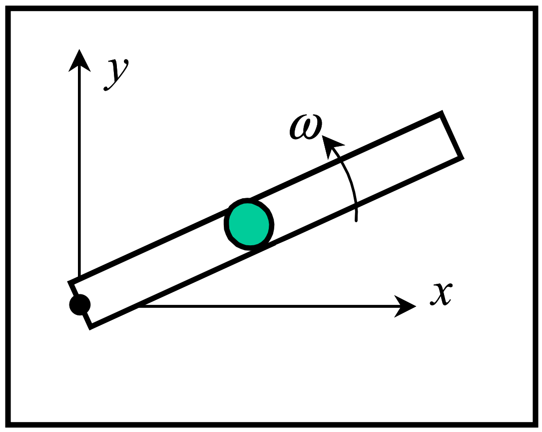
Lecture #8

Examples Using Lagrange's Equations

Example

Given: Catapult rotating at a constant rate (frictionless, in the horizontal plane)

Find the EOM of the particle as it leaves the tube.



Derivatives:

$$\frac{\partial T}{\partial \dot{r}} = m\dot{r}, \quad \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{r}}\right) = m\ddot{r}, \quad \frac{\partial T}{\partial r} = mr\omega^2$$

External forces: None

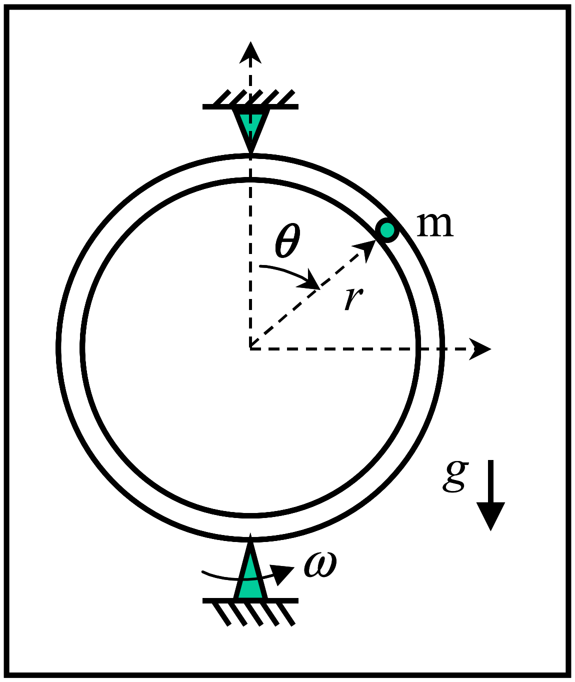
Lagrange's equation gives the equation of motion as $\ddot{r} - r\omega^2 = 0$

What do we get if we solve this via Newton's method?

Example

Mass particle in a frictionless spinning ring.

Ring spins at constant rate ω



Spherical coordinate set (2-11)

Two holonomic constraints

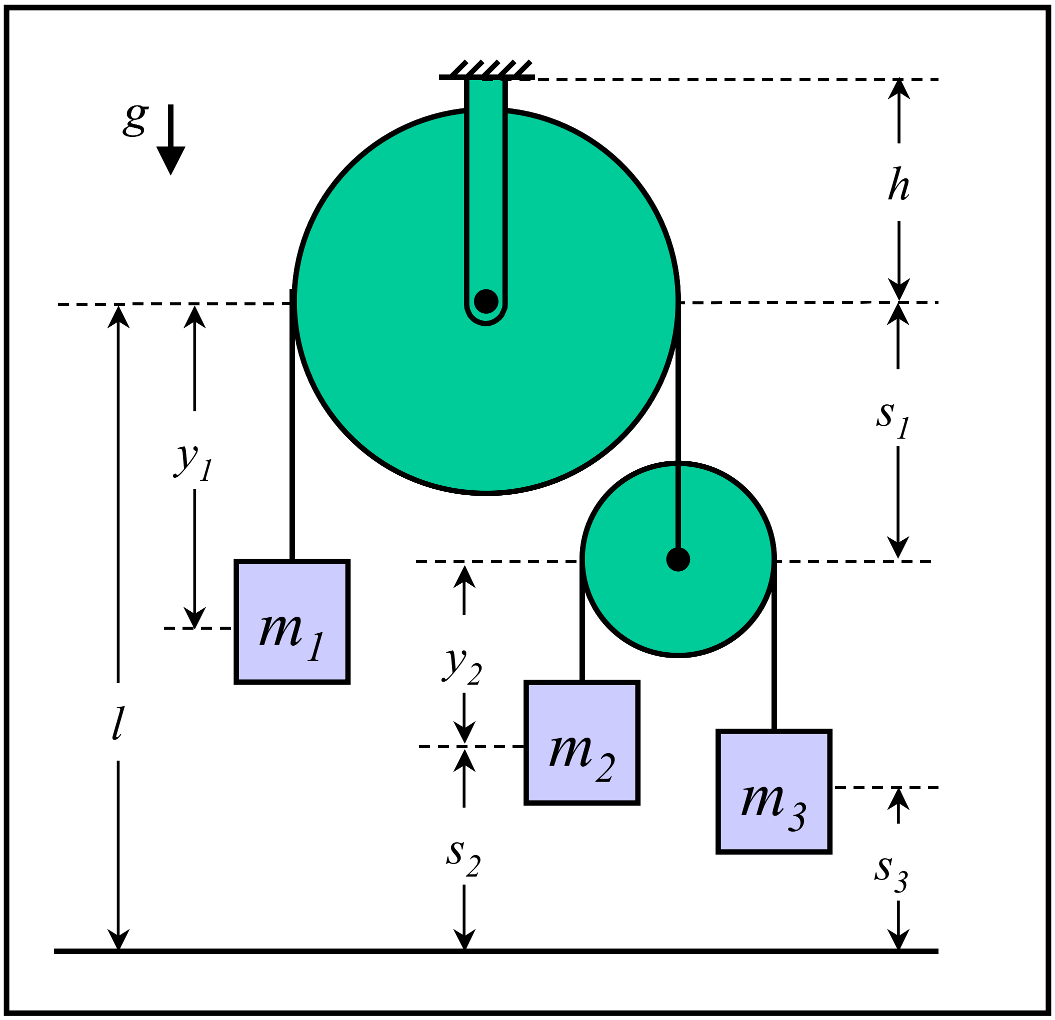
- r = constant
- $\phi = \omega t + \phi_0$ which gives the spin rate of the tube

So only 1 DOF ➔ use θ as the generalized coordinate

Example

System of 3 "particles" suspended by pulleys.

(Neglect mass of pulleys.)



Example

2 particles in a frictionless tube held by springs. Assume that

s = 0 and a = 0