Lecture 16: Solid Propellants: Design Goals and Constraints

Solid Propellants

Read Sutton's, Chapter 12

Double Base (DB) Nitrocellulose + Nitroglycerine + Additives (for opacity, plasticity, ...). Both NC and NG are explosives, dangerous sometimes

JPN NC 51.5%, NG 43%, Diethyl phthalate 3.2%, Ethyl centralite 1%, H_2SO_4 1.2% + carbon black + candelilla wax

Composite Modified Double base (CMBD) DB + Ammonium perchlorate (AP) or Aluminium (Al)

Composite (C) AP (sometimes A Nitr) + Synthetic Rubber binder (fuel) + Al. Safer than DB

Other Composite contain nitramine explosive (RDX, HMX), replacing some AP

Type	I_{sp} (s) at 1000/14.7 psi	T_c (K)	ρ_p (g/cm^3)	Al %	r (cm/s) @1000 psi	η	Fabrication
DB	220-230	2530	1.60	0	1.14	0.3	Extruded
DB-AP-Al	260-265	3866	1.79	20-21	1.98	0.4	Extruded
CTPB/AP/Al	260-265	3370-3490	1.76	15-17	1.14	0.4	Cast
HTPB/AP/Al	260-265	3370-3490	1.85	4-17	1.02	0.40	Cast

The addition of Aluminum is not necessarily beneficial, as the following example shows:

Problem 3. Adding Aluminum to the formulation of a solid rocket propellant increases the gas temperature, but incurs performance penalties related to the solid particles that are generated.

Consider a simple model for the effect of adding a mass fraction x_{Al} of Aluminium, of the form

$$\frac{T_c}{T_{c_o}} = 1 + rx \quad ; \quad x = 1.85\, x_{Al} \qquad (1)$$

where r $\cong$ 1.41 is a separately calculated coefficient, T_{co} is the flame temperature without aluminum (~2500K), and x is the solids fraction in the gas (the 1.85 factor accounts for the oxygen in the Al_2O_3 particles).

Consider also a linearized model for the effect of the particulates, of the form

$$\frac{u_e}{u_{e_o}} = 1 - fx \qquad \text{(at fixed } T_c\text{)} \qquad (2)$$

where f is as derived in class:

$$f = \begin{cases} \dfrac{1}{2} - \dfrac{c_s}{2c_{pg}}\left[1 + \dfrac{(1-\eta)\ln(1-\eta)}{\eta}\right]; \;\; \eta = 1 - \left(\dfrac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}} \text{ (small particles)} \\ 1 - - - - - - - - - - - - - - - - - \text{ (large particles)} \end{cases}$$

(a) Show that the optimum loading is given by

$$x_{OPT} = \frac{r-2f}{3rf} \;\; ; \;\; (x_{Al})_{OPT} = \frac{x_{OPT}}{1.85} \qquad (3)$$

(b) For $\frac{P_e}{P_c} = 0.01$, $\gamma_g = 1.25$, $M_g = 18\text{g}/\text{mol}$, $c_s = 1260\ \frac{J}{KgK}$, calculate $(x_{Al})_{OPT}$ for both small and large particulates. Comment on results.

Problem 3 – Solution

Ignoring the exit pressure effect (or at matched conditions),

$$g\ I_{sp} = v_e = \sqrt{2C_pT_c\left[1 - \left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

which is proportional to $\sqrt{T_c}$. The rest of the dependence (γ, c_p) are affected by particulates, but that is counted separately in the loss analysis. So we have (counting both effects)

(a) $\qquad v_e \sim (1 - fx)\sqrt{1 + rx}$

To optimize, take the logarithmic derivative and equate to zero

$$\frac{-f}{1-fx} + \frac{1}{2}\ \frac{r}{1+rx} = 0$$

$$2f\,(1+rx) = r\,(1-fx) \qquad\qquad r - 2f = 3rfx$$

$$\boxed{x_{OPT} = \frac{r-2f}{3rf}} \qquad \text{and then } \left(x_{Al}\right)_{OPT} = \frac{x_{OPT}}{1.85}$$

(b) For small particulate $f = \frac{1}{2}\left\{1 - \frac{c_s}{c_{pg}}\left[1 + \frac{(1-\eta)\,\ln(1-\eta)}{\eta}\right]\right\}$, and using the given values,

$$\eta = 1 - (0.01)^{0.25/1.25} = 0.6012 \quad ; \qquad C_{pg} = \frac{1.25}{0.25}\ \frac{8.314}{0.018} = 2309\ J/Kg/K$$

$$f = \frac{1}{2}\left\{1 - \frac{1260}{2309}\left[1 + \frac{0.3988\,\ln(0.3988)}{0.6012}\right]\right\} = 0.3934$$

$$x_{OPT} = \frac{1.41 - 2\times 0.3934}{3\times 1.41\times 0.3934} \qquad \boxed{x_{OPT} = 0.3746}$$

and then the Aluminum fraction should be

$$\boxed{\left(X_{Al}\right)_{OPT} = \frac{0.3746}{1.85} = 0.2025} \qquad 20.3\%\ \text{Al loading}$$

For the case of larger particle, the class derivation showed f=1, and so

$$x_{OPT} = \frac{1.41 - 2\times 1}{3\times 1.41\times 1} < 0$$

This nonsensical result simply means there is no good Al loading in this case. The losses due to the particles are stronger than the gains due to increased temperature, so no Aluminum should be added.

Fortunately, the particles are small, not large.