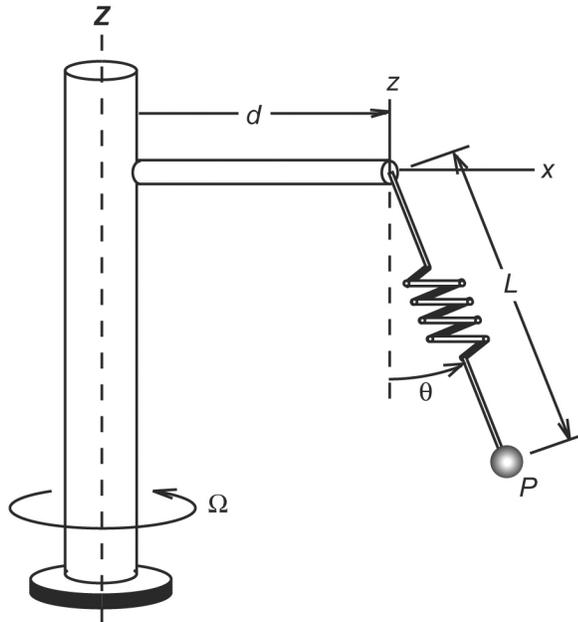


## 16.61 Homework Assignment #4

1. Consider the spring pendulum analyzed before. The arm attached to the rotating shaft has length  $d = 0.8\text{m}$ , as shown in the figure. The shaft is rotating with a constant angular velocity  $\Omega = 0.4 \text{ rad/sec}$ , but the pendulum is free to change length ( $L$ ) and swing ( $\theta$ ). Given that the spring constant is  $k$  and the mass of point P is  $m$ , find the equations of motion for this system. Note that gravity acts.



Pay special attention to Frame selection and FARM.

2. A pendulum of mass  $m$  and length  $l$  is in a uniform gravitational field. The support point is moving in the vertical direction with a displacement  $d(t)$ , which is a *known* function of time (Both  $\dot{d}(t)$  and  $\ddot{d}(t)$  are known as well).
  - (a) Use Newton's method to solve for the equations of motion of this system. What frames would be useful to select to solve this problem? Assume small angles.
  - (b) For use in the Lagrangian approach, what is the kinetic energy of the pendulum bob?

- (c) What is the potential energy of the pendulum bob? Clearly indicate the reference point for your calculation of the potential energy.
- (d) As we did in class, use Lagrange's method to derive the equations of motion for the system.
- (e) Discuss the final equations of motion – do they make physical sense?
3. The bob of a simple pendulum swings in a horizontal circle of radius  $r$  at a constant angular rate. Solve for the equations of motion, and show that the period of the period of revolution is given by

$$T = 2\pi \left[ \frac{l^2 - r^2}{g^2} \right]^{1/4}$$

where  $l$  is the length of the pendulum. Focus on setting up the frames, angles, rates, and matrix forms for the inertial acceleration. Then find the forces in the problem. Note that there is an interesting relationship for the half-cone angle  $\phi$ :

$$\cos \phi = \frac{\sqrt{l^2 - r^2}}{l}$$

(but you should show this).

4. Derive the equations of motion for the following double pendulum system with two equal masses. Use the angles shown as the generalized coordinates. Pay special attention to the calculation of  $V$ .

