Lecture #AC–3

Aircraft Lateral Dynamics

Spiral, Roll, and Dutch Roll Modes

Aircraft Lateral Dynamics

- Using a procedure similar to the longitudinal case, we can develop the equations of motion for the **lateral dynamics**

$$\dot{x} = Ax + Bu \; , \; x = \begin{bmatrix} v \\ p \\ r \\ \phi \end{bmatrix} \; , \; u = \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix}$$

and $\dot{\psi} = r \sec \theta_0$

$$A = \begin{bmatrix} \frac{Y_v}{m} & \frac{Y_p}{m} & \frac{Y_r}{m} - U_0 & g \cos \theta_0 \\ (\frac{L_v}{I'_{\text{xx}}} + I'_{\text{zx}} N_v) & (\frac{L_p}{I'_{\text{xx}}} + I'_{\text{zx}} N_p) & (\frac{L_r}{I'_{\text{xx}}} + I'_{\text{zx}} N_r) & 0 \\ (I'_{\text{zx}} L_v + \frac{N_v}{I'_{\text{zz}}}) & (I'_{\text{zx}} L_p + \frac{N_p}{I'_{\text{zz}}}) & (I'_{\text{zx}} L_r + \frac{N_r}{I'_{\text{zz}}}) & 0 \\ 0 & 1 & \tan \theta_0 & 0 \end{bmatrix}$$

where

$$\begin{aligned} I'_{\text{xx}} &= (I_{\text{xx}} I_{\text{zz}} - I_{\text{zx}}^2)/I_{\text{zz}} \\ I'_{\text{zz}} &= (I_{\text{xx}} I_{\text{zz}} - I_{\text{zx}}^2)/I_{\text{xx}} \\ I'_{\text{zx}} &= I_{\text{zx}}/(I_{\text{xx}} I_{\text{zz}} - I_{\text{zx}}^2) \end{aligned}$$

and

$$B = \begin{bmatrix} (m)^{-1} & 0 & 0 \\ 0 & (I'_{\text{xx}})^{-1} & I'_{\text{zx}} \\ 0 & I'_{\text{zx}} & (I'_{\text{zz}})^{-1} \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} Y_{\delta_a} & Y_{\delta_r} \\ L_{\delta_a} & L_{\delta_r} \\ N_{\delta_a} & N_{\delta_r} \end{bmatrix}$$

- The code gives the numerical values for all of the stability derivatives. Can solve for the eigenvalues of the matrix A to find the modes of the system.

$$-0.0331 \pm 0.9470i$$
$$-0.5633$$
$$-0.0073$$

  – Stable, but there is one very slow pole.

- There are 3 modes, but they are a **lot more complicated** than the longitudinal case.

Slow mode	-0.0073	$\Rightarrow$	**Spiral Mode**
Fast real	-0.5633	$\Rightarrow$	**Roll Damping**
Oscillatory	$-0.0331 \pm 0.9470i$	$\Rightarrow$	**Dutch Roll**

Can look at normalized eigenvectors:

	Spiral	Roll	Dutch Roll	
β	0.0067	-0.0197	0.3269	-28°
$\hat{p}$	-0.0009	-0.0712	0.1198	92°
$\hat{r}$	0.0052	0.0040	0.0368	-112°
ϕ	1.0000	1.0000	1.0000	0°

Not as enlightening as the longitudinal case.

Lateral Modes

Roll Damping - well damped.

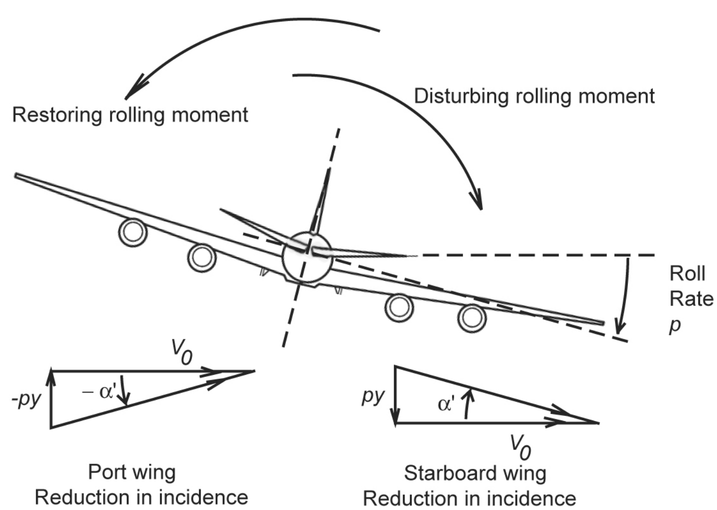
– As the plane rolls, the wing going down has an increased α (wind is effectively "coming up" more at the wing)

– Opposite effect for other wing.

– There is a difference in the lift generated by both wings $\rightarrow$ more on side going down

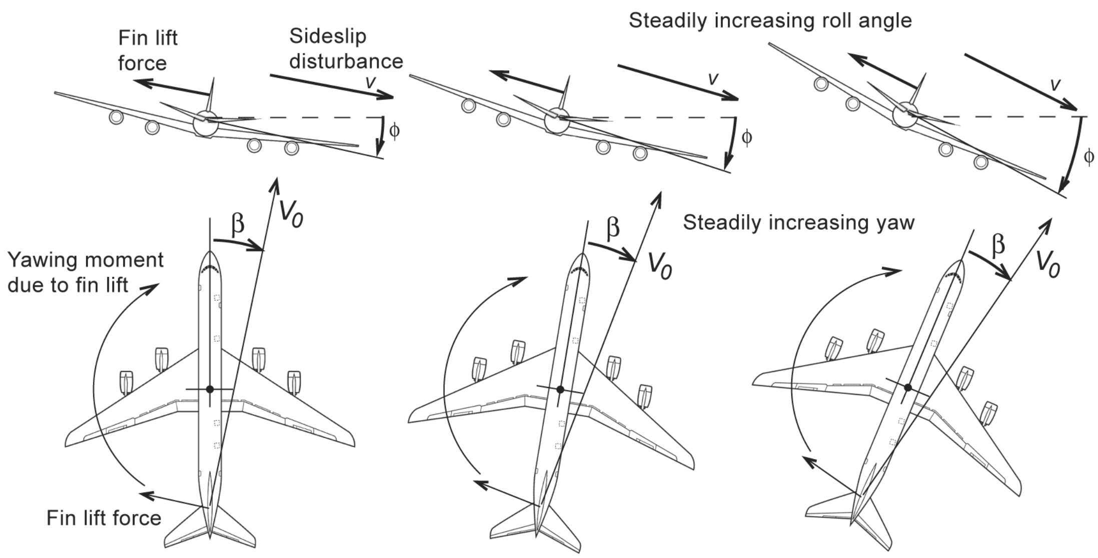
– The differential lift creates a **moment** that tends to **restore** the equilibrium

– After a disturbance, the roll rate builds up exponentially until the restoring moment balances the disturbing moment, and a steady roll is established.



Spiral Mode - slow, often unstable.

- From level flight, consider a disturbance that creates a small roll angle $\phi > 0$
- This results in a small side-slip v (vehicle *slides downhill*)
- Now the tail fin hits on the oncoming air at an incidence angle β $\rightarrow$ extra tail lift $\rightarrow$ yawing moment
- The positive yawing moment tends to increase the side-slip $\rightarrow$ makes things worse.
- If unstable and left unchecked, the aircraft would fly a slowly diverging path in roll, yaw, and altitude $\Rightarrow$ it would tend to *spiral* into the ground!!



- Can get a restoring torque from the wing **dihedral**
- Want a small tail to reduce the impact of the spiral mode.

Dutch Roll - damped oscillation in yaw, that couples into roll.

- Frequency similar to longitudinal short period mode, not as well damped (fin less effect than the horizontal tail).
- Do you know the origins on the name of the mode?
- Consider a disturbance from straight-level flight

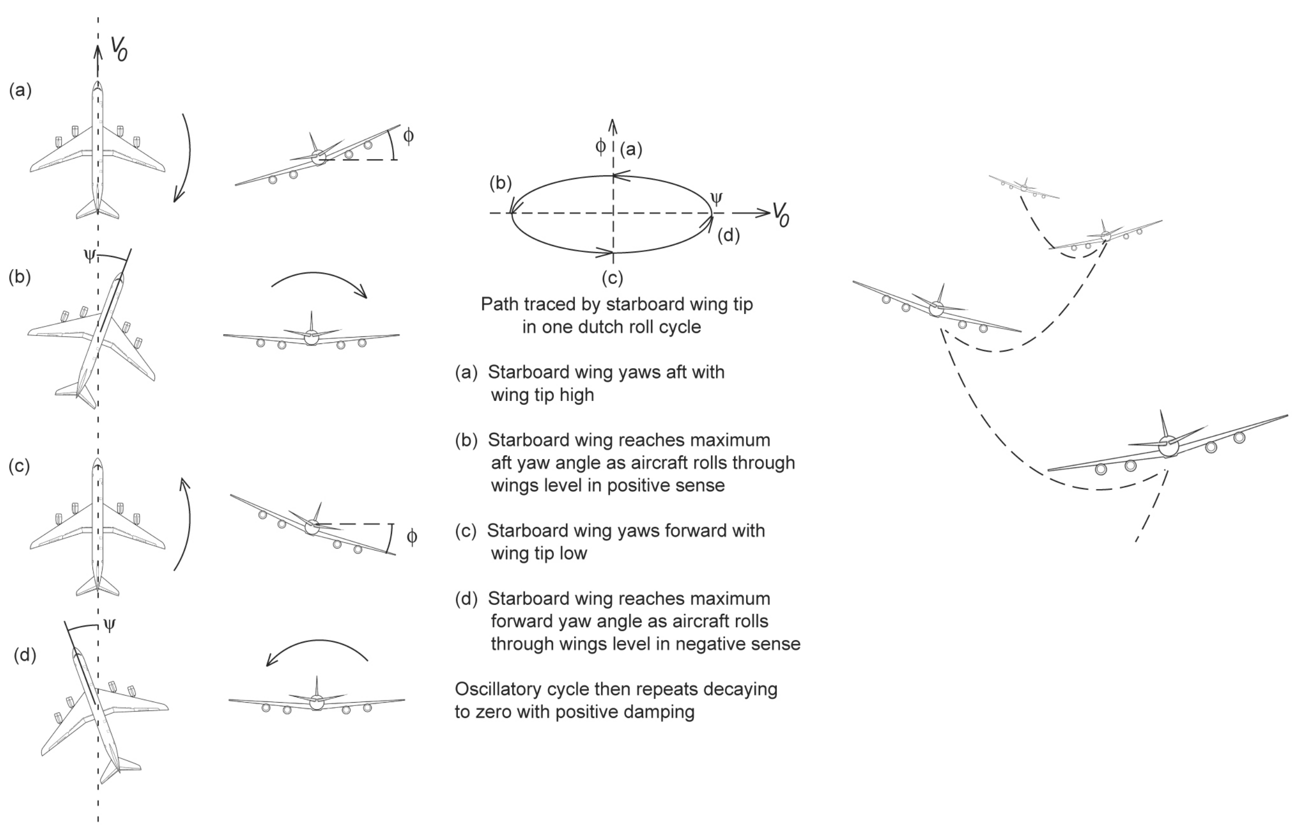
  → Oscillation in yaw ψ (fin provides the *aerodynamic stiffness*)

  → Wings moving back and forth due to yaw motion result in oscillatory differential Lift/Drag (wing moving forward generates more lift)

  → Oscillation in roll ϕ that lags ψ by approximately 90°

  ⇒ *Forward going wing is low*

  Oscillating roll → sideslip in direction of low wing.



- Damp the Dutch roll mode with a large tail fin.

Aircraft Actuator Influence

- Transfer functions dominated by lightly damped Dutch-roll mode.
  - Note the rudder is physically quite high, so it also influences the A/C roll.
  - Ailerons influence the Yaw because of the differential drag
- Impulse response for the two inputs:
  - **Rudder input**
    - ◇ β shows a very lightly damped decay.
    - ◇ p, r clearly excited as well.
    - ◇ ϕ oscillates around 2.5°
    - ⇒ Dutch-roll oscillations are clear.
    - ⇒ Spiral mode ultimately dominates $\phi \rightarrow 0$ after 250 sec.
  - **Aileron input**
    - ◇ Large impact on p
    - ◇ Causes large change to ϕ
    - ◇ Very small change to remaining variables.
    - ◇ Influence smaller than Rudder.
- Lateral approximate models are much harder to make (see discussion in Etkin and Reid). Not worth discussing at length.

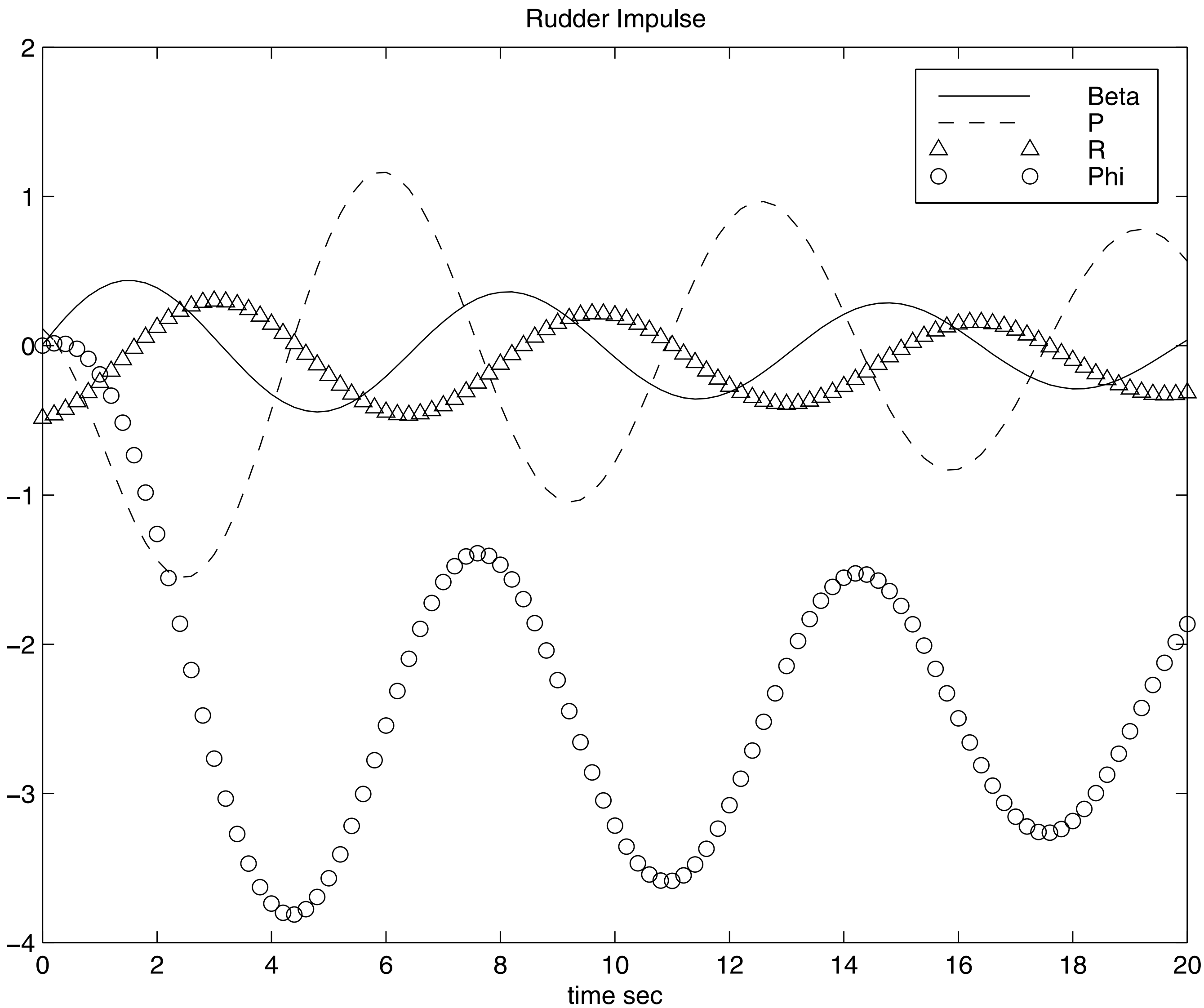


Figure 1: Rudder impulse to flight variables. The rudder excites all modes. Dutch roll oscillations dominate initially. The spiral mode dominates longer term.

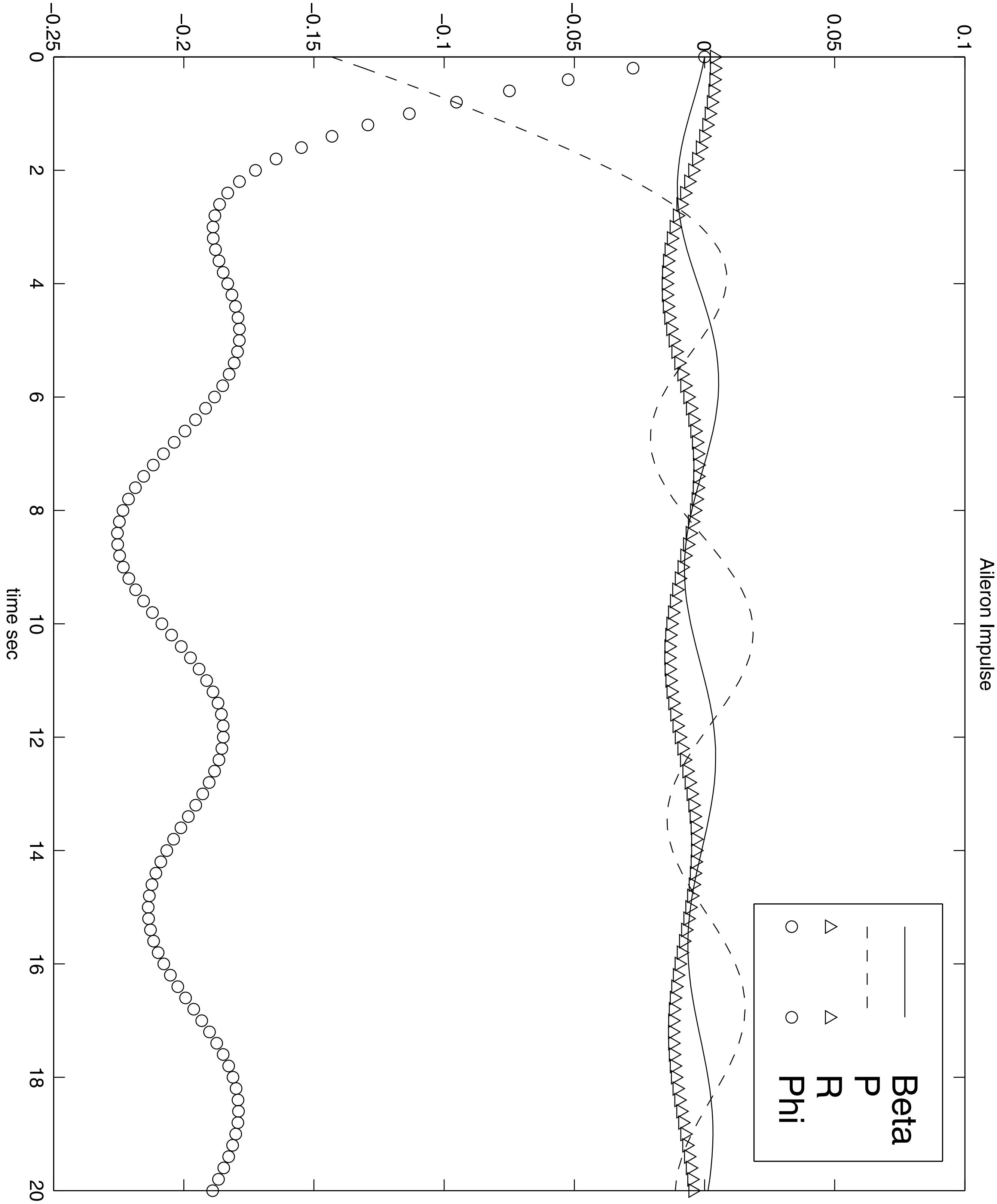


Figure 2: Aileron impulse to flight variables. Response primarily in ϕ.