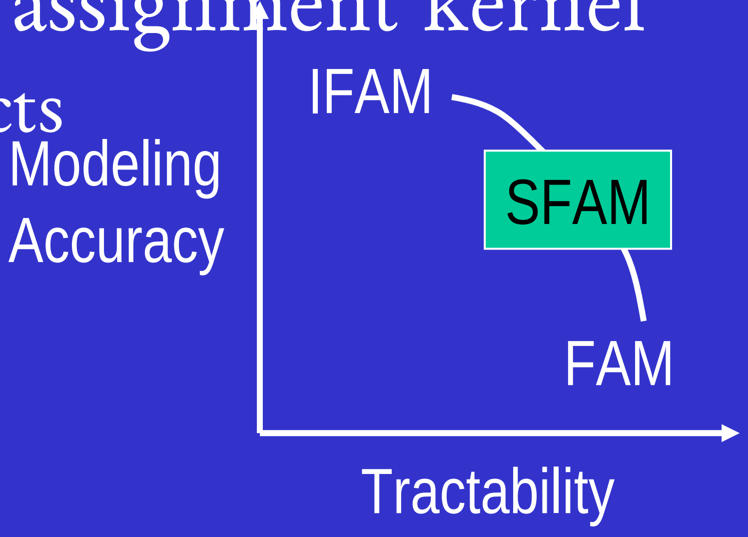
	FAM	IFAM
Problem size		
# of columns	18,487	77,284
# of rows	7,827	10,905
# of non-zero entries	50,034	128,864
Strength of formulation		
Root node LP solution	21,401,658	21,302,460
Best IP solution	21,401,622	21,066,811
Difference	36	235,649
Solution time [sec]	974	>100,000
Contribution [$/day]	21,178,815	21,066,811

IFAM Contributions

- Annual improvements over basic FAM
  - Network Effects: ~$30 million
  - Recapture: ~$70 million
- These estimates are upper bounds on achievable improvements

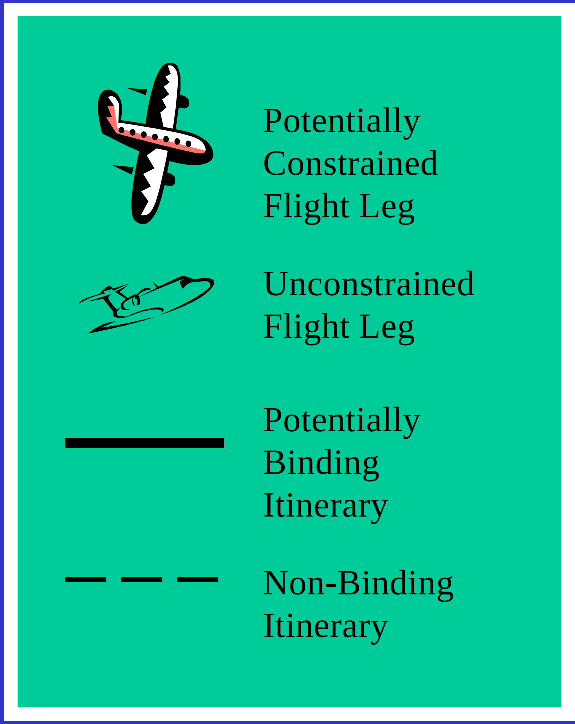
Subnetwork-Based FAM

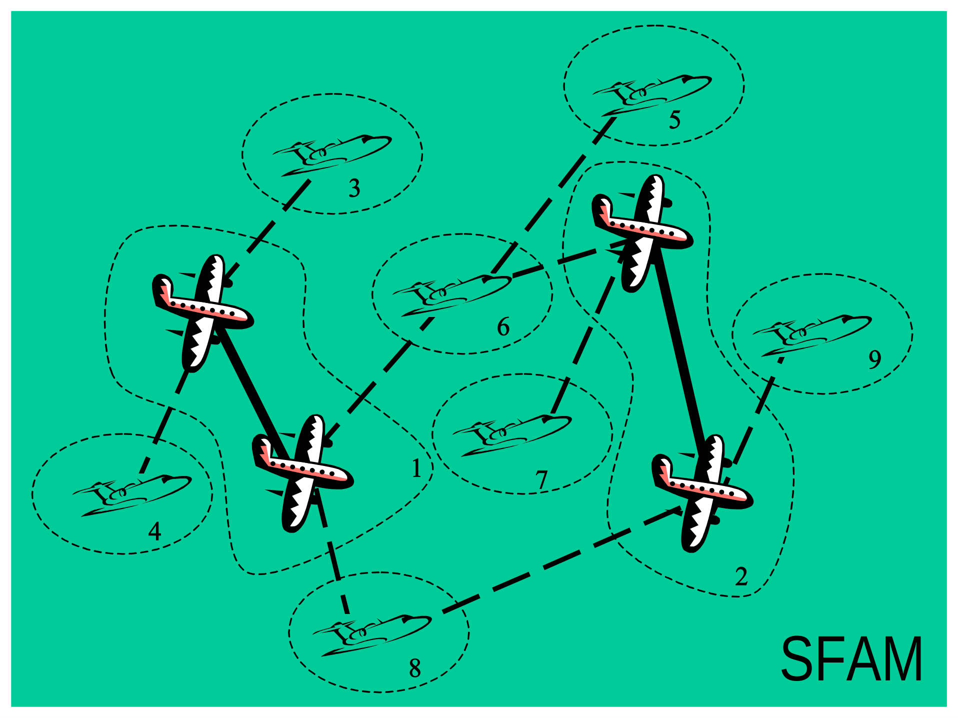
- IFAM has limited opportunity for expansion to include schedule design decisions
  - Fractionality of solution to LP relaxation is a big issue
- Need alternative fleet assignment kernel
  - Capture network effects
  - Maintains tractability



Basic Concept

- Isolate network effects
  - Spill occurs only on *constrained legs*

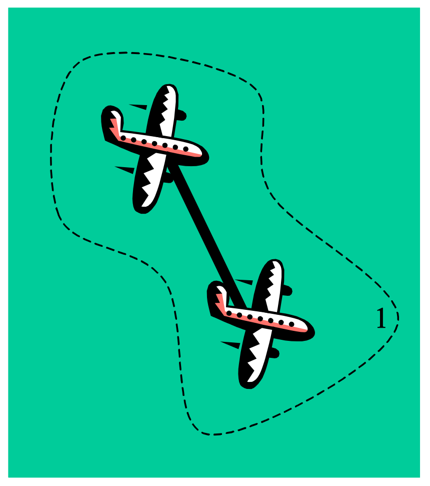




- < 30% of total legs are potentially constrained
- < 6% of total itineraries are potentially binding

Modeling Challenges

- Utilize composite variables (Armacost, 2000; Barnhart, Farahat and Lohatepanont, 2001)



1	2	3	4	5	6	7	8	9
A	**A**	**A**	**B**	**B**	**B**	**C**	**C**	**C**
A	**B**	**C**	**A**	**B**	**C**	**A**	**B**	**C**

3 Fleet Types: A, B, and C

- Challenges
  - Efficient column enumeration

SFAM Formulation

$$Min \sum_{m=1}^{M^S} \sum_{n=1}^{\eta_{\Pi^S}^m} \left(C_{\Pi^S}^m\right)_n \left(f_{\Pi^S}^m\right)_n$$

Subject to:

$$\sum_{m=1}^{M^S} \sum_{n=1}^{\eta_{\Pi^S}^m} \left(\delta_{\Pi^S}^m\right)_n^i \left(f_{\Pi^S}^m\right)_n = 1 \qquad \forall i \in L$$

$$y_{k,o,t^-} + \sum_{i \in I(k,o,t)} \sum_{m=1}^{M^S} \sum_{n=1}^{\eta_{\Pi^S}^m} \left(\kappa_{\Pi^S}^m\right)_n^{k,i} \left(f_{\Pi^S}^m\right)_n - y_{k,o,t^+} - \sum_{i \in O(k,o,t)} \sum_{m=1}^{M^S} \sum_{n=1}^{\eta_{\Pi^S}^m} \left(\kappa_{\Pi^S}^m\right)_n^{k,i} \left(f_{\Pi^S}^m\right)_n = 0 \qquad \forall k,o,t$$

$$\sum_{o \in A} y_{k,o,t_n} + \sum_{i \in CL(k)} \sum_{m=1}^{M^S} \sum_{n=1}^{\eta_{\Pi^S}^m} \left(\gamma_{\Pi^S}^m\right)_n^k \left(f_{\Pi^S}^m\right)_n \leq N_k \qquad \forall k \in K$$

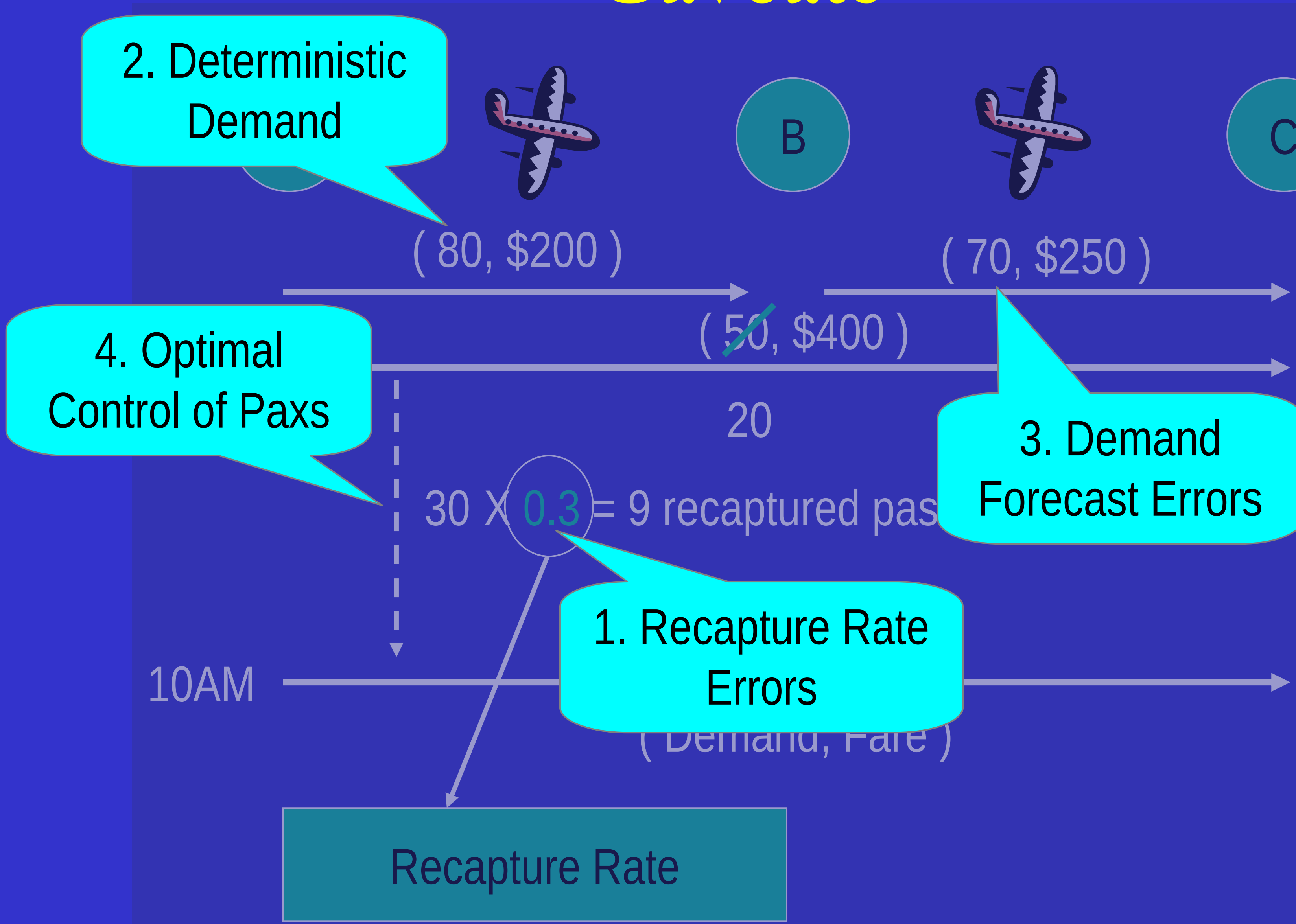
$$\left(f_{\Pi^S}^m\right)_n \in \{0,1\} \qquad y_{k,o,t} \geq 0$$

FAM solution algorithm applies

SFAM Results

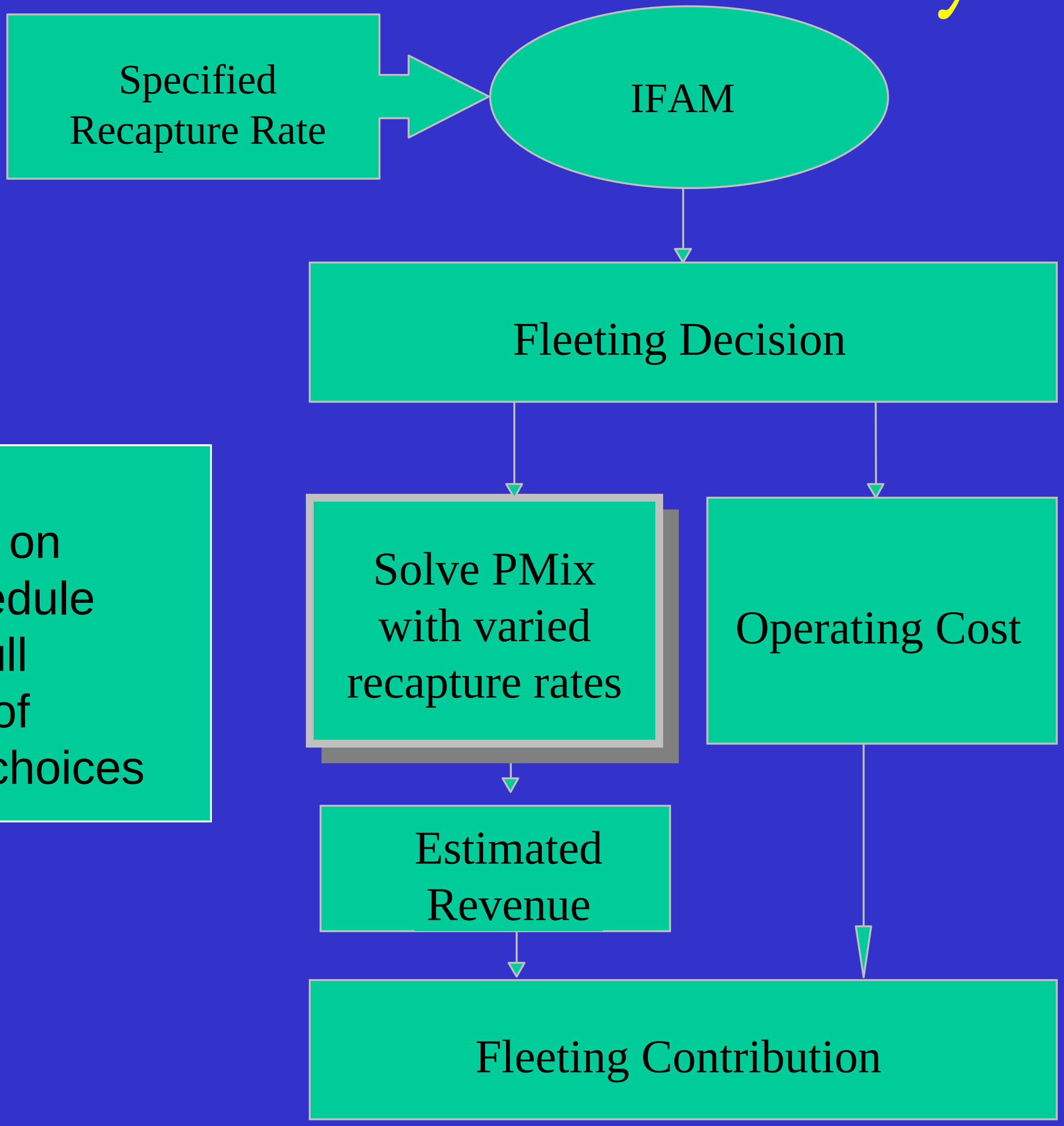
- Testing performed on full size schedules
  - Runtimes similar to FAM, much faster than itinerary-based approaches
    - Tight LP relaxations
  - SFAM achieve improved solutions relative to FAM and itinerary-based approach
- SFAM has potential for further integration or extension
  - Time windows, stochastic demand, schedule design

Caveats



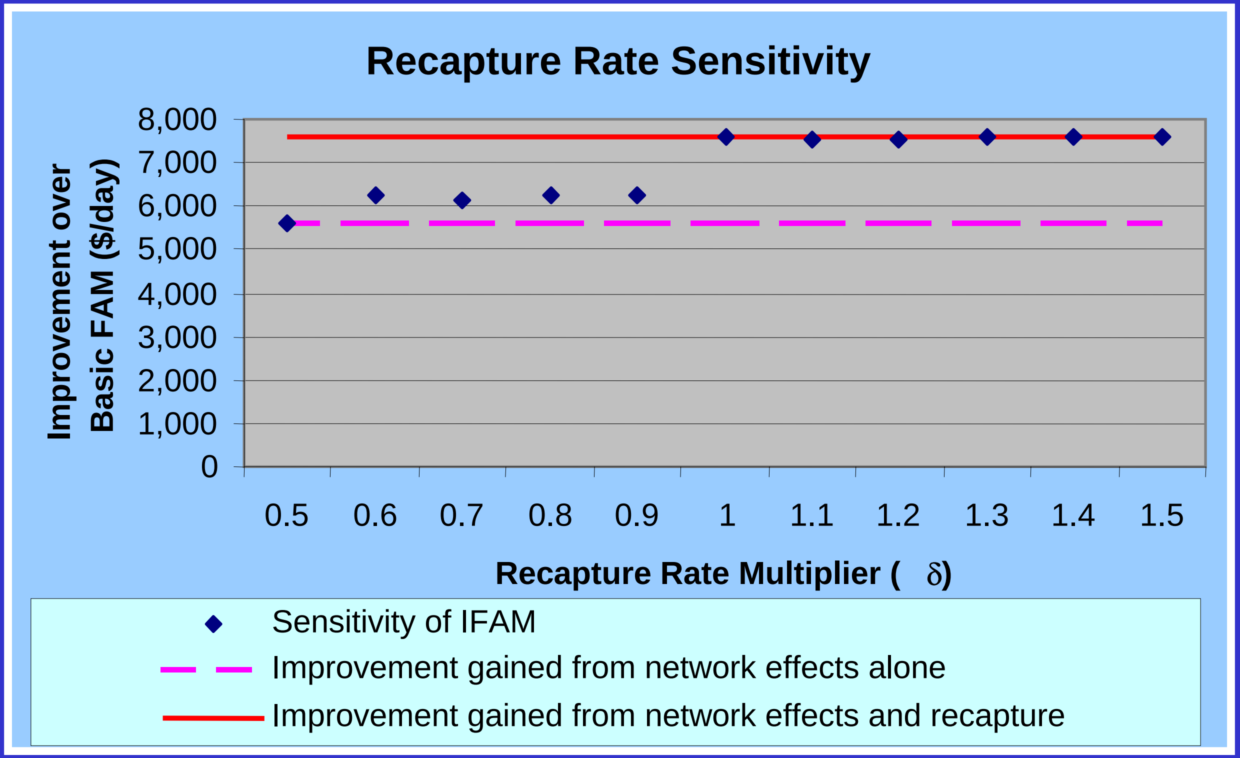
2. Deterministic Demand
B
C
(80, $200)
(70, $250)
(50, $400)
4. Optimal Control of Paxs
20
3. Demand Forecast Errors
30 X 0.3 = 9 recaptured pas
1. Recapture Rate Errors
10AM
Recapture Rate

Recapture Rate Sensitivity



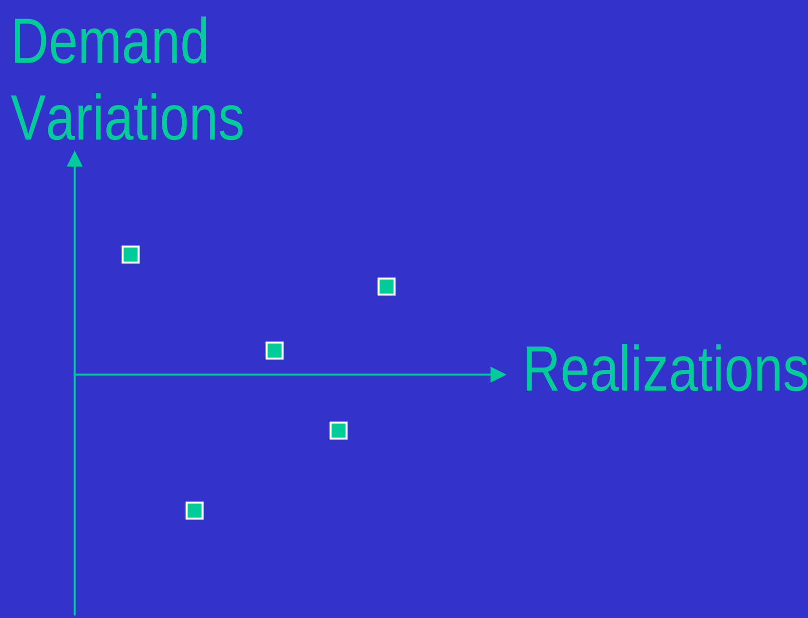
PMix flows passengers on fleeted schedule assuming full knowledge of passenger choices

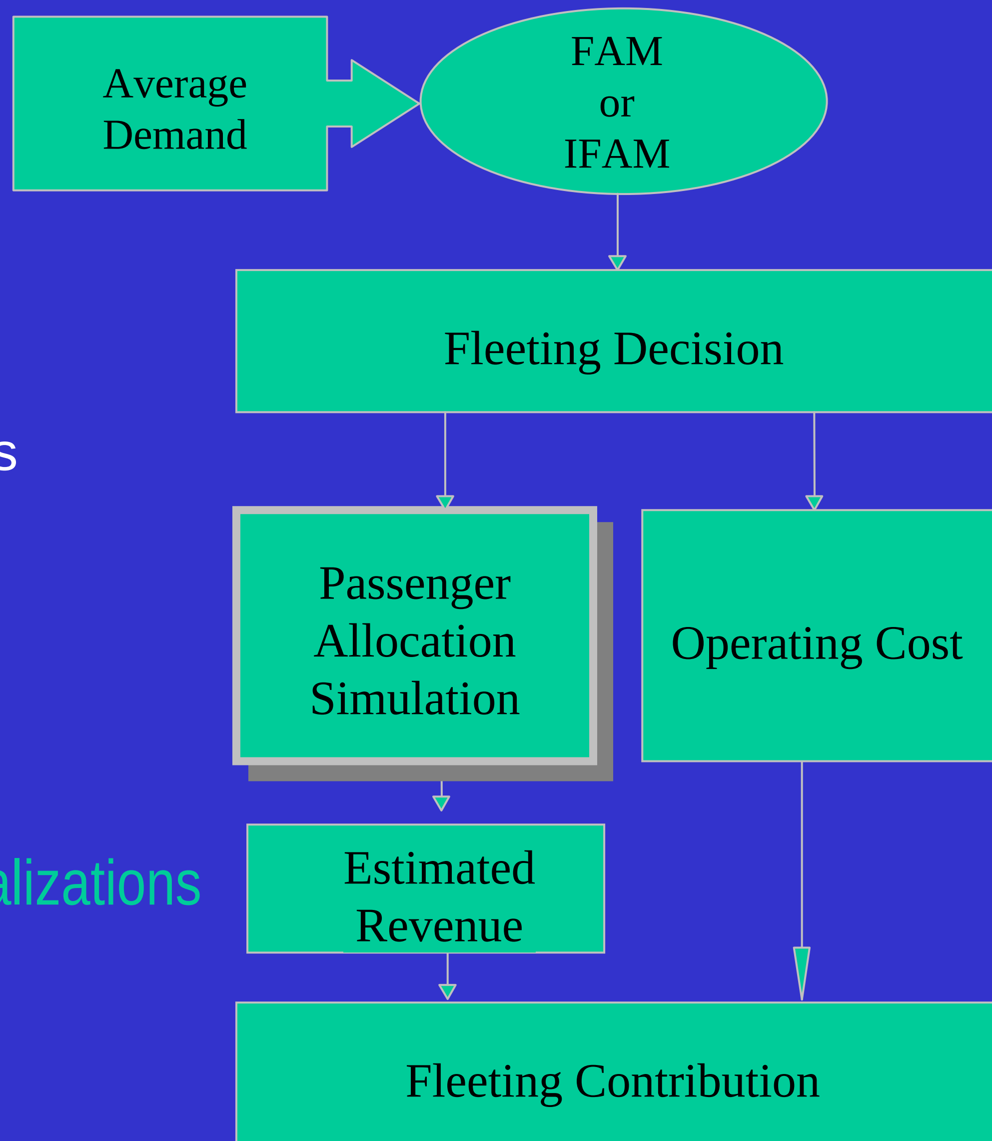
Recapture Rate Sensitivity



IFAM Sensitivity Analysis

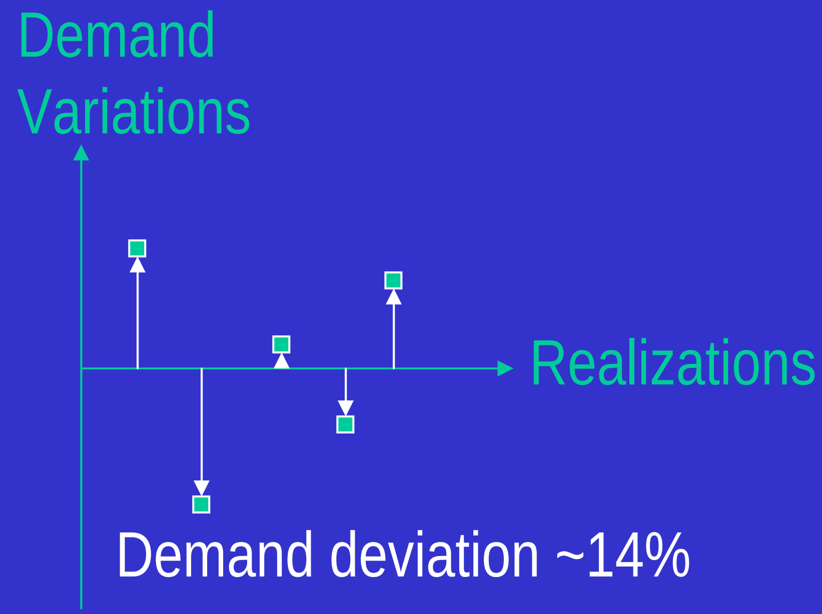
- Simulations
  - Simulate 500 realizations of demand based on Poisson distributions





IFAM vs. FAM

Demand Stochasticity



		FAM		IFAM		Difference (IFAM-FAM)
Problem 1N-3A						
Revenue	$	4,858,089	$	4,918,691	$	60,602
Operating Cost	$	2,020,959	$	2,021,300	$	341
Contribution	$	2,837,130	$	2,897,391	$	60,261
Problem 2N-3A						
Revenue	$	3,526,622	$	3,513,996	$	(12,626)
Operating Cost	$	2,255,254	$	2,234,172	$	(21,082)
Contribution	$	1,271,368	$	1,279,823	$	8,455

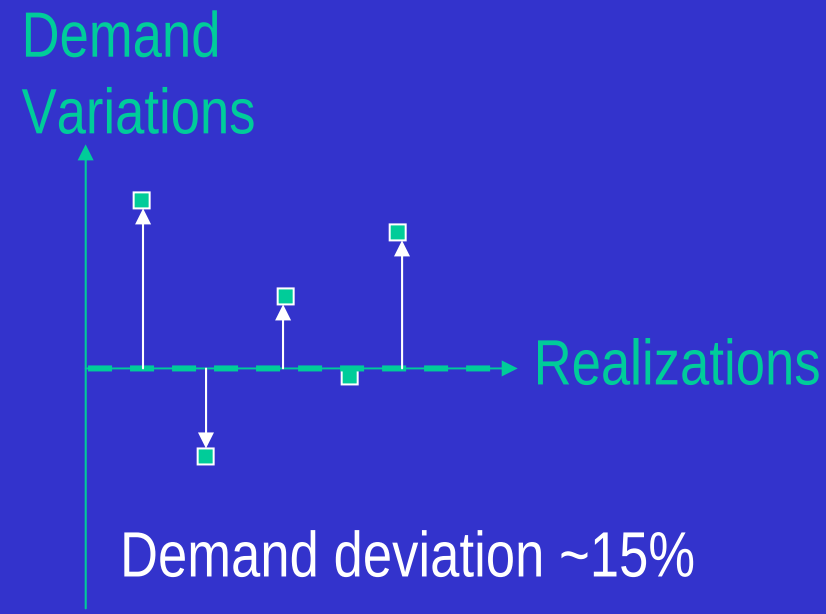
$/day

IFAM vs. FAM

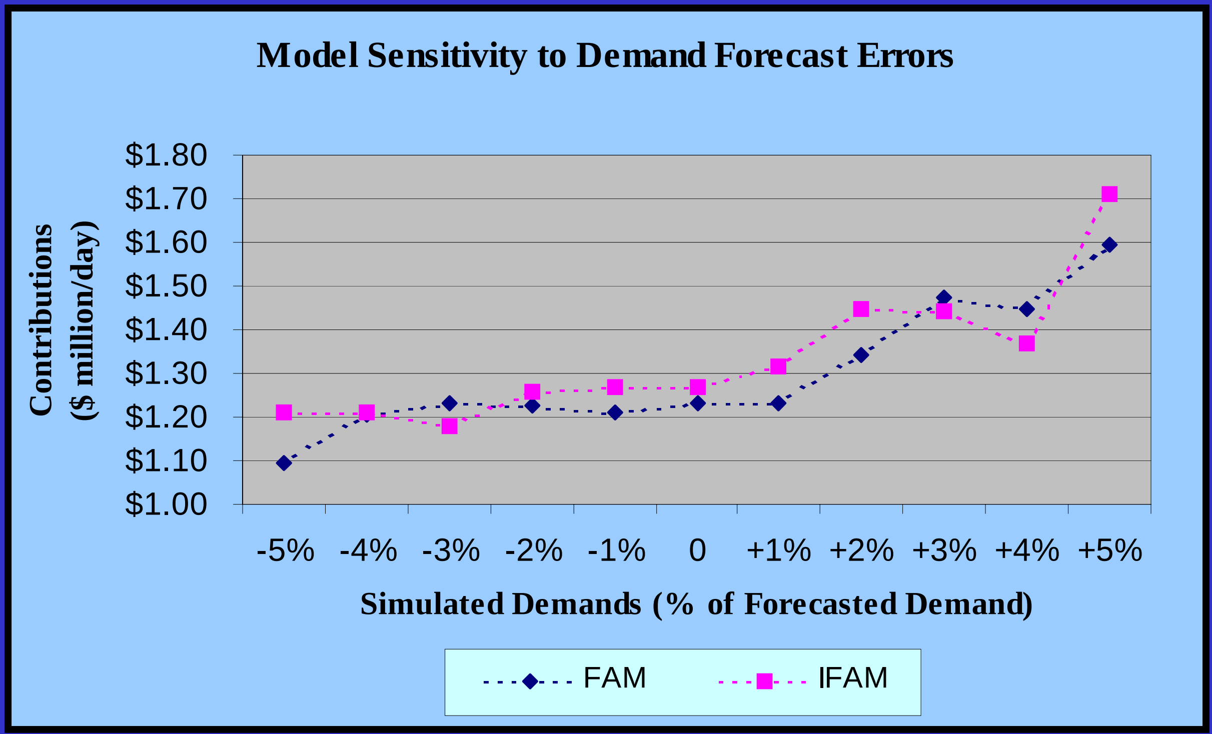
Demand Stochasticity

Forecast Errors

Data Quality Issue



Demand deviation ~15%



Extending Fleet Assignment Models to Include "Incremental" Schedule Design…

Airline Schedule Planning

Schedule Design

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Fleet Assignment

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Aircraft Routing

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Crew Scheduling

Select optimal set of *flight legs* in a schedule

Assign aircraft types to flight legs such that *contribution* is maximized

Schedule Design: Fixed Flight Network, Flexible Schedule Approach

- Fleet assignment model with time windows
  - Allows flights to be re-timed slightly (plus/ minus 10 minutes) to allow for improved utilization of aircraft and improved capacity assignments

➢ Initial step in integrating flight schedule design and fleet assignment decisions

Example: Results
Aircraft Utilization

Do time windows allow a reduction in the number of required aircraft?

	TW = 0		TW = 20	
	a/c req'd	cost	a/c req'd	cost
P1	*365*	*28,261,302*	*363*	*28,114,913*
P2	*428*	*29,000,175*	*426*	*28,965,409*

Results

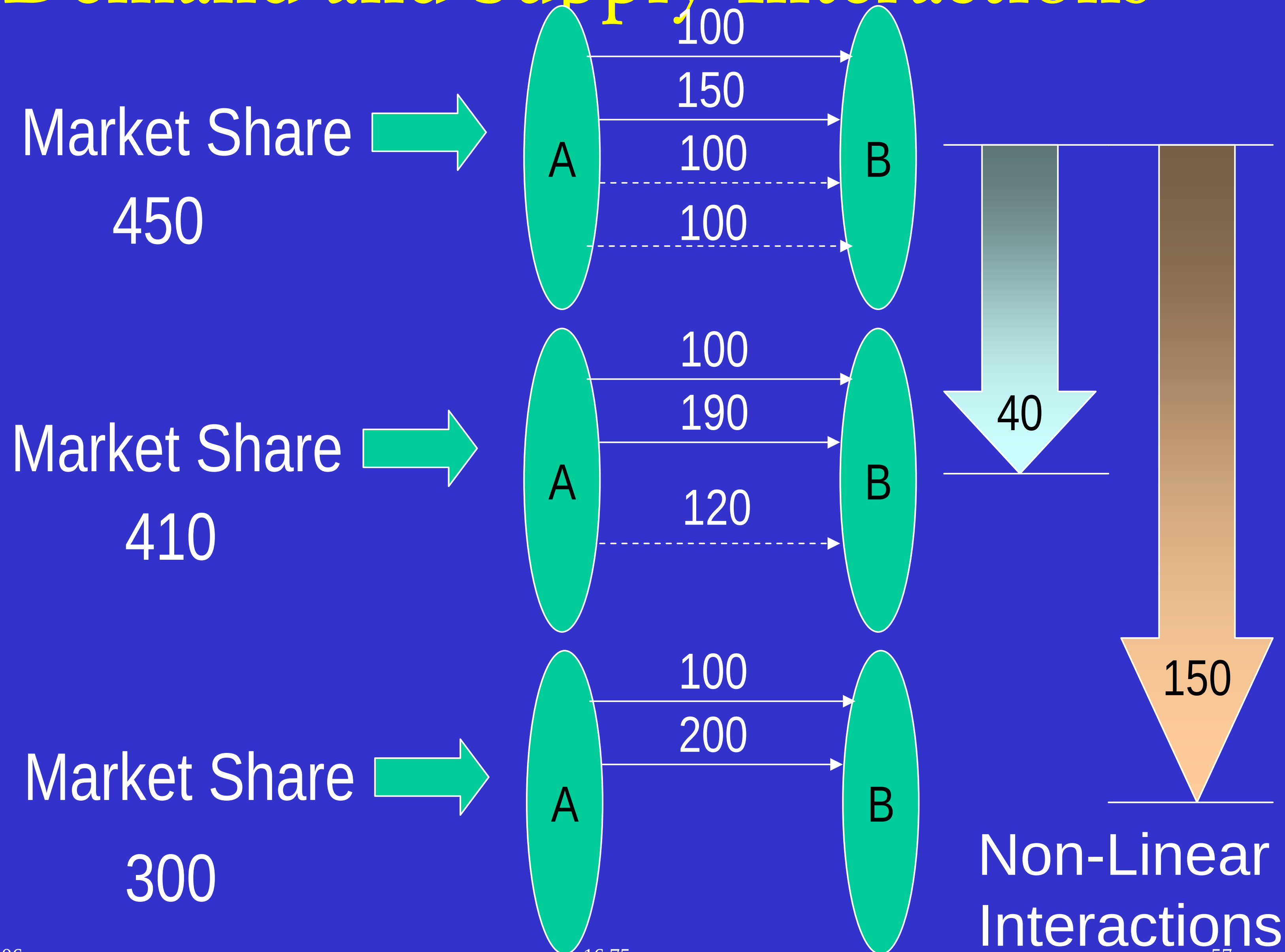
- Time windows can provide significant cost savings, as well as a potential for freeing aircraft
  - $50 million in operating costs alone for one U.S. airline

Schedule Design: Optional Flights, Flexible Schedule Approach

- Fleet assignment with “optional” flight legs
  - Additional flight legs representing varying flight departure times
  - Additional flight legs representing new flights
  - Option to eliminate existing flights from future flight network

➢ Incremental Schedule Design

Demand and Supply Interactions



Formulation

Schedule Design

Fleet Assignment

Spill + Recapture

$$t_p^r \geq 0 \qquad f_{k,i} \in \{0,1\} \qquad y_{k,o,t} \geq 0$$

Lohatepanont, M. and Barnhart, Cynthia, "Airline Schedule Planning: Integrated Models and Algorithms for Schedule Design and Fleet Assignment," Transportation Science, *future*.

Schedule Design: Results

- Demand and supply interactions
  - Tractability potentially a big issue
- Resulting schedules operate fewer flights
  - Lower operating costs
  - Fewer aircraft required
- Order of magnitude impact: ~ \$100 - \$350 million improvement annually for variable market demand
  - Rough estimates: sensitive to quality of data, spill and recapture assumptions, demand forecasts and stochasticity
  - Comparison to *planners' schedules*
  - Excludes benefits from saved aircraft