

Airline Demand Analysis and Spill Modeling

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Stochastic and Cyclical Nature of Demand

- The total demand for a particular flight even at a given time of day fluctuates by day of week and season of the year. In addition to these more predictable or “cyclical” fluctuations in demand, there are also less predictable or “stochastic” variations in demand around the mean or expected value for a flight:
 - We can represent the total uncertainty of demand for a future flight departure with a probability distribution (density) of expected demand.
 - Historically, a Gaussian (Normal) distribution of demand has been assumed, with a mean and standard deviation that depend on the market being studied and on the nature of its traffic.
 - Based on many empirical studies of actual airline data, the standard deviation of total demand for a flight relative to the mean demand is typically between 0.20 and 0.40. This measure is also known as the “coefficient of variation” or “k-factor” of demand, defined as:
$$\text{k-factor} = \text{standard deviation} / \text{mean demand}$$
 - The “k-factor” of demand for a given flight will depend on the variability of demand over a series of days, weeks or months. Leisure markets typically will have higher k-factors of demand than business or mixed travel markets with more stable demand patterns.
- It is important to understand that, in practice, it is difficult if not impossible to observe the actual “demand” for a flight. This is especially true if at least some of the departures of a series of flights on the same route at the same time of day and/or day of week depart full:
 - Airlines have no way of keeping track of how many requests for bookings on a given flight departure were turned away or rejected if there was not enough space on the aircraft to accommodate all passengers that wished to travel on the flight.
 - The notion of “total demand” for a particular flight or set of flights operated over a period of time is therefore a theoretical concept. Thus, the analysis of “total demand” (as opposed to the number of passengers who actually were transported) requires models and assumptions, as described below.

Terms and Definitions

- **DEMAND**: The total number of potential passengers wishing to make a reservation on a particular scheduled flight leg. In line with our definition of “demand” for an origin-destination market in Air Transportation Economics, the “demand” for a flight leg reflects a maximum potential, independent of the capacity being offered on the flight.
- **LOAD**: The total number of passengers who are actually carried on the flight leg. Because the demand for a flight can sometimes be greater than its capacity, it must be the case that load is always less than or equal to demand:
 - When demand is less than capacity, then load is equal to demand, as all potential passengers are accommodated and carried.
 - When demand exceeds capacity, then the load is equal to capacity, as some of the potential passengers cannot travel and must be rejected by the airline.
- **SPILL**: The total number of potential passengers who cannot obtain a reservation and travel on a given flight due to insufficient capacity. “Spill” is also known as “rejected demand”, since these passengers are rejected by the airline because the number of seats on the aircraft assigned to the flight is less than total potential demand.
 - Spill is by definition equal to total demand minus the total load of a flight.
 - When demand is less than capacity, load is equal to demand, and spill is zero.
 - When demand exceeds capacity, load is equal to capacity and spill is equal to demand minus capacity (load).
- “Spill” occurs as the result of greater potential demand for a flight than the physical capacity of the aircraft assigned to operate on the flight leg in question. “Spill” has little direct relationship to overbooking, and must not be confused with “denied boardings”. We will explore overbooking and denied boarding issues in much greater detail in Module 6. In the meantime, the most important differences between the two concepts can be summarized as follows:
 - “Spill” is the rejected demand resulting from operating two small an aircraft on a flight leg. It can occur whether or not the airline is using the practice of flight overbooking. For example, even if we assume no overbooking, an aircraft that departs with a load equal to capacity will likely have experienced spill.
 - “Denied boardings” occur when the airline overbooks its flights, and more passengers show up than there are physical seats available on the aircraft. Denied boardings can occur even if no spill occurred during the booking process (i.e., all

potential travelers obtained reservations for their desired flight, but when they showed up at the airport, there were not enough physical seats on the airplane to accommodate them. Denied boardings occurred because the airline overbooked too aggressively, not because the aircraft was too small for the demand.

- Perhaps the best way to distinguish between “spill” and “denied boardings” is to recognize that spill can occur during the booking process for a flight, as reservations requests are rejected because the aircraft is too small. On the other hand, denied boardings occur at the airport, just before departure.

Example: Individual Flight Departures

- It is common in the airline scheduling and fleet assignment process for the airline to assign the same aircraft type (capacity) to a series of departures of the same flight leg over a period of weeks or even months. Consider an example in which a 9:00 a.m. flight is scheduled to operate from Montreal (YUL) to Los Angeles (LAX) with a physical capacity of 125 seats. Below is a hypothetical set of loads as observed by the airline, for a sample of 5 Friday morning departures:

<u>DATE</u>	<u>LOAD</u>	<u>CAPACITY</u>	<u>LOAD FACTOR</u>	<u>SPILL?</u>
01 APR	92	125	73.6%	NO
08 APR	125	125	100.0%	LIKELY
15 APR	108	125	86.4%	NO
22 APR	83	125	66.4%	NO
29 APR	123	125	98.4%	POSSIBLY

- For this sample of five Friday morning flight departures, the airline can observe the following:
 - The Average Load Factor (ALF) of this flight leg over the 5 departures is 85.0%.
 - Three of the departures (on April 1, 15 and 22) had an observed load substantially less than capacity, so we can assume that no spill occurred on these departures and that total demand is well represented by the observed load.
 - On April 8, the flight departed full, meaning that potential demand for the flight was at least 125, and likely greater than 125. We assume that a full flight is an indication of rejected demand or passenger spill having occurred.
 - The April 29 flight departed with a load of 123, yet the table above suggests that spill was “possible” on this flight. This is because flights that depart with very high load factors likely reached their physical and/or booking capacity at some point close to departure, resulting in spill. Passenger no-shows and/or last-minute cancellations of reservations in the real world can lead to a load slightly below capacity, even though demand was rejected.

- For the purposes of fleet assignment, the question that this example raises is whether the 125-seat aircraft assigned to this flight leg for the period of 5 Fridays in question was the best choice of available aircraft types in terms of its capacity, as opposed to being “too big” or “too small”:
 - The 125-seat aircraft was clearly adequate for the three departures with observed load factors well below 100%.
 - However, the possibility exists that the 125-seat aircraft was too small to accommodate all of the potential demand on the other 2 departures identified as candidates for having rejected demand.
- The challenge of spill analysis for fleet assignment decision making is to try to estimate what the actual “unconstrained” demand might have been for the departures with inadequate capacity. The question is, “if the flight is/was operated with a bigger capacity, how much would spill decrease and by how much would the total revenue of the flight increase?”
 - Estimation of unconstrained demand requires a sample of observed load data for multiple departures operated with a given aircraft capacity. We cannot estimate the amount of spill or the unconstrained demand based on observed load data for a single departure.
 - For example, because the April 8 flight departed with a 100% load factor, it is very likely that demand was rejected during the booking process for that flight. However, all we know is that unconstrained demand potential for that departure was greater than (or equal to) 125 – we have no way of estimating how much greater it was.
 - With a sample of multiple departures, we can use statistical techniques for estimating average spill and average unconstrained demand on a “typical” departure. These techniques are known as “spill models” and are describe in detail in the next section.

Spill Model for Estimating Spill and Unconstrained Demand

- Variations of the “spill model” for airline demand analysis, specifically for estimating spill and unconstrained demand, have been developed both at MIT and by Boeing. The basic spill model makes the following assumptions:
 - Total demand for a flight departure or series of flight departures can be represented by a Gaussian distribution.
 - The demand distribution has a mean and standard deviation that is known or which can be estimated from a sample observed historical load data for the same or similar flights.

- The estimated demand distribution can represent the magnitude and variability of demand for future flight departures, if properly adjusted for trends and/or seasonal changes in demand.
- The details of the Boeing Spill Model are described in “Load Factor Analysis: The Relationship Between Flight Load and Passenger Turnaway” (1978). In the remainder of this section, the use of the spill model approach to estimate spill and demand for the previous simple example of five flight departures is explained.
- In our example, based on the sample of five flight departures, we can calculate that the mean load was 106.2 passengers per flight, with a sample standard deviation of 18.6:
 - Based on these measures, we can calculate the average load factor as:

$$ALF = \text{Mean Load} / \text{Capacity} = 106.2 / 125 = 85.0\%$$
 - The k-factor of the observed loads in the sample is:

$$K = \text{Standard deviation} / \text{Mean Load} = 18.6 / 106.2 = 0.175$$
- However, we know that 2 of the flights were full or close to full, suggesting that spill occurred on these departures. This further means that the following must be true:
 - The observed mean load per flight of 106.2 is less than the “true” or unconstrained total mean demand for these flights (since there was some spill).
 - The observed standard deviation of loads is less than the “true” standard deviation of demand for these flights (since on 2 of the flights, the capacity constraint limited our ability to observe how variable the actual demand was).
- The Boeing Spill Model approach relies on the properties of the Gaussian distribution to estimate spill and unconstrained demand, in one of several ways:
 - Normalized “spill tables” are the simplest way to estimate spill and unconstrained demand, and will be described here.
 - Use of “normal probability paper” to plot observed loads and estimate the mean and standard deviation of total demand is more complicated and is described in the Boeing paper.
 - Use of iterative statistical estimation methods is even more complicated, and well beyond the scope of this course.

- To make use of the “Spill Table” approach to estimation, the following additional terms and definitions are required:
 - DEMAND FACTOR is the mean total (unconstrained) demand per flight divided by the aircraft capacity. Unlike average load factor, the demand factor can exceed 1.0, as it is possible for mean total demand to exceed the assigned aircraft capacity. It is the total mean demand and/or the demand factor that we are trying to estimate, given an observed average load factor.
 - SPILL FACTOR is the average (or “expected”) spilled passengers per flight divided by the aircraft capacity. Again, estimation of mean spill and/or the spill factor is the goal of our estimation effort.
 - SPILL RATE is the average (or “expected”) spilled passengers per flight divided by the mean total demand for the flight. Thus, it is a measure of the probability or likelihood that a random passenger wishing to make a reservation for a flight will not be able to due to inadequate aircraft capacity. It is also the proportion of total demand that is rejected.
- Given these definitions and those introduced earlier the following relationships must apply for a sample of flight departures with any degree of variability in demand (i.e., with a standard deviation greater than zero):
 - Mean demand is greater than or equal to the mean observed load. Mean demand is equal to mean load only if none of the flight departures in the sample experienced spill.
 - Demand factor is greater than or equal to average load factor. Demand factor is equal to average load factor only if none of the flight departures in the sample experienced spill.
 - Mean demand is always equal to mean observed load plus mean spill per flight.
 - Demand factor is always equal to average load factor plus spill factor.
 - Spill rate and spill factor must both be greater than zero if any of the flight departures in the sample experienced spill.

Example: Use of Spill Tables

- The Boeing Spill Tables contain “pre-calculated” relationships between demand factor, load factor, and spill factor, based on the assumed Gaussian distribution of total demand for a flight. We can use these tables to answer two types of questions:
 - Given an observed mean load factor and an assumed k-factor of unconstrained total demand, what is the corresponding demand factor and spill factor?

- Given an estimate of demand factor for a flight and an assumed k-factor of unconstrained total demand, what would be the corresponding average load factor and spill factor?
- Thus, the spill table approach requires the user to assume a k-factor of unconstrained total demand for a flight. This assumption is essentially a “best guess”, based on previous experience with demand variability for a given flight or within a particular market. This requirement is a minor limitation of the spill table approach that is not shared by the more complicated estimation methods mentioned earlier.
 - For this example, we will assume a “typical” k-factor of total demand of 0.35.
 - The Boeing Spill Table for demand factor $k=0.35$ is attached.
- For our small example of 5 departures, we have the following information:
 - Mean observed load = 106.2
 - Average load factor = 85.0%
- Now, using the Boeing Spill Table for $k=0.35$, we can read the following:
 - For average load factor 85.0%, the estimated demand factor is 0.972 and the estimated spill factor is 0.122. Note that $DF = LF + SF$ in all cases!
- These estimated values from the Spill table allow us to make the following additional calculations for this flight sample:
 - Mean total demand = Demand Factor * Capacity = $0.972 * 125 = 121.5$
 - Standard deviation of demand = $0.35 * \text{Mean demand} = 0.35 * 121.5 = 42.5$
 - Mean spill per flight = Spill Factor * Capacity = $0.122 * 125 = 15.25$
 - Spill rate per flight = Mean Spill/Mean Demand = $15.25/121.5 = 12.55\%$
- If we believe that our (admittedly small) sample of 5 departures of this flight is representative of the demand for the flight, then we can estimate that if the airline continues to use a 125-seat aircraft, the loss of demand and revenue due to spill appears to be substantial:
 - As a general rule, spill rates much higher than approximately 5% are regarded by most airline managers as being “too high”. A loss of potential revenue of substantially more than 5% is a significant lost opportunity for airlines that typically struggle to exceed a 5% operating margin.

- In our example, it appears that the use of a 125-seat aircraft to serve this estimated demand distribution results in substantial spill and loss of potential revenue. The spill table approach can be used to estimate how much spill might be reduced if the airline were to assign a 140-seat aircraft to this flight for future departures:
 - With a mean total demand of 121.5, the demand factor for a 140-seat aircraft would be $DF = 121.5/140 = 0.868$.
 - NOTE: The mean total unconstrained demand does not change with a change in aircraft size, as long as there are no differences in passenger preferences between the alternative aircraft types.
 - Given a new demand factor of 0.868, the spill table shows an estimated load factor of about 0.802 and an estimated spill factor of 0.066 (some “interpolation” is required between the demand factors of 0.865 and 0.870 shown on the table).
- Therefore, if the airline were to switch from a 125- to a 140-seat aircraft to serve the demand for this flight, it could expect the following:
 - An average load factor of 80.2%, as read from the Spill Table
 - An average observed load equal to $0.802 * 140 = 112.3$ passengers per flight, an increase of 6.1 passengers compared to the 125-seat aircraft.
 - Average spill per flight equal to $0.066 * 140 = 9.24$ passengers per flight, a decrease of just over 6 spilled passengers compared to the 125-seat aircraft.
 - A spill rate of $9.24/121.5 = 7.6\%$, down from 12.6% with the smaller aircraft.
- Clearly, assignment of a bigger aircraft results in higher loads (but lower load factors), increased revenues, and reduced spill and spill rate, as expected. In this case, given our assumption of $k=0.35$ for total unconstrained demand, the demand distribution reflects quite high (but not atypical) demand variability. With such high variation of demand relative to the mean even a 15-seat increase in capacity does not eliminate spill:
 - The fleet assignment question for the airline is whether the increase in revenue from the increased capacity to accommodate on average 6 additional passengers per flight exceeds the additional operating cost of the 140-seat aircraft compared to the 125-seat aircraft used currently. If so, then the airline will increase its operating profit by assigning the larger aircraft to this particular flight leg.