



# Introduction to Revenue Management: Flight Leg Revenue Optimization

## ***16.75 Airline Management***

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## Lecture Outline

- 1. Airline Revenue Maximization**
  - Pricing vs. Yield (Revenue) Management
- 2. Computerized RM Systems**
  - RM System in ePODS
- 3. Single-leg Fare Class Seat Allocation Problem**
  - Partitioned vs. Serial Nesting of Booking Classes
  - Deterministic vs. Probabilistic Demand
- 4. EMSRb Model for Seat Protection**
  - Example of Calculations



# 1. Airline Revenue Maximization

- **Two components of airline revenue maximization:**

## **Differential Pricing:**

- Various “fare products” offered at different prices for travel in the same O-D market

## **Yield Management (YM):**

- Determines the number of seats to be made available to each “fare class” on a flight, by setting booking limits on low fare seats
- **Typically, YM takes a set of differentiated prices/products and flight capacity as given:**
  - With high proportion of fixed operating costs for a committed flight schedule, revenue maximization to maximize profits



## Why Call it “Yield Management”?

- Main objective of YM is to protect seats for later-booking, high-fare business passengers.
- YM involves tactical control of airline’s seat inventory:
  - But too much emphasis on yield (revenue per RPM) can lead to overly severe limits on low fares, and lower overall load factors
  - Too many seats sold at lower fares will increase load factors but reduce yield, adversely affective total revenues
- Revenue maximization is proper goal:
  - Requires proper balance of load factor and yield
- Many airlines now refer to “Revenue Management” (RM) instead of “Yield Management”



# Seat Inventory Control Approaches

EXAMPLE: 2100 MILE FLIGHT LEG

CAPACITY = 200

## NUMBER OF SEATS SOLD:

FARE CLASS	AVERAGE REVENUE	YIELD EMPHASIS	LOAD FACTOR EMPHASIS	REVENUE EMPHASIS
Y	\$420	20	10	17
B	\$360	23	13	23
H	\$230	22	14	19
V	\$180	30	55	37
Q	\$120	15	68	40
TOTAL PASSENGERS		110	160	136
LOAD FACTOR		55%	<b>80%</b>	68%
TOTAL REVENUE		\$28,940	\$30,160	<b>\$31,250</b>
AVERAGE FARE		\$263	\$189	\$230
YIELD (CENTS/RPM)		<b>12.53</b>	8.98	10.94



# Revenue Management Techniques

- **Overbooking**
  - Accept reservations in excess of aircraft capacity to overcome loss of revenues due to passenger “no-show” effects
- **Fare Class Mix (Flight Leg Optimization)**
  - Determine revenue-maximizing mix of seats available to each booking (fare) class on each flight departure
- **Traffic Flow (O-D) Control (Network Optimization)**
  - Further distinguish between seats available to short-haul (one-leg) vs. long-haul (connecting) passengers, to maximize total network revenues
  - Currently under development by some airlines



## 2. Computerized RM Systems

- **Size and complexity of a typical airline's seat inventory control problem requires a computerized RM system**
- **Consider a US Major airline with:**
  - 2000 flight legs per day
  - 10 booking classes
  - 300 days of bookings before departure
- **At any point in time, this airline's seat inventory consists of 6 million booking limits:**
  - This inventory represents the airline's potential for profitable operation, depending on the revenues obtained
  - Far too large a problem for human analysts to monitor alone

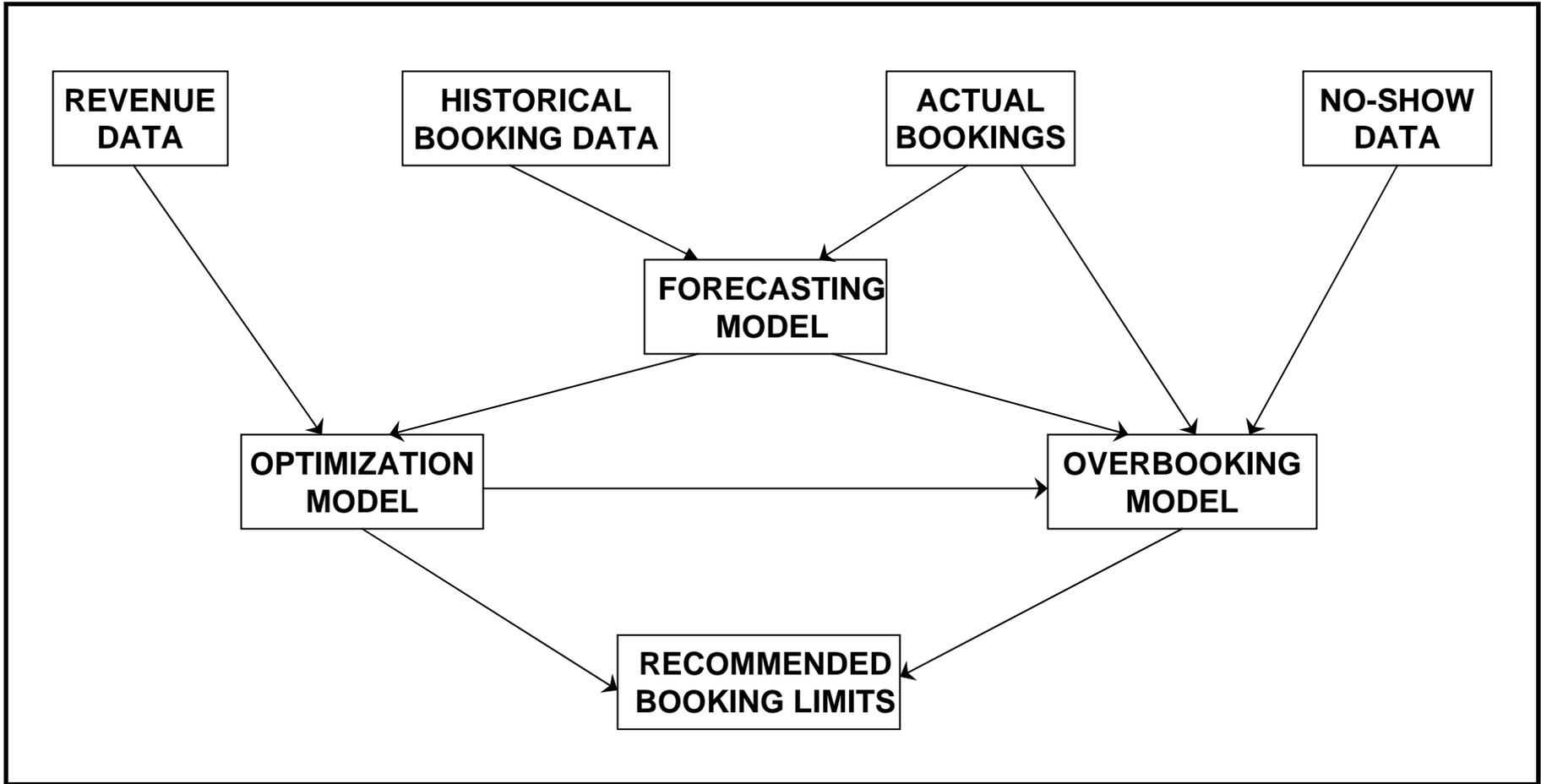


## Typical 3rd Generation RM System

- **Collects and maintains historical booking data by flight and fare class, for each past departure date.**
- **Forecasts future booking demand and no-show rates by flight departure date and fare class.**
- **Calculates limits to maximize total flight revenues:**
  - **Overbooking levels to minimize costs of spoilage/denied boardings**
  - **Booking class limits on low-value classes to protect high-fare seats**
- **Interactive decision support for RM analysts:**
  - **Can review, accept or reject recommendations**



## Example of Third Generation RM System





## Dynamic Revision and Intervention

- **RM systems revise forecasts and re-optimize booking limits at numerous “checkpoints” of the booking process:**
  - Monitor actual bookings vs. previously forecasted demand
  - Re-forecast demand and re-optimize at fixed checkpoints or when unexpected booking activity occurs
  - Can mean substantial changes in fare class availability from one day to the next, even for the same flight departure
- **Substantial proportion of fare mix revenue gain comes from dynamic revision of booking limits:**
  - Human intervention is important in unusual circumstances, such as “unexplained” surges in demand due to special events



## Current State of RM Practice

- **Most of the top 25 world airlines (in terms of revenue) have implemented 3rd generation RM systems.**
- **Many smaller carriers are still trying to make effective use of leg/fare class RM**
  - Lack of company-wide understanding of RM principles
  - Historical emphasis on load factor or yield, not revenue
  - Excessive influence and/or RM abuse by dominant sales and marketing departments
  - Issues of regulation, organization and culture
- **About a dozen leading airlines are looking toward network O-D control development and implementation**
  - These carriers could achieve a 2-5 year competitive advantage with advanced revenue management systems



## **“Vanilla” RM System in ePODS**

- **Airlines’ RM systems forecast fare class demand for each flight leg departure:**
  - Simple “pick-up” forecasts of bookings still to come
  - Unconstraining of closed observations based on booking curve probabilities.
- **Optimization is leg-based EMSRb seat protection algorithm:**
  - Booking limits set for each fare class on each flight leg departure, revised 16 times during booking process.
- **No overbooking or no-shows in ePODS.**

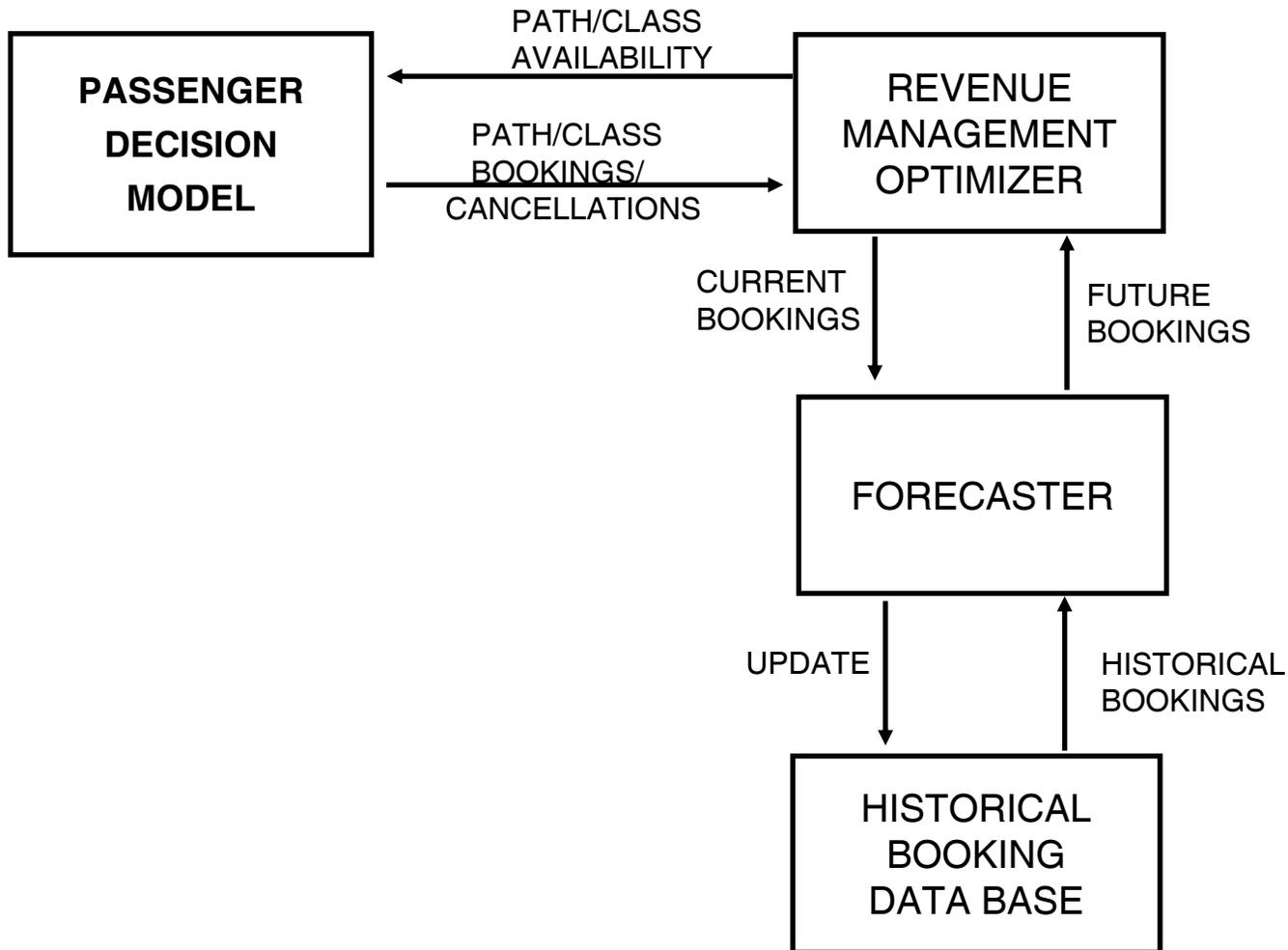


## Revenue Management Intervention

- **ePODS replicates airline RM system actions over time, taking into account previous interventions:**
  - **Previously applied booking limits affect actual passenger loads and, in turn, future demand forecasts**
- **“Historical” booking data is used to generate forecasts for “future” departures.**
- **RM system only uses data available from past observations.**



## PODS Simulation: Basic Schematic





### 3. Single-Leg Seat Allocation Problem

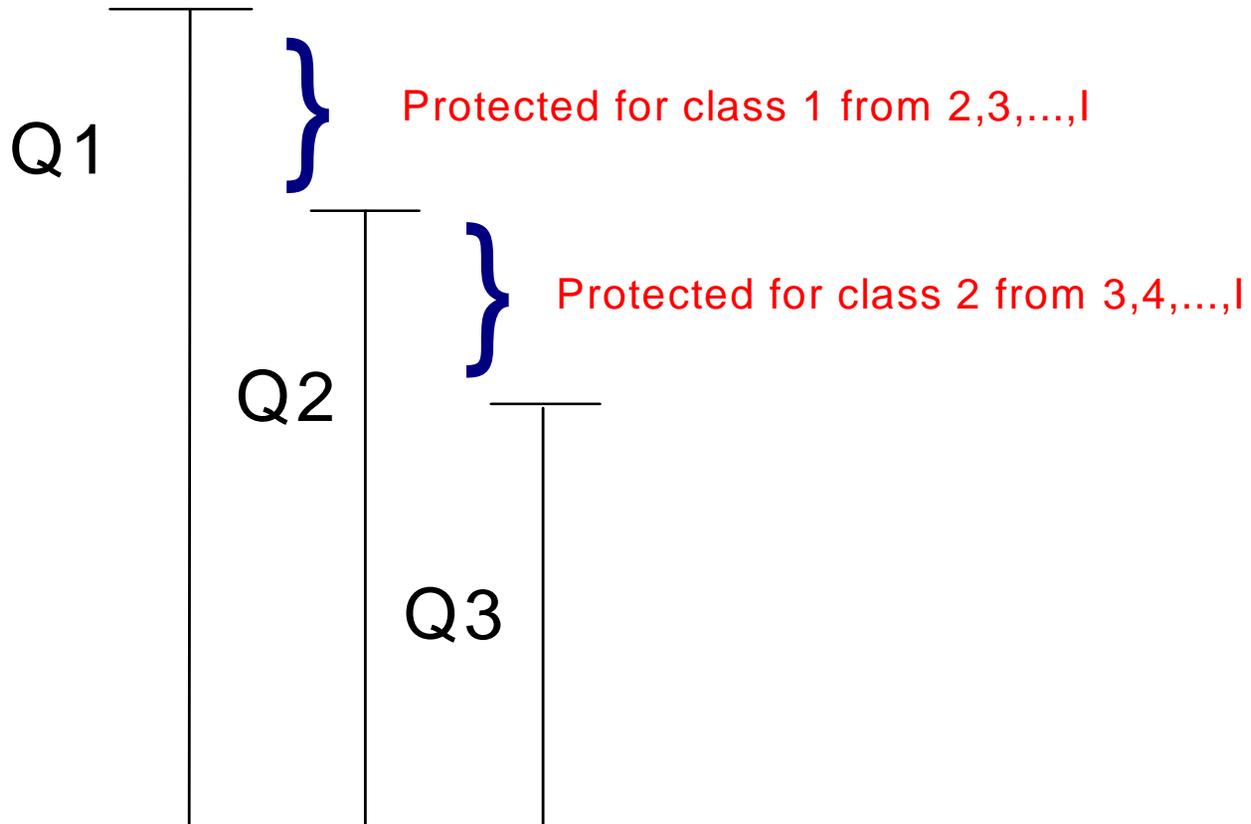
- **Given for a future flight leg departure:**
  - Total booking capacity of (typically) the coach compartment
  - Several fare (booking) classes that share the same inventory of seats in the compartment
  - Forecasts of future booking demand by fare class
  - Revenue estimates for each fare (booking) class
- **Objective is to maximize total expected revenue:**
  - Allocate seats to each fare class based on value



## Partitioned vs. Serial Nesting

- In a partitioned CRS inventory structure, allocations to each booking class are made separately from all the other classes.
- **EXAMPLE** (assuming uncertain demand):
  - Given the following allocations for each of 3 classes--Y = 30, B = 40, M = 70 for an aircraft coach cabin with booking capacity = 140.
  - If 31 Y customers request a seat, the airline would reject the 31<sup>st</sup> request because it exceeds the allocation for the Y class
  - It is possible that airline would reject the 31<sup>st</sup> Y class customer, even though it might not have sold all of the (lower-valued) B or M seats yet!
- Under serial nesting of booking classes, the airline would never turn down a Y fare request, as long as there are any seats (Y, B or M) left for sale.

# Serially Nested Buckets





## Deterministic Seat Allocation/Protection

- **If we assume that demand is deterministic (or known with certainty), it would be simple to determine the fare class seat allocations**
  - Start with highest fare class and allocate/protect exactly the number of seats predicted for that class, and continue with the next lower fare class until capacity is reached.
- **EXAMPLE: 3 fare classes (Y, B, M)**
  - Demand for Y = 30, B = 40, M = 85
  - Capacity = 140
- **Deterministic decision: Protect 30 for Y, 40 for B, and allocated 70 for M (i.e., spill 15 M requests)**
- **Nested booking limits Y=140 B=110 M=70**



# EMSRb Model for Seat Protection: Assumptions

- **Basic modeling assumptions for serially nested classes:**
  - a) demand for each class is separate and independent of demand in other classes.**
  - b) demand for each class is stochastic and can be represented by a probability distribution**
  - c) lowest class books first, in its entirety, followed by the next lowest class, etc.**
  - d) booking limits are only determined once (i.e., static optimization model)**



## EMSRb Model Calculations

- **Because higher classes have access to unused lower class seats, the problem is to find seat protection levels for higher classes, and booking limits on lower classes**
- **To calculate the optimal protection levels:**
  - Define  $P_i(S_i) = \text{probability that } X_i \geq S_i$ ,
  - where  $S_i$  is the number of seats made available to class  $i$ ,  $X_i$  is the random demand for class  $i$



## EMSRb Calculations (cont'd)

- **The expected marginal revenue of making the Sth seat available to class i is:**

$$\text{EMSR}_i(S_i) = R_i * P_i(S_i) \text{ where } R_i \text{ is the average revenue (or fare) from class } i$$

- **The optimal protection level,  $\pi_1$  for class 1 from class 2 satisfies:**

$$\text{EMSR}_1(\pi_1) = R_1 * P_1(\pi_1) = R_2$$

- **Once  $\pi_1$  is found, set  $BL_2 = \text{Capacity} - \pi_1$ . Of course,  $BL_1 = \text{Capacity}$  (authorized capacity if overbooking)**



## Example Calculation

Consider the following flight leg example:

<u>Class</u>	<u>Mean Fcst.</u>	<u>Std. Dev.</u>	<u>Fare</u>
Y	10	3	1000
B	15	5	700
M	20	7	500
Q	30	10	350

- To find the protection for the Y fare class, we want to find the largest value of  $\pi_Y$  for which
$$\text{EMSR}_Y(\pi_Y) = R_Y * P_Y(\pi_Y) \geq R_B$$



## Example (cont'd)

$$\text{EMSR}_Y(\pi_Y) = 1000 * P_Y(\pi_Y) \geq 700$$
$$P_Y(\pi_Y) \geq 0.70$$

where  $P_Y(\pi_Y)$  = probability that  $X_Y \geq \pi_Y$ .

- **If we assume demand in Y class is *normally* distributed with mean, standard deviation given earlier, then we can create a standardized normal random variable as  $(X_Y - 10)/3$ .**



## Probability Calculations

- **Next, we use Excel or go to the Standard Normal Cumulative Probability Table for different “guesses” for  $\pi_Y$ . For example,**
  - for  $\pi_Y = 7$ ,  $\text{Prob} \{ (X_Y - 10)/3 \geq (7 - 10)/3 \} = 0.841$
  - for  $\pi_Y = 8$ ,  $\text{Prob} \{ (X_Y - 10)/3 \geq (8 - 10)/3 \} = 0.747$
  - for  $\pi_Y = 9$ ,  $\text{Prob} \{ (X_Y - 10)/3 \geq (9 - 10)/3 \} = 0.63$
- **So, we can see that  $\pi_Y = 8$  is the largest integer value of  $\pi_Y$  that gives a probability  $\geq 0.7$  and therefore we will protect 8 seats for Y class!**



## Joint Protection for Classes 1 and 2

- How many seats to protect jointly for classes 1 and 2 from class 3?
- The following calculations are necessary:

$$\overline{X}_{1,2} = \overline{X}_1 + \overline{X}_2$$

$$\hat{\sigma}_{1,2} = \sqrt{\hat{\sigma}_1^2 + \hat{\sigma}_2^2}$$

$$R_{1,2} = \frac{R_1 * \overline{X}_1 + R_2 * \overline{X}_2}{\overline{X}_{1,2}}$$

$$P_{1,2}(S) = \Pr ob(X_1 + X_2 > S)$$



## Protection for Y+B Classes

- To find the protection for the Y and B fare classes from M, we want to find the largest value of  $\pi_{YB}$  that makes

$$\text{EMSR}_{YB}(\pi_{YB}) = R_{YB} * P_{YB}(\pi_{YB}) \geq R_M$$

- Intermediate Calculations:

$$R_{YB} = (10*1000 + 15 *700) / (10+15) = 820$$

$$\overline{X}_{Y,B} = \overline{X}_Y + \overline{X}_B = 10 + 15 = 25$$

$$\hat{\sigma}_{Y,B} = \sqrt{\hat{\sigma}_Y^2 + \hat{\sigma}_B^2} = \sqrt{3^2 + 5^2} = \sqrt{34} = 5.83$$



## Example: Joint Protection

- **The protection level for Y+B classes satisfies:**

$$820 * P_{YB}(\pi_{YB}) \geq 500$$

$$P_{YB}(\pi_{YB}) \geq .6098$$

- **Again, we can make different “guesses” for  $\pi_{YB}$ .**

$$\text{for } \pi_{YB} = 20, \text{ Prob } \{ (X_{YB} - 25)/5.83 \geq (20 - 25)/5.83 \} = 0.805$$

$$\text{for } \pi_{YB} = 22, \text{ Prob } \{ (X_{YB} - 25)/5.83 \geq (22 - 25)/5.83 \} = 0.697$$

$$\text{for } \pi_{YB} = 23, \text{ Prob } \{ (X_{YB} - 25)/5.83 \geq (23 - 25)/5.83 \} = 0.633$$

$$\text{for } \pi_{YB} = 24, \text{ Prob } \{ (X_{YB} - 25)/5.83 \geq (24 - 25)/5.83 \} = 0.5675$$



## Joint Protection for Y+B

- So, we can see that  $\pi_{YB} = 23$  is the largest integer value of  $\pi_{YB}$  that gives a probability  $\geq 0.6098$  and therefore we will jointly protect 23 seats for Y and B class from class M!
- Suppose we had an aircraft with authorized booking capacity 80 seats, our Booking Limits would be:
  - $BL_Y = 80$
  - $BL_B = 80 - 8 = 72$
  - $BL_M = 80 - 23 = 57$

## General Case for Class n

- How many seats to protect jointly for classes 1 through n from class n+1?
- The following calculations are necessary:

$$\overline{X}_{1,n} = \sum_{i=1}^n \overline{X}_i$$

$$\hat{\sigma}_{1,n} = \sqrt{\sum_{i=1}^n \hat{\sigma}_i^2}$$

$$R_{1,n} = \frac{\sum_{i=1}^n R_i * \overline{X}_i}{\overline{X}_{1,n}}$$



## General Case (cont'd)

- We then find the value of  $\pi_n$  that makes

$$\text{EMSR}_{1,n}(\pi_n) = R_{1,n} * P_{1,n}(\pi_n) = R_{n+1}$$

- Once  $\pi_n$  is found, set  $\text{BL}_{n+1} = \text{Capacity} - \pi_n$