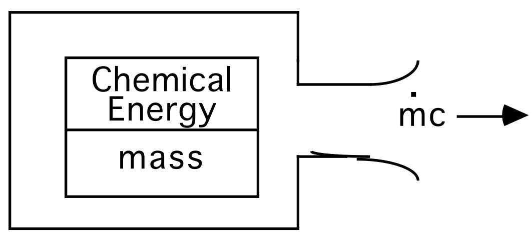
16.50 Lecture 5

Subjects: Non-Chemical rockets; Optimum exhaust velocity

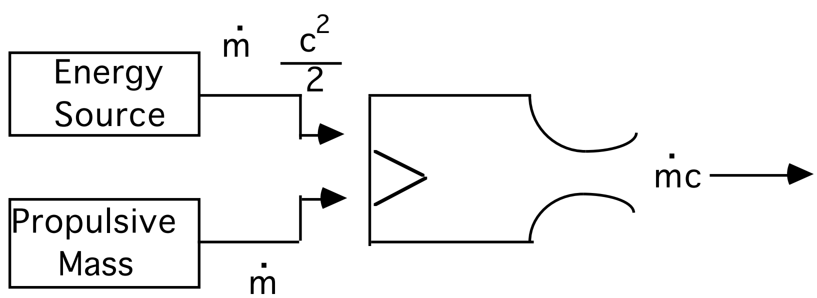
1) Non-chemical rockets

A shared characteristic of all non-chemical propulsion systems is that the energy and propellant mass are separate initially

Chemical



Non-chemical



There are several possible energy sources:

1) Solar
   a) Photovoltaic
   b) Solar thermal
   c) Solar pressure

2) Nuclear
   a) Fission
   b) Radioisotope
   c) Fusion?

There are also many ways to bring the mass and energy together to produce thrust, but all behave according to the rocket equation.

$$\frac{m_{final}}{m_{tot}} = e^{-\frac{\Delta V}{c}}$$

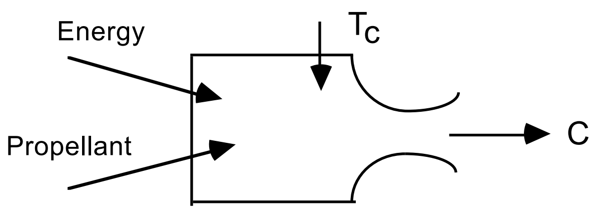
or breaking the final mass into its constituent parts,

$$\frac{m_{pay}}{m_{tot}} = e^{-\frac{\Delta V}{c}} - \frac{m_{propsys} + m_{struct}}{m_{tot}}$$

There are 2 general categories of systems, Thermal and Electrical, separable according to whether the energy is available in electrical or mechanical form, or only as thermal energy at some limiting temperature.

A. Thermal

Here the energy is used directly to heat the propellant, which is then expanded through a nozzle to produce thrust.



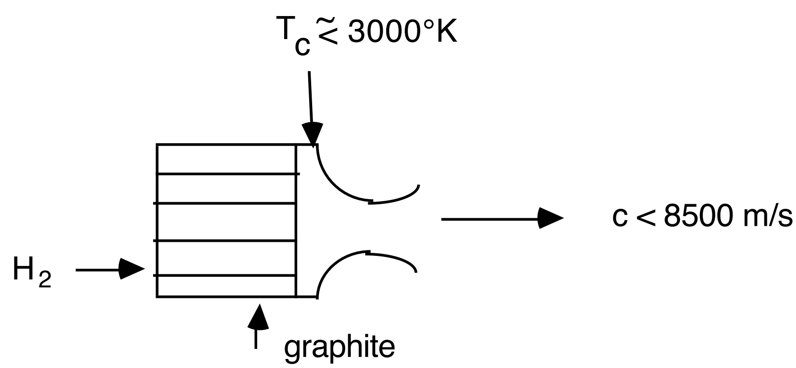
Now there is a chamber temperature T_c, limited by the energy source, and the exhaust velocity is given approximately by:

$$c_p T_c = \frac{c^2}{2} \qquad \text{or} \qquad c \approx \sqrt{2 c_p T_c} \approx \sqrt{\frac{2\gamma}{\gamma - 1} \frac{R}{M} T_c}$$

What limits T_c?

1) Source Temperature e.g. $T_{sun} = 6000°K$
2) Materials

So for these we generally want low M, e.g., $H_2 \rightarrow 2H$. For a nuclear thermal rocket

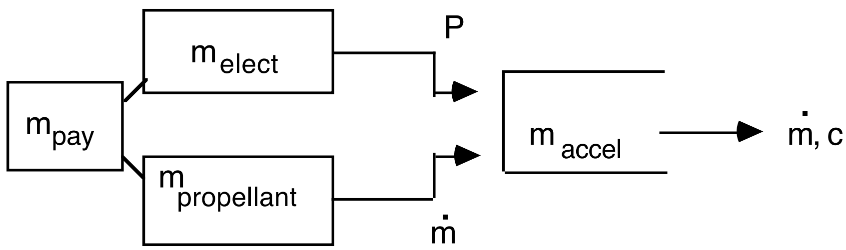


Figures of merit are: a) Specific impulse, c; b) Thrust per unit mass, $\frac{F}{m_{eng} g}$

B. Electrical

If the energy is available in electrical form, then there is no limit in principle on c (other than the speed of light) and in practice we can achieve very high c with good efficiency by using any of a number of electrical accelerators.

The system requirement is to produce a ΔV on a payload, m_{pay}. In the absence of gravity loss, $\frac{m_{final}}{m_{tot}} = e^{-\frac{\Delta V}{c}}$, so why not make it very close to 1 by increasing c? To see the answer we must analyze the whole system, and take account of the mass of the energy source.



The total mass of the system can be broken out as:

$$m_{tot} = m_{pay} + m_{elect} + m_{prop} + m_{eng} + m_{struct}$$

so that ratio of final mass to initial mass is

$$\frac{m_{final}}{m_{tot}} = e^{-\frac{\Delta V}{c}} = \frac{m_{pay} + m_{elect} + m_{eng} + m_{struct}}{m_{tot}}$$

The Figure of Merit for such a system is

$$\frac{m_{pay}}{m_{tot}} = e^{-\frac{\Delta V}{c}} - \frac{m_{elect} + m_{eng} + m_{struct}}{m_{tot}}$$

Let us neglect $m_{eng} + m_{struct}$ for the moment, compared to m_{elect} (or simply redefine m_{pay} to include $m_{eng} + m_{struct}$, which makes sense for some missions).

We know that:

$$F = \dot{m}c$$

and the power P is

$$P \geq \dot{m}\frac{c^2}{2} = \frac{Fc}{2}$$

Define a specific weight $\alpha_e \equiv \frac{m_{elect}}{P}$, and an initial acceleration $a_o = \frac{F}{m_{tot}}$. Then

$$\frac{m_{pay}}{m_{tot}} = e^{-\frac{\Delta V}{c}} - \frac{\alpha_e P}{m_{tot}} = e^{-\frac{\Delta V}{c}} - \frac{\alpha_e Fc}{2m_{tot}}$$

or finally in terms of the minimum number of dimensionless parameters,

$$\frac{m_{pay}}{m_{tot}} = e^{-\frac{\Delta V}{c}} - \left(\frac{\alpha_e a_o \Delta V}{2}\right)\left(\frac{c}{\Delta V}\right) \quad (1)$$

Here the group $(\frac{\alpha_e a_o \Delta V}{2})$ is determined by technology level (α_e), the mission requirement (ΔV) and how fast we want to achieve it (a_o). So we should consider this relation a way to find c_{opt} to maximize m_{pay}/m_{tot}, given ΔV, a_o and α_e.

Differentiating,

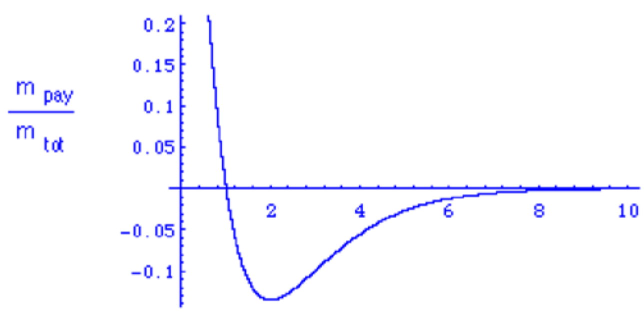
$$\frac{\partial(\frac{m_{pay}}{m_{tot}})}{\partial(\frac{\Delta V}{c})} = -e^{-\frac{\Delta V}{c}} + (\frac{\alpha a_o \Delta V}{2})(\frac{c}{\Delta V})^2 = 0$$

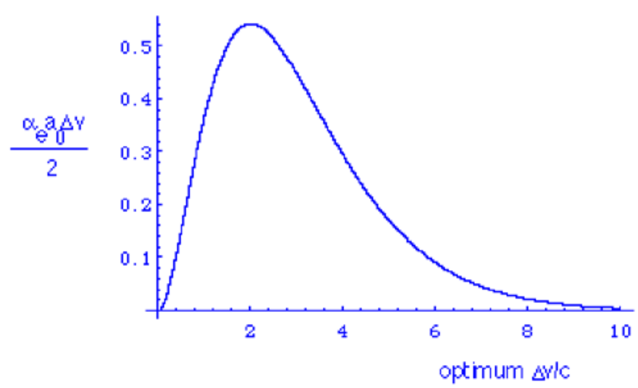
$$(\frac{\Delta V}{c})^2_{opt} e^{-(\frac{\Delta V}{c})_{opt}} = (\frac{\alpha a_o \Delta V}{2}) \qquad (2)$$

which we must solve for the optimum c/ΔV). For graphical presentation, let us eliminate the group $(\frac{\alpha a_o \Delta V}{2})$ between (2) and (1):

$$\frac{m_{pay}}{m_{tot}} = e^{-\frac{\Delta V}{c}}(1 - \frac{\Delta V}{c}) \qquad (3)$$

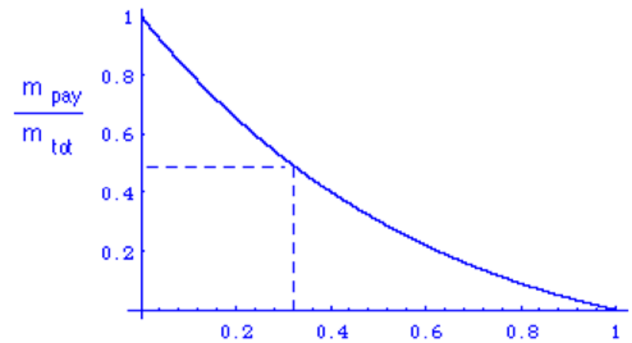
Equations (3) and (2) are represented below over a broad range of ΔV/c:

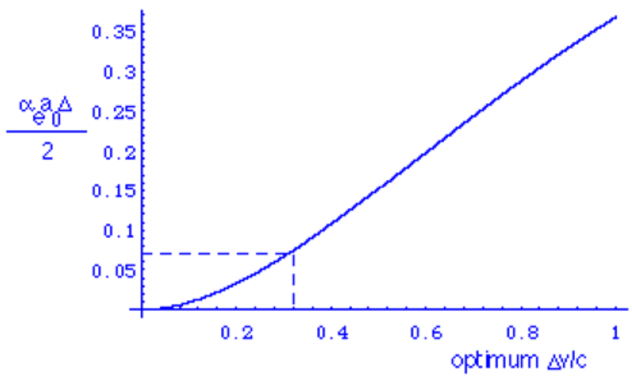




So we see that it <u>only</u> makes sense to choose $\frac{\Delta V}{c} < 1$ or $\frac{c}{\Delta V} > 1$ for such systems. This is because for $\frac{\Delta V}{c} > 1$ the exponential is so small it outweighs the term representing m_{elect}.

Expanding the range $0 < (\frac{\Delta V}{c})_{opt} < 1$,





Let us take a look at the meaning of these results:

1) If we <u>choose</u> a_o and have given α_e and ΔV, this gives us the $(\frac{\Delta V}{c})_{opt}$, and in turn the maximum $\frac{m_{pay}}{m_{tot}}$.

2) For given ΔV and α_e, increasing a_o (for a faster mission) increases $(\frac{\Delta V}{c})_{opt}$, which reduces $\frac{m_{pay}}{m_{tot}}$.

3) For given ΔV and a_0, <u>reducing α_e</u> (lighter power plant) increases $\frac{m_{pay}}{m_{tot}}$.

<u>Take an example</u>:

Suppose that the mission gives as a requirement $\Delta V = 10^4$ m/s and technology enables $\alpha_e = \frac{m_{elect}}{P}$ = 20 kg/kW = 0.020 kg/W

Then $\frac{\alpha_e a_o \Delta V}{2}$ = 100 a_o where a_o is in m/s^2.

We can still choose how fast we want to do the mission, within limits. We know that the upper limit of $\frac{\alpha_e a_o \Delta V}{2}$ = 1/e= .368. So for the assumed mission and technology,

$100a_o = \frac{\alpha_e a_o \Delta V}{2} \leq .368$. This implies $a_o \leq .00368$ m/s^2 or 3.8×10^{-4} g's and for this maximum available acceleration, $\frac{m_{pay}}{m_{tot}} = 0$, not a very useful result! Suppose we insist on $\frac{m_{pay}}{m_{tot}} = 0.5$. This gives $(\frac{\Delta v}{c})_{opt}$ = .3, which in turn implies $\frac{\alpha_e a_o \Delta v}{2}$ = .07. The acceleration is then a_o = $\frac{.07}{100}$ = 7×10^{-4} m/s^2 = 7×10^{-5} g's. This is only about 1/5 the maximum acceleration, but now we have lots of payload.

The time required to achieve the ΔV is

$$t = \frac{m_{propellant}}{\dot{m}} = \frac{m_0(1-e^{-\frac{\Delta V}{c}})}{F/c} = \frac{c}{a_0}(1-e^{-\frac{\Delta V}{c}})$$

and for our example,

$$t = \frac{10^4/0.3}{7 \times 10^{-4}}(1-e^{-0.3}) \simeq 1.23 \times 10^7 s. = 142\ days$$

This type of propulsion requires patience! Note that for a $H_2, 0_2$ rocket,

$$\frac{m_{pay}}{m_{tot}} \approx e^{-\frac{\Delta V}{c}} - \frac{m_{struct}}{m_{tot}} \approx e^{-\frac{10^4}{4500}} - .1 \approx .0084$$

So we would probably use 2 stages. But the main point is the very much smaller payload to total mass ratio of the chemical system. In addition, if the coasting period for a chemical rocket is very long, as in an interplanetary transfer, a continuous low thrust can in many cases accelerate the mission.