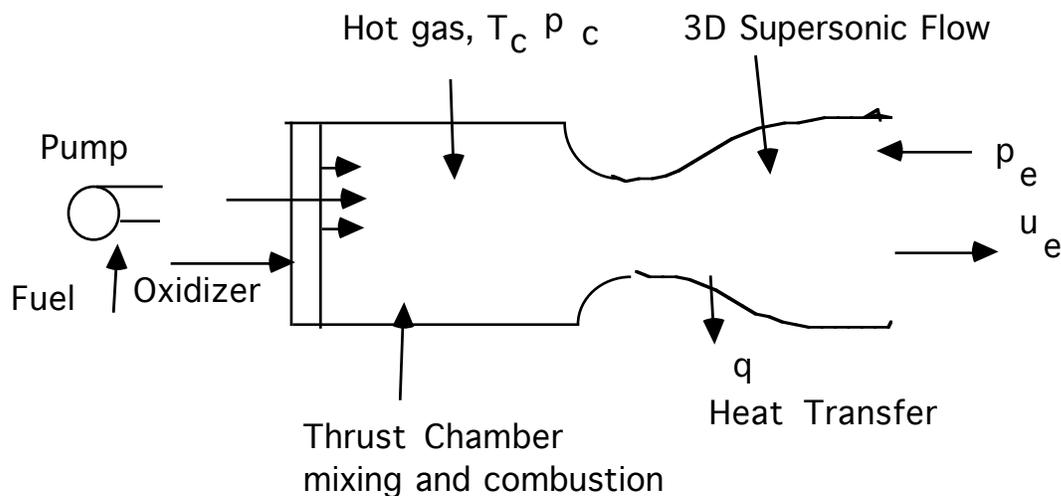


16.50 Lecture 6

Subject: Modeling of Thermal Rocket Engines; Nozzle flow; Control of mass flow

Though conceptually simple, a rocket engine is in fact physically a very complex device and difficult to represent quantitatively by mathematical models. But the difficulty and high cost of arriving at a successful design makes it essential that the performance-limiting phenomena be described as accurately as possible. It is no longer feasible to approach the development of a new rocket engine by a purely cut-and-try method.

The range of phenomena that must be dealt with is suggested by the sketch. They range from the fluid dynamics of the pumping system through the very complex phenomena of combustion to structures with heat transfer rates beyond those experienced anywhere other than in reentry from space.

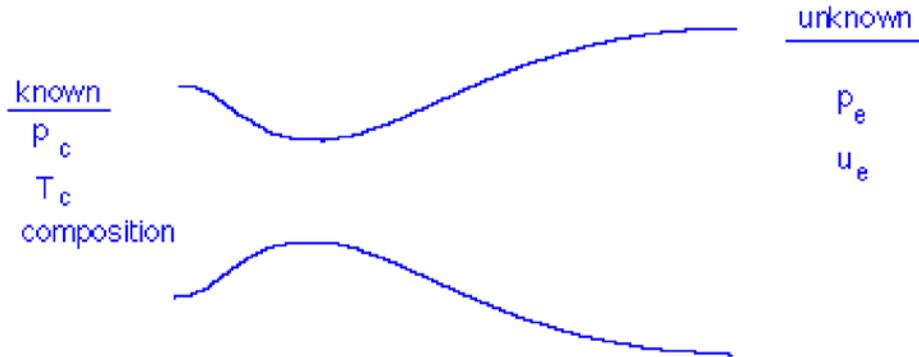


To deal with this complex situation, we use models that represent adequately the phenomena we want to deal with, while suppressing other phenomena. Some models we will find useful are those for:

- 1) Turbopumps
- 2) Injectors
- 3) Combustion
- 4) Nozzle Flow
- 5) Heat Transfer
- 6) Structure of combustion chambers and nozzles

Nozzle Flow

Let us focus on the Nozzle Flow first. For this purpose we regard the combustor as just a source of hot gas of known chemical composition, stagnation temperature T_c and stagnation pressure p_c .



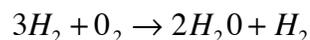
We have already used a very simple model to represent the result of this flow process. Assuming that the expansion was to zero pressure and therefore zero temperature we used conservation of energy to show that

$$c = \sqrt{2c_p T_c} = \sqrt{2 \frac{\gamma}{\gamma - 1} R T_c} \quad (\text{model 1})$$

where c was defined as the "exhaust velocity" meaning the velocity which, multiplied by the mass flow rate, gives the thrust. Now we want to explore the fluid mechanical processes in more detail, and also represent the characteristics of the gas more accurately.

The gas is in fact a mixture of gases resulting from the combustion process Its composition is specified in terms of mole fractions y_i of species i or mass fractions x_i of species i .

For example if the rocket burns H_2 and O_2 in a mixture ratio of 3 moles of H_2 to 1 mole of O_2 to produce H_2O and H_2 , (by mass $O/F=32/(3 \times 2)=5.3$, comparable to $O/F=6$ in the Shuttle SSME engine),



then the mole fractions and mass fractions of H_2O and H_2 are as shown in the table.

	<u>H_2</u>	<u>H_2O</u>
x	1/19	18/19
y	1/3	2/3

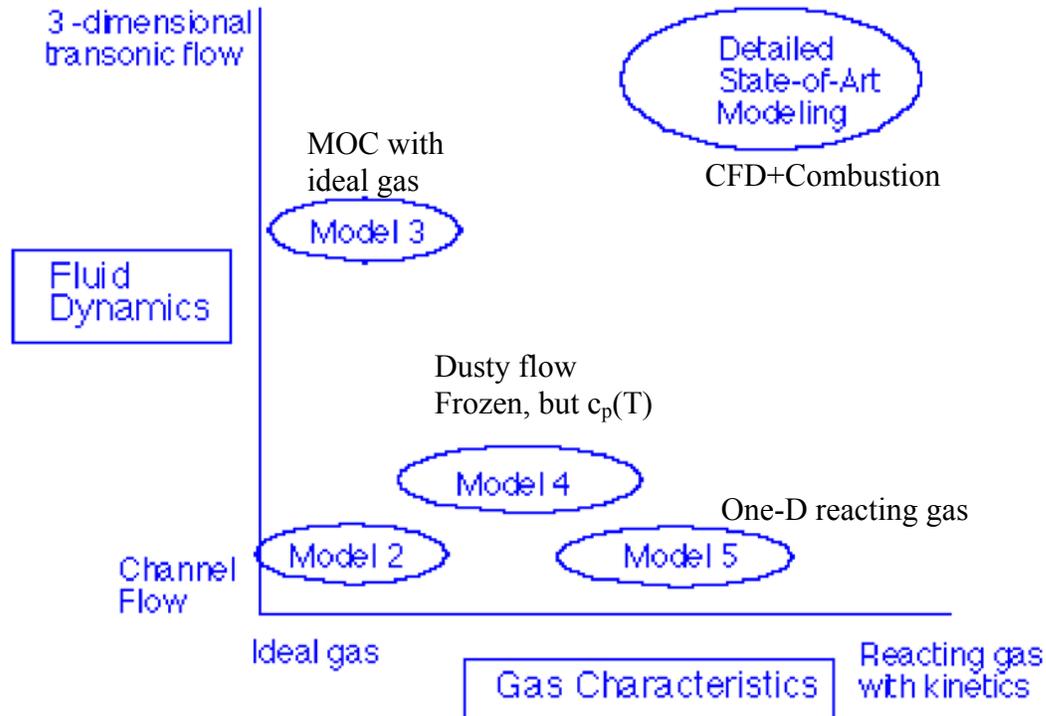
With these mole or mass fractions we can compute the properties of the gas mixture, such as the specific heats, the gas constant and γ .

The fluid-mechanical and heat transfer phenomena that are of importance to describe are:

- 1) The effects of nozzle shape on flow
- 2) The effects of chemical reactions during the flow
- 3) The heat transfer from the flow to the nozzle

Effect of Nozzle Shape

The phenomena that influence the coupling between nozzle shape and the flow can be divided into two general classes, first those associated with the gas dynamics and second those associated with the properties of the gas. Depending on how accurately we attempt to model each of these, our model of the flow can vary enormously in complexity. The range of possibilities is shown schematically in the figure, where sophistication of the modeling increases away from the origin along two axes, one measuring gas dynamics, the other fluid properties.



Along the gas dynamic axis the description can range from the simple channel flow model that we will discuss here to very sophisticated three-dimensional descriptions of the transonic flow. Similarly along the properties axis the range is from the thermally and calorically perfect gas model we will use (often called the "ideal gas") to full kinetic description of the effects of chemical reactions that occur as the gas flows through the nozzle. The simplest model of all (Model 1) equates the kinetic energy of the exhaust flow to the thermal energy in the chamber

Here we will limit the discussion of the coupling of flow to the geometry of the nozzle to a simple model (Model 2) at the origin. The implications of the more complex phenomena will then be discussed separately through two more models. Model 5 will deal with the effects of chemical reactions, and Model 3 with the effects of nozzle shape other than the simple area variation. Model 4 will represent some special chemical and fluid mechanical aspects of solid rocket motors.

Model 2: Channel Flow of Perfect Gas

The channel flow model (often erroneously referred to as "one-dimensional flow") is one in which the entire flow is considered to be in a single stream tube having a mass flow that is constant along the axis of the stream tube. The stream tube is considered to be narrow enough that the angle of the walls to the axis may be considered small, the gas properties nearly constant across the stream tube, and the components of velocity normal to the axis small enough that their squares are negligible compared to that of the streamwise velocity.

If the cross-sectional area of the streamtube is denoted $A(x)$, the conservation of mass becomes

$$\rho u A(x) = \dot{m} = \text{constant} \quad (1)$$

The conservation of energy becomes

$$h + \frac{u^2}{2} = h_c \quad (\text{Note: } \frac{v^2}{u^2} \ll 1)$$

or if the specific heat is assumed constant.

$$c_p T + \frac{u^2}{2} = c_p T_c \quad (2)$$

The final relationship defining the flow is a connection between two of the thermodynamic properties of the fluid. In general this is a description of the variation of the entropy of the fluid along the flow direction. For our simple model we assume that the flow is Isentropic (Adiabatic and Reversible). The entropy is referred to a reference state which we take to be the chamber condition. Conventionally, we assume that the velocity in the chamber is low enough that this is also the stagnation condition in the chamber.

$$s - s_c = \int_{T_c}^T c_p \frac{dT}{T} - R \ln \frac{P}{P_c} = 0$$

so if s is constant, and c_p is as well,

$$c_p \ln \frac{T}{T_c} - R \ln \frac{P}{P_c} = 0$$

and from this it follows that

$$\frac{T}{T_c} = \left(\frac{P}{P_c}\right)^{\frac{R}{c_p}} = \left(\frac{P}{P_c}\right)^{\frac{\gamma-1}{\gamma}} \quad (3)$$

where

$$\gamma \equiv \frac{c_p}{c_v} ; \quad c_p - c_v = R$$

We want to manipulate these three relationships to arrive at an expression giving the variation of Mach number with A , hence with x . We begin by expressing equation (2) in the form:

$$\frac{T_c}{T} = 1 + \frac{u^2}{2c_p T} = 1 + \frac{\gamma-1}{2} M^2, \quad M^2 \equiv \frac{u^2}{\gamma R T}$$

and then from (3)

$$\frac{p_c}{p} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}}$$

With these we can write (1) in the form,

$$\begin{aligned} \dot{m} &= \rho u A = \frac{p}{RT} u A = \frac{p}{p_c} \frac{T_c}{T} \frac{1}{R} M \sqrt{\gamma R T_c} \sqrt{\frac{T}{T_c}} \frac{p_c}{T_c} \\ &= \left(1 + \frac{\gamma-1}{2} M^2\right)^{-\frac{\gamma}{\gamma-1} + \frac{1}{2}} M \frac{p_c}{T_c} \sqrt{\gamma T_c / R A} \end{aligned}$$

$$\frac{\dot{m}}{A} = p_c \sqrt{\frac{\gamma}{R T_c}} \frac{M}{\left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}}$$

For given (and constant) p_c and T_c , this expression gives the mass flow per unit area as a function of the Mach number M . Since the mass flow is constant, we have the desired expression for M as a function of $A(x)$.

We see that the mass flow per unit area, ρu , is proportional to the stagnation pressure and inversely proportional to the square root of the stagnation temperature.

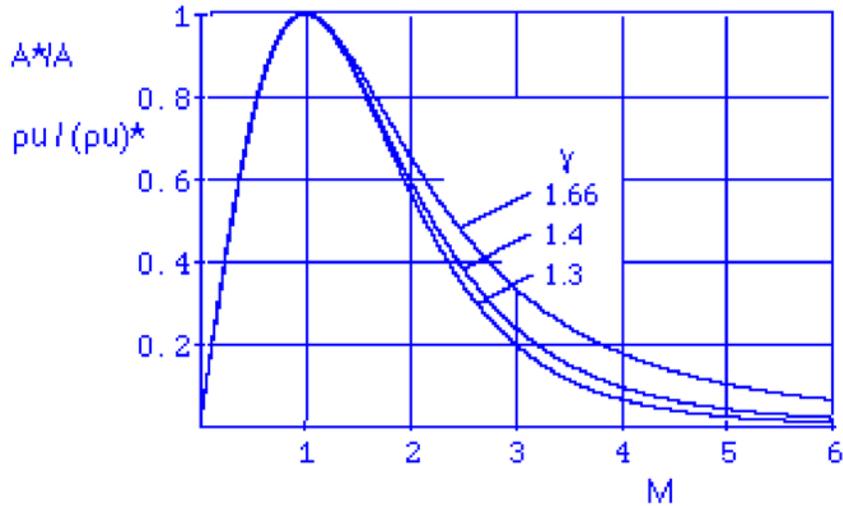
It is both conventional and useful to plot the function of Mach number in ratio to its value at the point in the flow where $M=1$. The value of ρu at this point we call ρu^* . It is given by

$$(\rho u)^* = p_c \sqrt{\frac{\gamma}{R T_c}} \left(\frac{\gamma+1}{2}\right)^{-\frac{\gamma+1}{2(\gamma-1)}}$$

Then the ratio is

$$\frac{\rho u}{\rho u^*} = \frac{A^*}{A} = M \left(\frac{\frac{\gamma+1}{2}}{1 + \frac{\gamma-1}{2} M^2} \right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

This function of M is shown in the figure.



Some of the dominant features are:

- 1) There is a maximum in the mass flow per unit area at $M=1$.
- 2) For $M < 1$, ρu (and also u) increases with M , so M increases with decreasing A .
- 3) For $M > 1$, ρu (and also u) decreases with M , so M increases with increasing A .

We note however that p always decreases with increasing M . It follows that if we want to increase M from a small value to one in excess of 1, as we do in a rocket nozzle, the nozzle should be shaped as in the sketch at the beginning of the lecture, with a convergence to a minimum area "throat", followed by a divergence.

For given p_c and T_c , the area of the throat controls the mass flow.

To get an idea of the required throat size we can evaluate the (dimensional) factor in front of the function of M for a set of typical values.

Say $p_c = 100 \text{ atm} = 10^7 \text{ N/m}^2$
 $T_c = 3000^\circ\text{K}$
 $\gamma = 1.2$
 $R = \frac{8.32 \times 10^3}{13} = 640 \text{ J/kg/K}$

$$p_c \sqrt{\frac{\gamma}{RT_c}} = 10^7 \sqrt{\frac{1.2}{(640)(3000)}} = 7.91 \times 10^3 \text{ kg/m}^2/\text{s}$$

We define the throat of the nozzle as the point of minimum area, where $M=1$ and call the area there A_t . Then

$$\frac{\dot{m}}{p_c A_t} = \sqrt{\frac{\gamma}{RT_c}} \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

The conclusion that the point of minimum area in the nozzle controls the mass flow is valid beyond the limits of our simple ideal gas mode.. More generally, we define a quantity c^* by

$\dot{m} \equiv \frac{p_c A_t}{c^*}$ that is a measure of the mass flow per unit area at the throat. For the ideal gas model,

$$c^* = \sqrt{\frac{RT_c}{\gamma}} \left(\frac{\gamma+1}{2} \right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

For the example above,

$$c^* = \sqrt{\frac{(640)3000}{1.2}} \left(\frac{2.2}{2} \right)^{5.5} \approx 2138 \text{ m/s}$$

In general c^* depends only on the propellant combination and p_c .

One further condensation of these formulas that is sometimes useful is to define the

quantity $\Gamma = \sqrt{\gamma} \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{2(\gamma-1)}}$, which is close to 0.65 for most values of γ , and then write

$$c^* = \frac{\sqrt{RT_c}}{\Gamma} \quad ; \quad \dot{m} = \frac{p_c A_t}{c^*}$$

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