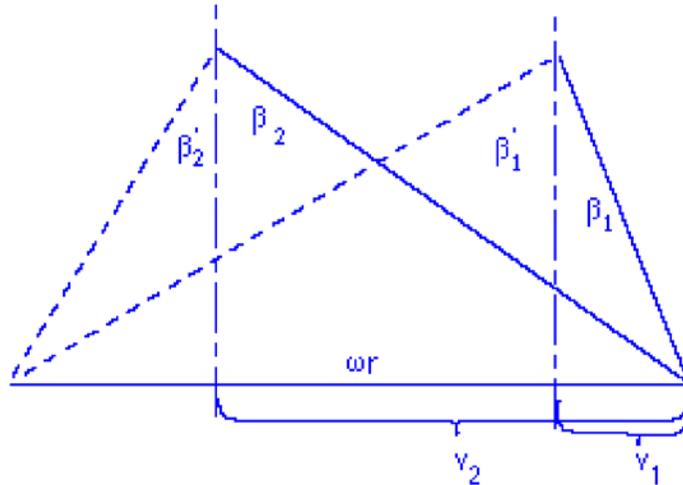


## 16.50 Lecture 25

### Subjects: Velocity triangles; Compressor performance maps

In the last lecture we discussed the basic mechanisms of energy exchange in compressors and drew some simple velocity triangles to show how we go from the stationary coordinate system to one in the moving blades. In more detail, the velocity triangle is:



Now we can write

$$v_1 = w_1 \tan \beta_1$$

$$v_2 = w_2 \tan \beta_2 = \omega r - w_2 \tan \beta_2'$$

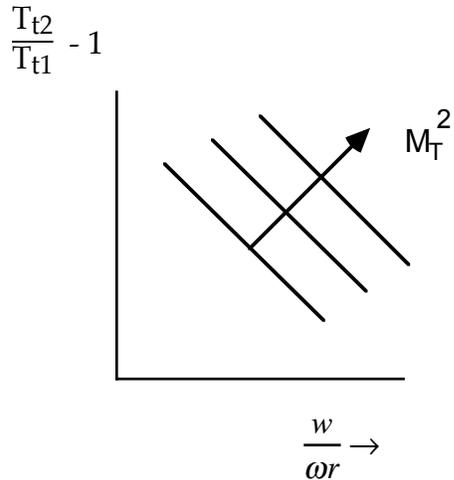
where  $\beta_2'$  is the flow angle in the rotating coordinate system of the rotor blades. The advantage of the latter form for  $v_2$  is that it is  $\beta_2'$  which is nearly set by the shape of the blades, i.e.  $\beta_2'$  is nearly constant. The difference between the flow angle and the angle of the chord line at the trailing edge of the blades is called the deviation, and is usually a small number of degrees.

Taking advantage of this small deviation, we can write the Euler equation as

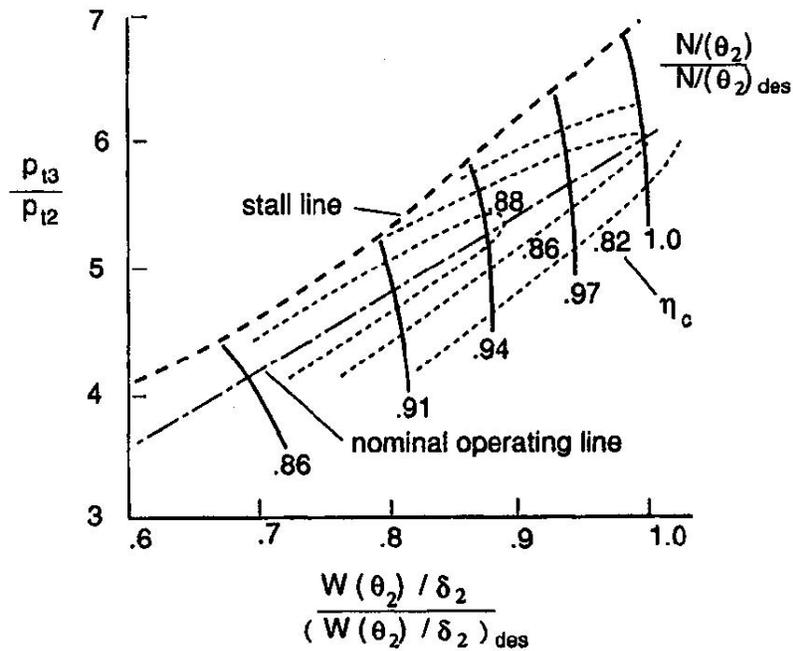
$$\frac{T_{t2}}{T_{t1}} = 1 + \frac{(\gamma - 1)M_T^2}{1 + \frac{\gamma - 1}{2}M_1^2} \left[ 1 - \frac{w_2 \tan \beta_2'}{\omega r} - \frac{w_1 \tan \beta_1}{\omega r} \right]$$

where  $M_1$  is the flow Mach number upstream and  $M_T$  is the tangential Mach number of the blade motion.

Since the two flow angles expressed this way are nearly constant, the temperature ratio becomes a function primarily of two variables, the tangential Mach number  $M_T$  and the ratio of the axial flow velocity to the blade speed. For the usual case of  $\beta_2' > 0$ ,  $\beta_1 > 0$  we get a functional dependence something like:



Actually, we usually plot this in terms of the pressure ratio rather than the temperature ratio, and include superimposed lines of efficiency, so that the map looks like:



From Kerrebrock, Jack L. *Aircraft Engines and Gas Turbines*. 2nd edition. MIT Press, 1992. © Massachusetts Institute of Technology. Used with permission.

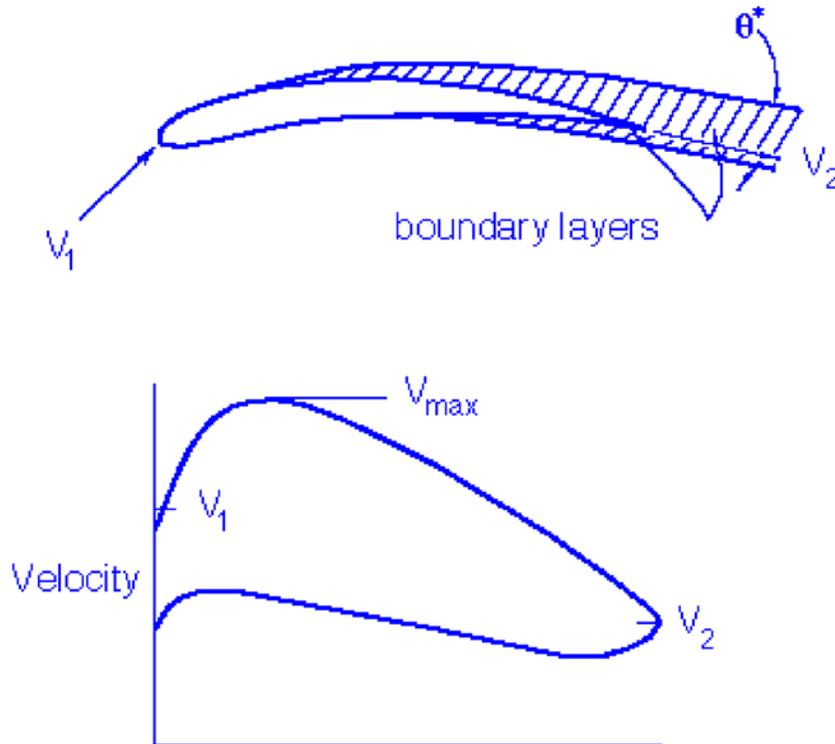
Here the compressor efficiency is defined as:

$$\eta_c = \frac{(\Delta T_t)_{Ideal}}{(\Delta T_t)_{Actual}} = \frac{T_{t1} \left( \frac{P_{t2}}{P_{t1}} \right)^{\frac{\gamma-1}{\gamma}} - T_{t1}}{T_{t2} - T_{t1}} = \frac{\left( \frac{P_{t2}}{P_{t1}} \right)^{\frac{\gamma-1}{\gamma}} - 1}{T_{t2}/T_{t1} - 1}$$

Since  $\eta_c < 1$ , the total pressure rise for a given total temperature rise is less than it could be:

$$\pi_c = [1 + \eta_c(\tau_c - 1)]^{\frac{\gamma}{\gamma-1}} < \tau_c^{\frac{\gamma}{\gamma-1}}$$

By decreasing  $\beta_1$  and  $\beta_2'$ , or even making them negative, we can increase  $T_{t2}/T_{t1}$ . What limits this increase? The answer is best given in terms of a Diffusion Factor, which describes the tendencies for the boundary layer to separate under the influence of the pressure rise in the blade passage.



From Kerrebrock, Jack L. *Aircraft Engines and Gas Turbines*. 2nd edition. MIT Press, 1992. © Massachusetts Institute of Technology. Used with permission.

The critical region is the suction surface of the blade. This region feels a pressure rise due to the decrease of  $V'$  from  $V'_1$  to  $V'_2$ , (the prime is a reminder that these velocities in the relative frame) and also due to the acceleration, followed by deceleration on the suction surface. The Diffusion Factor, defined as

$$D = 1 - \frac{V'_2}{V'_1} + \frac{|v_2 - v_1|}{2\sigma V'_1}; \quad \sigma \equiv \frac{c}{s}$$

is a crude, but effective way to account for these two flow deceleration components. Here  $c$  is the chord of the blades and  $s$  is their spacing (in the peripheral direction).

A value of about 0.5 for  $D$  is the upper limit for good efficiency. It is conventional and useful to represent the loss in the blading in terms of a Loss Factor, defined as

$$\varpi_1 = \frac{p_{t1} - \bar{p}_{t2}}{p_{t1} - p_1}$$

We then find that we can correlate the losses in the form

$$\frac{\bar{\omega} \cos \beta'_2}{2\sigma} \left( \frac{\cos \beta'_2}{\cos \beta'_1} \right)^2$$

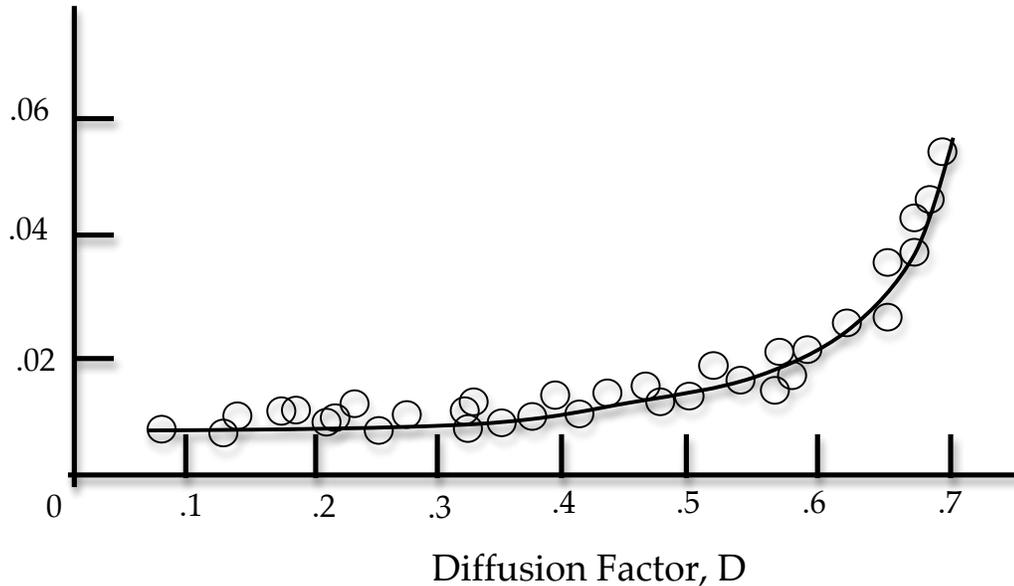


Image by Manuel Martinez-Sanchez. Adapted from Fig. 5.12 in Kerrebrock, Jack L. *Aircraft Engines and Gas Turbines*. 2nd edition. MIT Press, 1992.

### Compressor limitations. Rotating Stall. Surge.

As the incidence angles increase in the compressor a point is reached when the flow begins to separate from the blades, with serious consequences. Large incidence angles occur when the ratio (axial velocity)/(blade speed) is low, i.e., for a fixed rotational velocity, at reduced flow, and hence at increased pressure ratio. All compressors have a fairly well defined "stall line" that runs diagonally from low flow and low  $\pi_c$ , to high values of both. Operation must be restricted to the region below and to the right of this line. The operational line for the compressor runs roughly parallel to this stall line. As we saw in Lecture 20, The reason for the compressor to be restricted to his operation line have to do with flow continuity between it and the turbine and nozzle. It is a fortunate fact that the line thus defined tends to correspond to a constant blade incidence, which, when chosen properly, is close to that which optimizes blade performance, and so the operating line is more or less the peak compressor efficiency line.

The phenomena that occur at and beyond stall are complex, dynamical and highly nonlinear. A detailed understanding of these effects has only emerged in the last two decades, and remedial measures based on this understanding are still under evaluation. Much of the pioneering work in this field has been done at MIT (see E. Greitzer, Engineering for Power, 98, 2, (1976) and J. of Fluids Engineering, 103 (1981), Paduano et al., ASME Paper 91-GT-87, 1981). A more complete exposition of the principles is given in Kerrebrock's book, (*Aircraft*

Engines and Gas Turbines, (MIT Press, 1992, Sec. 5.7, pp. 3254-266), a short summary of which is presented below.

In most axial compressors, incipient separation first leads to the Rotating Stall phenomenon: sections of the stalling rotor then operate in deep stall, with almost zero flow, while the rest carries normal or high flow per unit frontal area. These regions move backwards in the rotating frame at  $\sim 0.4 - 0.6 \omega r$ , so that, when observed from rest they rotate forward, but only at a fraction of the rotor speed. The reason for the bimodal flow distribution is that adjacent streamtubes become unstable with respect to flow exchange: if the rotor as a whole is near stall conditions, and one streamtube loses some flow and diverts it to its neighbors, the streamtube with less flow goes into stall and loses even more flow, while the neighbors gain flow and remain stable. The reason the “stall cells” move backwards relative to the rotor is an elaboration of the same argument: if one flow passage stalls, the swirling incoming flow is re-routed such that the passage ahead of the one stalled sees a more axial flow, while the one behind sees a larger incidence angle. The stall moves to this trailing passage, and the stalled passage clears.

Rotating stall, since it moves rapidly about, tends to average out and, aside from high-frequency excitation, may not be dynamically significant. On the other hand, the net compressor performance drops strongly, and in addition, it is not easy to reverse once started, except by stopping and re-starting the engine. Detecting rotating stall and instituting the appropriate control reaction is very important. If the engine controls simply detect a loss of pumping performance (pressure loss) they may react by increasing fuel flow, which combined with the reduced airflow, may lead to overheating and burnout.

Under some conditions having to do mainly with the ratio of flow inertia to flow passage capacitance, the complete “pumping system” (compressor plus choked turbine nozzles) can enter a global oscillation, called Surge. Unlike rotating stall, surge involves deep oscillations or even reversals of the whole flow through the compressor, and can be mechanically destructive (certainly quite noticeable, in the form of loud, repeated bangs, accompanied by flame ejection from both ends of the engine). When stall is reached, the engine may go into either Rotating Stall or Surge. The detailed mechanisms that determine which of the phenomena will prevail are encapsulated in Greitzer’s “B parameter”

$$B = \frac{\omega r}{2a} \sqrt{\frac{V_p}{V_c}}$$

where  $a$  is the speed of sound in the burner,  $V_p$  is the “plenum” volume (mainly the burner volume) and  $V_c$  is the volume in the compressor flow passages. In a single-stage compressor, values of  $B$  below  $\sim 0.8$  lead to rotating stall, while higher values lead to surge. For multistage ( $N$ ) compressors,  $B_{crit}$  is lower by somewhere between  $\sqrt{N}$  and  $N$ . One favorable aspect of surge (as opposed to rotating stall) is that it can usually be cleared by simply reducing fuel flow (or, in a test stand, opening the downstream throttle).

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