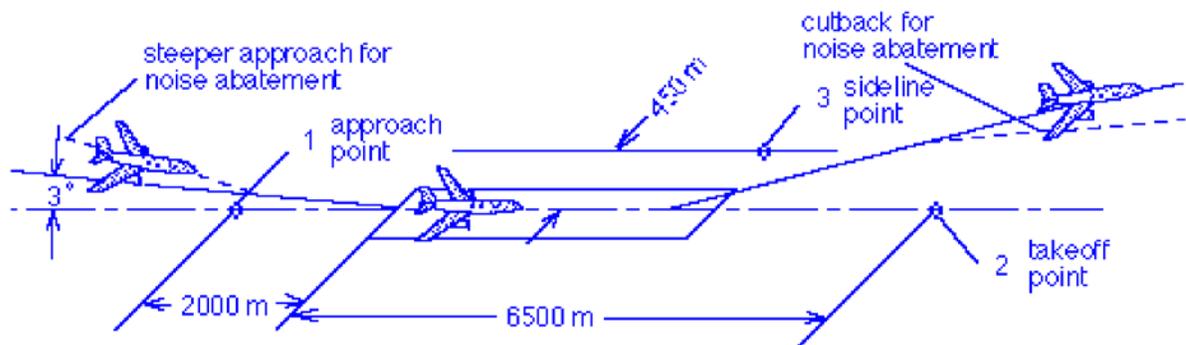


16.50 Lecture 36

Subjects discussed: Aircraft Engine Noise : Principles; Regulations

Noise generation in the neighborhoods of busy airports has been a serious problem since the advent of the jet-powered transport, in the late 1950's. Although piston-engined aircraft had caused some concerns prior to that, it was the turbojet-powered first generation jets (707, Comet, DC-8) with their jet noise that led to wide public concern, and to regulations by some airport authorities. With the continuing growth of airline travel, the problem has continued to expand, leading to rules promulgated by the FAA that limit the noise that any individual aircraft can make at each of 3 measuring stations.



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Noise is the human ear's response to pressure fluctuations in time, at the ear of the observer. As such it has both physical and psychological aspects. Thus, one person's music can be another person's noise. There is general agreement for example that the exhaust noise of motorcycles is annoying, but the biker will not agree. We will from this point on avoid the psychological and concentrate on the physical aspects of aircraft noise, without meaning to imply that the latter is more important.

Noise is transmitted from the source (aircraft) to the observer by sound waves. So let us begin with a brief review of sound propagation.

Conservation of mass

$$\frac{D\rho}{Dt} + \rho \nabla \cdot \bar{u} = 0 \quad (1)$$

Conservation of momentum

$$\rho \frac{D\bar{u}}{Dt} = -\nabla p \quad (2)$$

Suppose the velocity and pressure to be the sum of a large steady component and a small time-varying one

$$\vec{u} = \vec{u}_0 + \vec{u}'(t) \quad p = p_0 + p'(t) \quad \rho = \rho_0 + \rho'(t)$$

and further suppose that $\vec{u}_0 = 0$. Then to first order in small quantities (1) and (2) become

$$\frac{\partial \rho'}{\partial t} + \rho_0 \nabla \cdot \vec{u}' = 0 \quad (3)$$

$$\rho_0 \frac{\partial \vec{u}'}{\partial t} = -\nabla p' \quad (4)$$

In the absence of heat conduction and viscosity, ρ' and p' are related isentropically, i.e.

$$\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^\gamma \Rightarrow \frac{dp}{p_0} = \gamma \frac{d\rho}{\rho_0} \quad \text{or} \quad \frac{p'}{p_0} = \gamma \frac{\rho'}{\rho_0}$$

It follows that

$$p' = \gamma \frac{p_0}{\rho_0} \rho' = \gamma R T_0 \rho' \equiv a_0^2 \rho' \quad (5)$$

where a_0 is the speed of sound.

So in 1a

$$\frac{\partial \rho'}{\partial t} = \frac{1}{a_0^2} \frac{\partial p'}{\partial t}$$

Now taking $\frac{\partial}{\partial t}$ (1a) - $\nabla \cdot$ (2a) we have

$$\frac{1}{a_0^2} \frac{\partial^2 p'}{\partial t^2} - \nabla^2 p' = 0 \quad (6)$$

In one dimension (for a plane wave)

$$\frac{1}{a_0^2} \frac{\partial^2 p'}{\partial t^2} - \frac{\partial^2 p'}{\partial x^2} = 0$$

This is a Wave Equation, satisfied in general by solutions of the form $p' = p'(x \pm a_0 t)$, so that p' is constant for $x = \pm a_0 t$. Thus the solution for a plane wave would be of the form

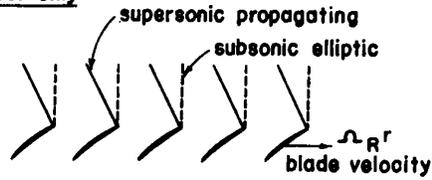
$$p' = P \cos k(x - a_0 t)$$

Excitation

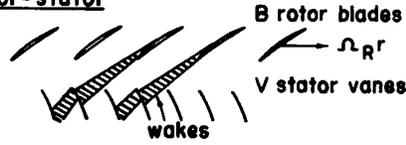
We can now ask how such waves are generated in an engine. Some of the main sound sources are schematized below:

TURBOMACHINE NOISE

Rotor only



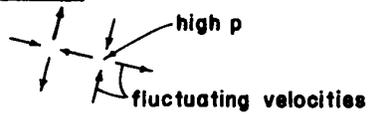
Rotor-stator



Turbulence

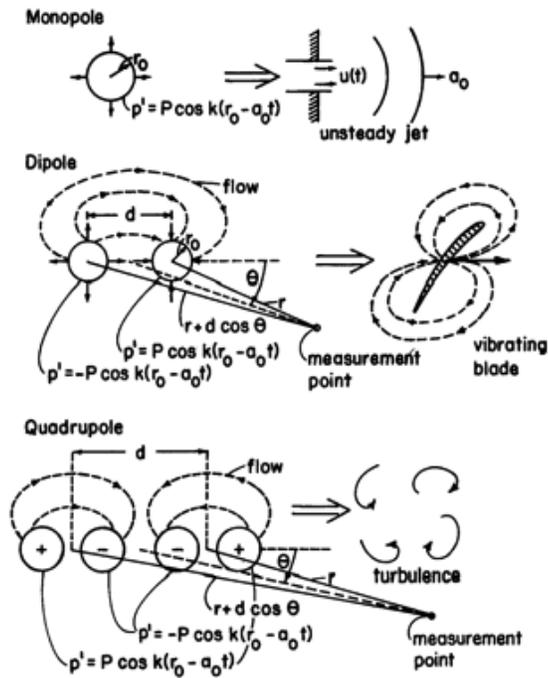


Jet turbulence



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Elementary sources. It is useful to examine the simplest acoustic sources, which are all configurations with an imposed pressure fluctuation or an imposed wall vibration. The first in a systematic series of such sources are shown below:



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(a) Monopoles

Suppose now the perturbation is due to a pulsating sphere, whose radius oscillates by some small δr_0 at a frequency ω . This sets up spherically symmetric pressure field that oscillates at ω as well, and propagates as a sound wave train. In spherical coordinates, the wave equation is

$$\frac{1}{a_0^2} \frac{\partial^2 p'}{\partial t^2} = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial p'}{\partial r} \right) \quad (7)$$

We expect acoustic energy to be conserved, and since at least the compression part of this energy varies as p'^2 , let us try a solution of the Monopole form:

$$p' = P \left(\frac{r_0}{r} \right) \cos k(r - a_0 t) \quad (8)$$

It can be seen by direct substitution that this does satisfy the wave equation. The quantity $k = \omega / a_0$ is called the wave number, and the wavelength is $\lambda = \frac{2\pi}{k} = 2\pi \frac{a_0}{\omega}$.

The velocity field (purely radial) can be calculated from (4)

$$\frac{\partial u_r}{\partial t} = -\frac{1}{\rho_0} \frac{\partial p'}{\partial r} \quad (4a)$$

and from (8), after integrating in time,

$$u'_r = \frac{P}{\rho_o a_o} \frac{r_o}{r} \left[\cos(r - a_o t) - \frac{\sin(r - a_o t)}{kr} \right] \quad (9)$$

The acoustic power flux (energy crossing unit area per unit time, averaged over one cycle) is the average of the work done by p' :

$$\Gamma = \frac{1}{\tau} \int_0^\tau p' u'_r dt = \frac{P^2}{2\rho_o a_o} \left(\frac{r_o}{r} \right)^2 \quad (10)$$

where $\tau = \frac{2\pi}{\omega} = \frac{2\pi}{ka_o}$.

The net acoustic radiated power is $\mathbf{P}_m = 4\pi r^2 \Gamma = 2\pi \frac{P^2 r_o^2}{\rho_o a_o}$, which is seen to be

independent of r , as it should. The monopole power radiated is independent of wave number $k = 2\pi/\lambda$, and only dependent on pressure amplitude. A possible physical implementation of a monopole source is a pulsating jet, such as produced by a pulse-jet (like that in the V-1 missile), or by the oscillations during an engine surge.

(b) Dipoles

Consider next two monopoles of equal strength P operating in counter-phase to each other, and spaced a small distance d along the x -direction. If an observer is located at a distance r from one of them, and at an angle ϑ from the x direction, its distance to the other will be (approximately) $r + d \cos \vartheta$. Then the pressure p' at the observer's location will vary as

$$p' = \frac{Pr_o}{r} \cos k(r - a_o t) - \frac{Pr_o}{r + d \cos \vartheta} \cos k(r + d \cos \vartheta - a_o t) \quad (11)$$

Expanding the second term and assuming d to be much smaller than both, the wavelength ($\approx 1/k$) and the observation distance r ,

$$p' = \frac{Pr_o}{r} (k d \cos \vartheta) \sin k(r - a_o t) + \left(\frac{d}{r} \cos \vartheta \right) \frac{Pr_o}{r} \cos k(r - a_o t) \quad (11b)$$

of the two terms in (11b), the first has the $1/r$ dependence that will ensure energy flux conservation at all distances, while the second will decay faster and will be negligible for distances $r \gg 1/k$; this is the "near-field" dipole sound, which can be important near the source. We concentrate here on the "far-field" contribution (first term in (11b)).

The calculation of the far-field power flux is as before, remembering that it is still a radial flux, even though there is an angular dependence in ϑ . So we still have the first equality in Eq. (10), and the calculation is straightforward. We obtain

$$\Gamma(r, \vartheta) = \frac{1}{2\rho_o a_o} \left(P \frac{r_o}{r} k d \cos \vartheta \right)^2 \quad (12)$$

This flux has now a $\frac{\cos^2 \vartheta}{r^2}$ distribution (no sound at 90° to the dipole axis, maximum along the dipole axis). The total radiated power is

$$\mathbf{P}_d = 2\pi \int_0^\pi \Gamma(r, \vartheta) r^2 \sin \vartheta d\vartheta = \frac{2}{3} \pi \frac{(\text{Pr}_o k d)^2}{\rho_o a_o} \quad (13)$$

Aside from the $\frac{1}{3}$ factor, this differs from \mathbf{P}_m , the monopole power, by the factor $(kd)^2 = (2\pi d/\lambda)^2$, which is, by assumption a small number, and becomes smaller the longer the wavelength is (or the lower the frequency $\omega = a_o/k$).

An important observation is that a physical dipole requires application of an external oscillatory force. A possible physical implementation of the radiating dipole is any vibrating compact object, such as a fan or a turbine blade. The dipole axis is then the direction of vibratory motion, and the surrounding air is forced back and forth as it would between the hypothetical two monopoles in counter-phase.

(c) Quadrupoles

Strongly turbulent flows, such as an engine exhaust jet, are known to be strong sources of acoustic radiation. If the jet is steady and subsonic, there is no possibility of macroscopic monopole (expansion/contraction) type of radiation, and since there is no external force in the body of the fluid, no dipole sources either. However, there are fluctuating pressures at different points (turbulent eddies), exerting forces on each other with zero net on the larger scale. The lowest order "multipole" with these features is the Quadrupole, which can be built up from two dipoles with a common axis, and separated by 2d and with opposing directions.

The detailed derivation is similar to that for a dipole. We calculate (in the far field)

$$p' = -2 \frac{\text{Pr}_o}{r} (k d \cos \vartheta)^2 \cos k(r - a_o t) \quad (14)$$

and for the acoustic power flux,

$$\Gamma = 2 \left(\frac{\text{Pr}_o}{r} \right)^2 \frac{(k d \cos \vartheta)^4}{\rho_o a_o} \quad (15)$$

which integrates for all directions to a radiated power

$$\mathbf{P}_q = \frac{8}{5} \pi \frac{(\text{Pr}_o)^2 (kd)^4}{\rho_o a_o} \quad (16)$$

The quadrupole radiator pattern is more sharply directional along the axis ($\cos^4 \vartheta$), and is an additional $(kd)^2$ weaker than the dipole, with an even stronger rate of increase with frequency.

It should be noted that the collinear-dipole type of quadrupole is not the only one possible, but all of them share the $(kd)^4$ feature (although with different angular patterns).

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