

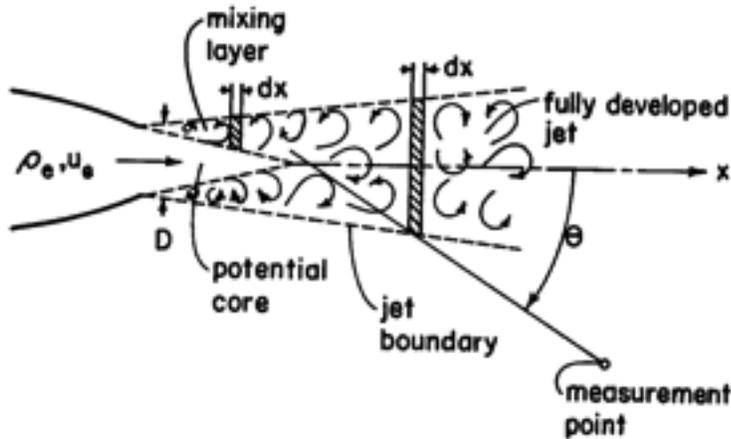
## 16.50 Lecture 37

### Subjects discussed: Jet noise, Turbomachinery noise

Having reviewed the mechanisms for noise propagation and the three types of acoustic sources - Monopoles, Dipoles and Quadrupoles, we can now examine aircraft engines as sources of noise.

#### Jet Noise

We begin with jet noise because historically it was the principal source of concern with the introduction of jet transports. Jet noise is generated by the turbulent mixing of the high speed exhaust jet with the ambient air. We can think of the mixing region as a region of strong turbulent flow,



From Kerrebrock, Jack L. *Aircraft Engines and Gas Turbines*. 2nd edition. MIT Press, 1992. © Massachusetts Institute of Technology. Used with permission.

that can be modeled as a random array of Quadrupoles, since there is no mass source, and no force in the fluid.

In a turbulent jet, in order of magnitude  $P \sim \rho_o u^2$  (u being the jet velocity) and the "size" of the fluctuating regions is some fraction of the diameter  $D$  of the jet. Similarly, the spacing  $d$  of these regions must also be some factor times  $D$ . The wavenumber  $k = \frac{\omega}{a_o}$  can be estimated by noting that  $\omega \sim u/D$ . We then have,

$$\mathbf{P}_q \sim \frac{(\rho_o u^2 D)^2}{\rho_o a_o} \left( \frac{u}{D a_o} D \right)^4 \sim \rho_o \frac{u^8 D^2}{a_o^5} \quad (17)$$

The striking feature is the scaling of this radiated power with the 8<sup>th</sup> power of the jet velocity (a result due to Lighthill (1963)). To compare it to the jet kinetic power, note that

$$\mathbf{P}_{jet} \sim (\rho_o u D^2) u^2 = \rho_o u^3 D^2 \quad (18)$$

so that

$$\frac{\mathbf{P}_q}{\mathbf{P}_{jet}} \sim \left( \frac{u}{a_o} \right)^5 = M_{jet}^5 \quad (19)$$

This very strong sensitivity to jet velocity (or Mach number) is one of the significant disadvantages of the pure turbojet as compared to the high bypass turbofan (the other being its lower propulsive efficiency, although that disappears at a high enough flight Mach number).

### Turbomachinery noise

Vibrations of lifting blades, or periodic passage of lifting blades past the observer, produce dipole-type radiation, which emits a power given by Eq. (13).

Estimating in this case  $r_o \sim d \sim c$  (blade chord),  $k \sim \frac{\omega}{a_o} \sim \frac{u}{a_o c}$ , where  $u$  is now the blade speed, and  $P = \rho_o u^2$ , the power radiated per blade is

$$\mathbf{P}_{bl} \sim \rho_o u^6 c^2 / a_o^3 \quad (20)$$

The force on one blade is of the order of  $F_{bl} \sim \rho_o u^2 c H \sim \rho_o u^2 c^2$ , and so the "acoustic efficiency" is

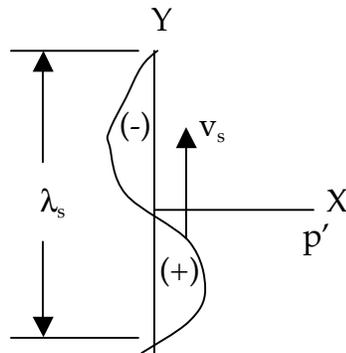
$$\eta_{bl.noise} = \frac{\mathbf{P}_{bl}}{F_{bl} u} \sim \left( \frac{u}{a_o} \right)^3 = M_T^3 \quad (21)$$

This is noise generated, but the duct, even with no special absorbing design, attenuates the sound, and in fact, forbids propagation for the lower range of frequencies. The frequency is here related to the blade Mach number  $M_T$  and the number  $B$  of blades:

$$\frac{\omega r_T}{a_o} = \frac{B \omega_{rot} r_T}{a_o} = B M_T \quad (22)$$

and so  $M_T$  must be higher than a certain threshold for blade noise to propagate.

In order to understand this phenomenon let us look at the simple case of sound emitted by a moving source, meaning that the pressure perturbation on a given surface  $x=0$  is a sliding sinusoid, moving a velocity  $v_s$  in the  $y$ -direction.



From the Cartesian wave equation (Lecture 36), a plane wave propagating in the direction of the “wave vector”  $\vec{k}$  has the general form

$$p' = \text{Re}[\hat{p}e^{-i(\omega t - k_x x - k_y y)}] \quad (23)$$

provided  $k_x$ ,  $k_y$  and  $\omega$  satisfy the “dispersion relation”, that is obtained by substituting (23) into the wave equation itself:

$$\omega^2 = a_0^2(k_x^2 + k_y^2) \quad (24)$$

Also, the moving boundary condition represented in the figure above is of the form (23), with  $x=0$ ,  $k_y=(2\pi/\lambda_s)$  and  $\omega = \frac{2\pi}{\lambda_s} v_s$ . Substituting these  $\omega$  and  $k_y$  in (24) and solving for  $k_x$

$$k_x = \pm \frac{2\pi}{\lambda_s} \sqrt{\left(\frac{v_s}{a_0}\right)^2 - 1} \quad (25)$$

and we can see that  $k_x$  is real only if  $v_s > a_0$ , i.e., only if the source is moving at a supersonic speed. The direction of the sound waves is given by

$$\theta = \tan^{-1}(k_y / k_x) = \sin^{-1}(a_0 / v_s) \quad (26)$$

namely, the Mach angle at the wall Mach number.

For a source moving at subsonic speed, the sound is in fact exponentially attenuated over a distance of the order of  $\lambda_s / 2\pi$ . Physically, the reason is that for a supersonic source the waves from the infinity of pressure peaks and troughs on the wall interfere constructively with each other when their direction is as in (26). For a subsonic source there is no such direction, and interference is always destructive.

In the case of turbomachinery, the supersonic relative velocity can come from two sources:

- a) Supersonic tip speed
- b) Subsonic tip speed, but interacting blade rows

To understand the second situation, consider a rotor operating in the vicinity of a stator. The stator blades produce a pressure variation

$$p'_{stator} \propto (\text{incidence}) e^{iV\theta}$$

where  $V$  is the number of stationary vanes.

The disturbances from the rotor are periodic in rotor coordinates, so

$$incidence \propto e^{+iB(\theta - \omega_R t)}$$

where B is the number of moving blades. It follows that

$$p'_{stator} = e^{i[(V - B)\theta - B\omega_R t]}$$

So the Effective Speed of Rotation of the disturbance is

$$\omega_{Eff} = \left(\frac{d\theta}{dt}\right)_{const. phase} = \frac{B\omega_R}{V - B}$$

In early engines there was a tendency to make  $V \approx B$  and they were very noisy. Now we use the rule  $V = 2B + 1$ , so that the rotor-stator interaction is no more likely to generate propagating disturbances than is the rotor rotation itself.

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