16.512, Rocket Propulsion
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Lecture 27: Turbines

1. Rocket turbine design emphasizes power density, because of the overriding concern for mass saving. Efficiency, while clearly a consideration, takes a less prominent role than in aircraft turbine design. This tends to favor low reaction turbines and when multi-staging, velocity compounding over pressure compounding.

   The degree of reaction of a turbine stage (stator nozzles plus rotor blades) is the fraction of the fluid static enthalpy drop which occurs in the rotor (see section 2. of this Lecture). In an <u>impulse turbine</u>, the degree of reaction is zero, meaning that the gas expands and accelerates as it turns in the stator passages, and then is merely redirected at constant thermodynamic state by the moving rotor blades. The stage velocity triangles are shown in Fig 1, which also includes the case of a 50% reaction turbine for comparison. In both cases, the flow leaves axially. The torque (and hence the power) is proportional to the change in the tangential component of the absolute velocity. Fig 1 shows that this change is the wheel speed ωR for the 50% reaction turbine, but is $2\omega R$ for the impulse turbine. It is also clear, however, that flow velocities are higher in the impulse case which will lead to larger viscous losses. Also, the lack of acceleration in the rotor passage will favor flow separation on the suction side. Altogether, the more powerful impulse stage is also less efficient.

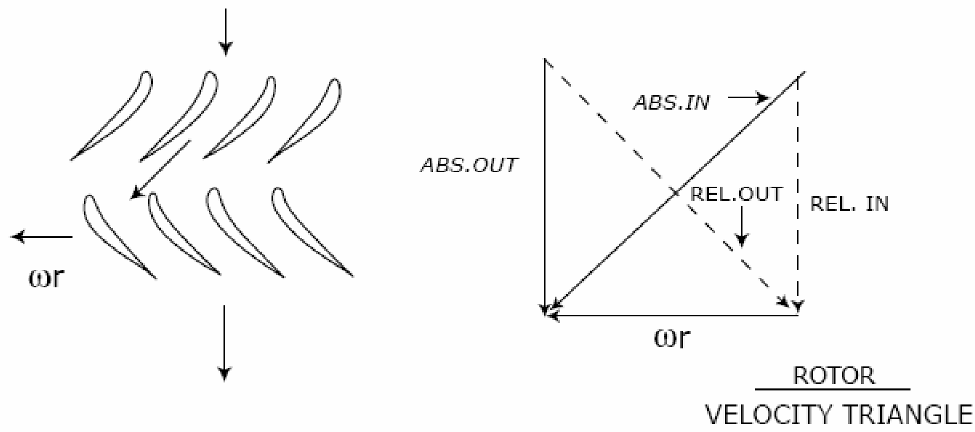


Fig. 1(a) A 50% reaction turbine.

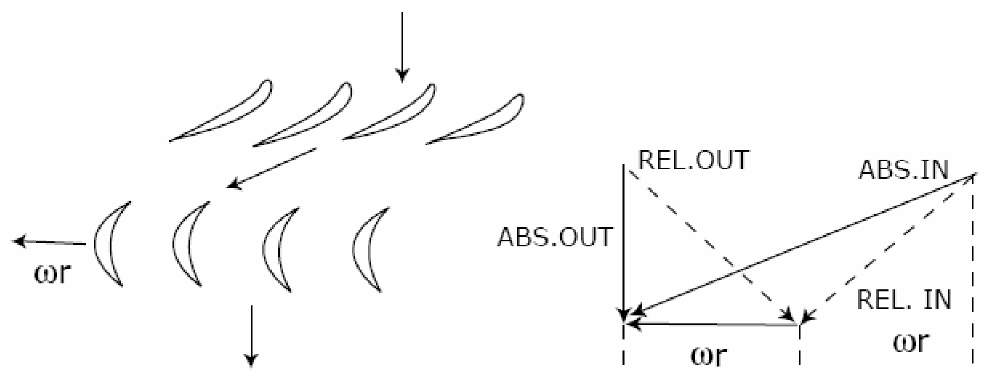


Fig. 1(b) A pure impulse turbine.

If two stages are used, one can over-turn the flow in the first rotor, so that it leaves with a component of velocity contrary to rotation. The second stator then can re-direct this flow without accelerating it, and send it into yet another pure impulse rotor. It can be shown that this <u>velocity compounding</u> (see Section 3. of this lecture) will yield four times as much torque as the single impulse stage such as those in Fig. 1b so that the flow is re-expanded in the second nozzle (pressure compounding). This only doubles the power of the single stage, but improves the efficiency. The SSME fuel turbopump turbine is essentially of this type, although there is some non zero reaction (R $\cong$ 0.22).

For preliminary design purpose, Ref [46] (Refer to the list of the References in Lecture 25) presents (n_s, d_s) diagrams similar to those in use for pumps. One of these is reproduced in Fig. 2. The quantities Q and gH in the definitions of d_s and n_s (Eqs. 3.34-3.35) are now to be specified more fully: the volumetric flow rate Q is at turbine exit static conditions, and the energy *gH* is the total-to-static isentropic enthalpy drop:

$$gH = c_p T_{T.ti}\left[1-\left(\frac{P_{te}}{P_{t.ti}}\right)^{\frac{\gamma-1}{\gamma}}\right] \tag{1}$$

Notice that, as in the case of a pump, the product $n_s d_s$ has a simple connotation:

$$n_s d_s = \sqrt{8}\,\frac{\omega R}{C_0} = \frac{2}{\sqrt{\psi}} \tag{2}$$

where $c_0 = \sqrt{2gH}$ is sometimes called the "spouting velocity". The velocity ratio $\omega R / c_0 = \dfrac{1}{\sqrt{2\psi}}$ is often used as a design parameter. For optimum efficiency, it should be about 0.4 in an impulse stage, 0.2 for a velocity-compound double stage, or 0.7 for a 50% reaction stage [40]. The optimum rotational speed of the turbine was in the past higher than that for the driven pumps, which required heavy gearing. The steady advancements in inducer design have more recently allowed mounting both turbine and pumps on one shaft, with major mass savings.

Whereas pump impellers (except for hydrogen) are rarely stress limited, the reverse is true for turbines, where the blade root axial stress and/or the disc rim hoop stress tend to limit speed. The radial root stress is approximately (see Sec. 4 of this Lecture)

$$\sigma_6 = \rho_b \omega^2 \frac{A}{2\pi} \tag{3}$$

where A is the annulus flow area and ρ_b is the density of the blade material, which is dictated by the flow rate and inlet gas density. There is thus an incentive to use materials, such as directionally solidified superalloys, that retain a high strength at the uncooled operating conditions.

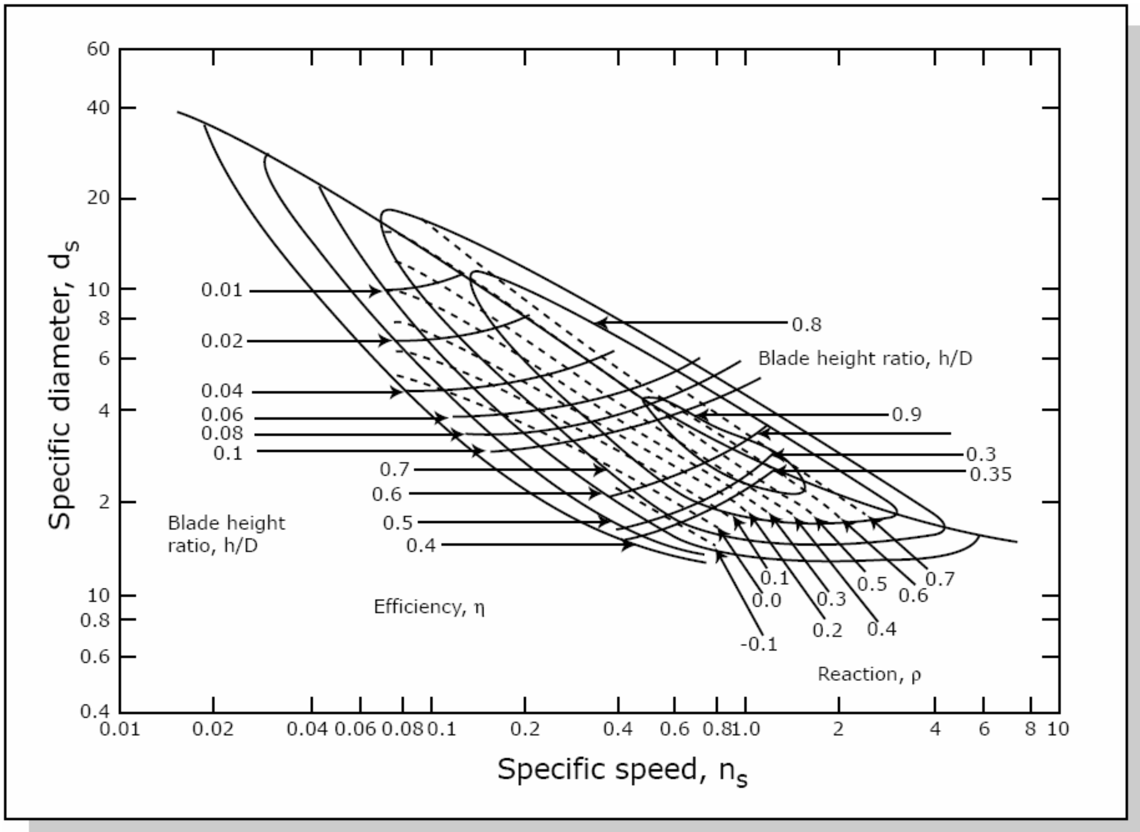


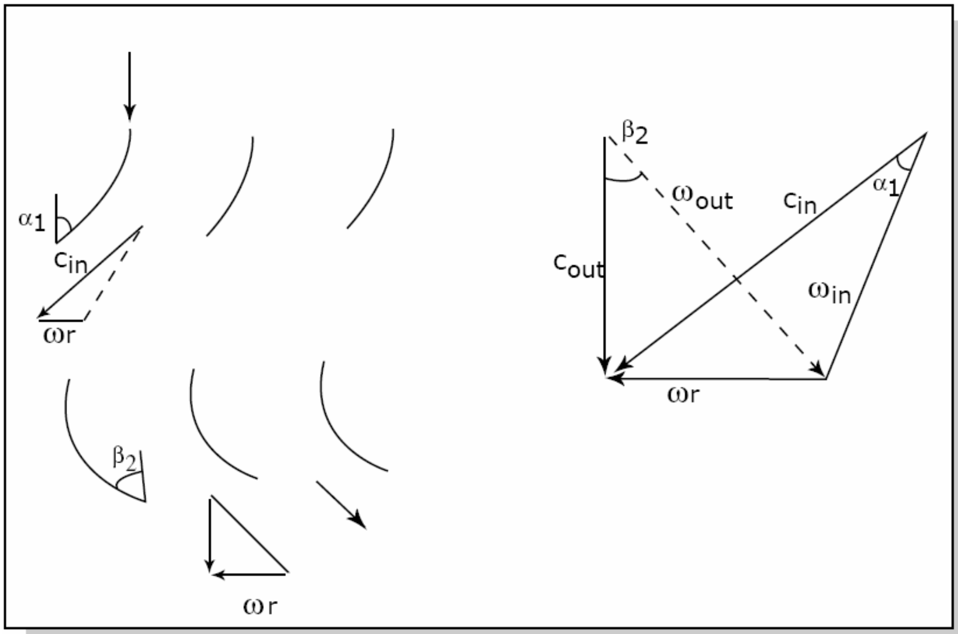
Fig. 2. $n_s d_s$ diagram for turbines calculated for minimum loss coefficients.

2. **Some More General Performance Relationships for Turbines**

Ignore inefficiencies: $\Delta h = \frac{\Delta P}{P}$

Define <u>Degree of Reaction</u>

$$R = \frac{(\Delta h)_{Rot.}}{(\Delta h)_{stage}} \simeq \frac{(\Delta P)_{Rot.}}{(\Delta P)_{stage}} \qquad \text{(low p.r. per stage, } \rho \simeq \text{const.)}$$



c=absolute vel.
ω=relative vel.

Assume axial exhaust (design cond.)

$$\frac{\text{Torque}}{\dot{m}} = c_{IN}\sin\alpha_1 r$$

$$\frac{\text{Power}}{\dot{m}} = (\omega r)c_{IN}\sin\alpha_1 = \frac{\Delta P_{stage}}{\rho}$$

$$= (\omega r)c_{OUT}\tan\alpha_1$$

$$c_{OUT} = c_{IN}\cos\alpha_1$$

c_x =const. through stage

$$\frac{(\Delta P)_{stator}}{\rho} = \frac{c_{IN}^2 - c_{OUT}^2}{2} = \frac{c_{OUT}^2}{2}\left(\frac{1}{\cos^2\alpha_1} - 1\right) = \frac{c_{OUT}^2}{2}\tan^2\alpha_1$$

$$1 - R = \frac{(\Delta P)_{stator}}{(\Delta P)_{stage}} = \frac{1}{2}\frac{c_{OUT}\tan\alpha_1}{\omega r}$$

$$\text{Also,}\quad \psi = \frac{\left(\Delta P/P\right)_{stage}}{(\omega r)^2} = \frac{c_{OUT}\tan\alpha_1}{\omega r}$$

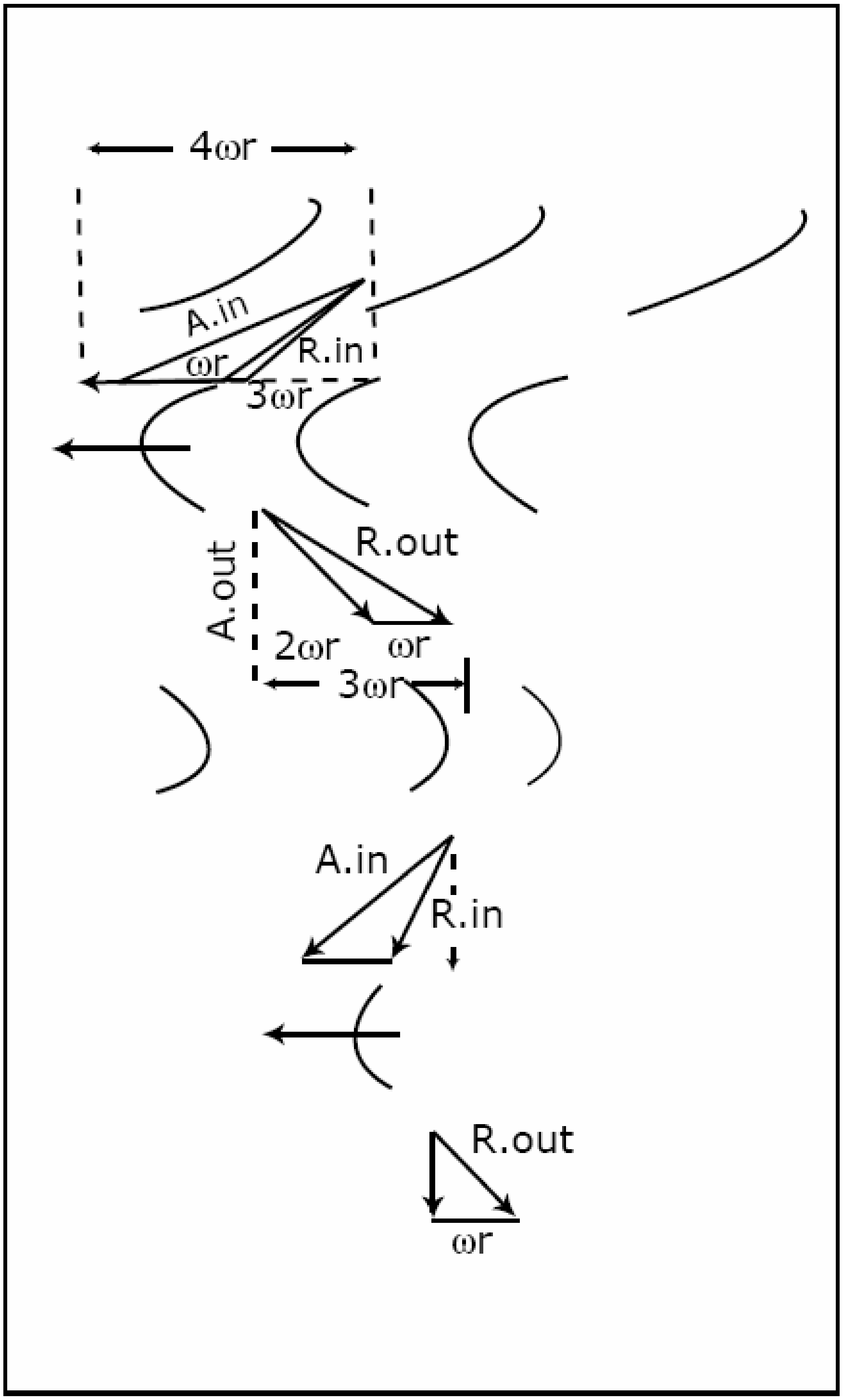
So, $1 - R = \frac{\psi}{2}$ $\boxed{\psi = 2(1-R)}$ More work/stage with less reaction

R=0 (impulse turbine) $\rightarrow \psi = 2$
R= $1/2$ (50% reaction) $\rightarrow \psi = 1$

Also, $\phi = \frac{c_x}{\omega r} = \frac{c_{OUT}}{\omega r}$, so, $\boxed{\begin{array}{l}\tan\alpha_1 = \frac{\psi}{\phi} \\ \tan\beta_2 = \frac{1}{\phi}\end{array}}$ For, 50% design, $\alpha_1 = \beta_2$

and $\tan\beta_2 = \frac{\omega r}{c_{OUT}}$ For impulse, $\tan\alpha_1 = 2\tan\beta_2$

3. **Velocity Compounding:** Follows this diagram starting from the exhaust (at bottom)



St. 1

Rotor 1 Power =
$\dot{m}(4\omega r + 2\omega r)R\omega = 6\dot{m}(\omega R)^2$

St. 2

Rotor 2 Power =
$\dot{m}(2\omega R)R\omega = 2\dot{m}(\omega R)^2$

Total = $8\dot{m}(\omega R)^2$ =
$4\times$ Single impulse

1st stator must give
<u>$4\omega r$</u> tangential velocity.

4. **Blade root stress**:

$$P_b\,\omega^2\,r\,\cancel{A}\,\cancel{dr} = -\frac{d}{dr}\left(\sigma\cancel{A}\right)\cancel{dr} \qquad \sigma = \sigma_1 - \frac{1}{2}\,P_b\,\omega^2\,(r_2^2 - r_1^2)$$

$$\sigma(r_2) = 0 \qquad \sigma_1 = \frac{1}{2}\,P_b\,\omega^2\,(r_2^2 - r_1^2)$$

$$A = \pi(r_2^2 - r_1^2) \qquad \boxed{\sigma_1 = \rho_b\,\omega^2\,\frac{A}{2\pi}}$$