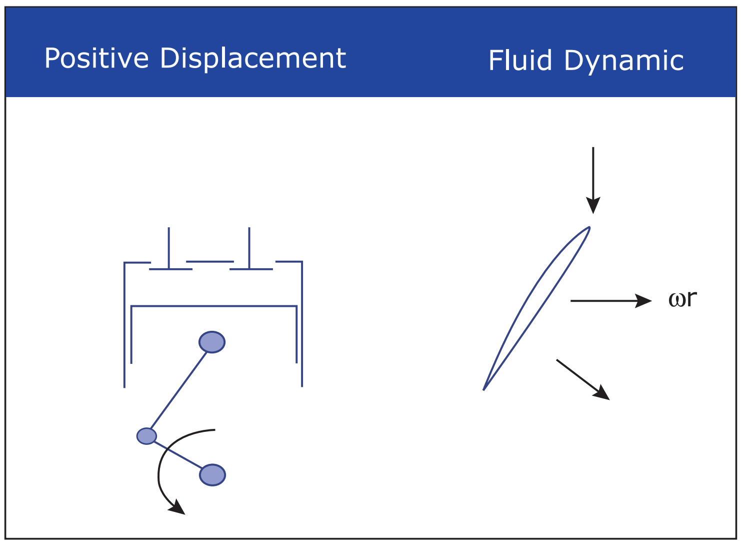
**16.50 Lecture 24**

**Subject: Compressors and Fans**

These are fluid dynamic devices, i.e., they depend on fluid accelerations to compress and expand gases, in contrast to positive displacement devices such as the familiar piston engine. The moving blades exert a force on the fluid by virtue of their motion, resulting in an added velocity of the fluid, and an increase in its energy in stationary coordinates. It is the fact that the force is dependent on the blades' velocity, roughly as the square, that makes this a dynamic machine. In contrast, the energy per unit mass added to the fluid in the positive-displacement piston-cylinder mechanism is nearly independent of the speed of the piston, depending only on the volumetric compression ratio.

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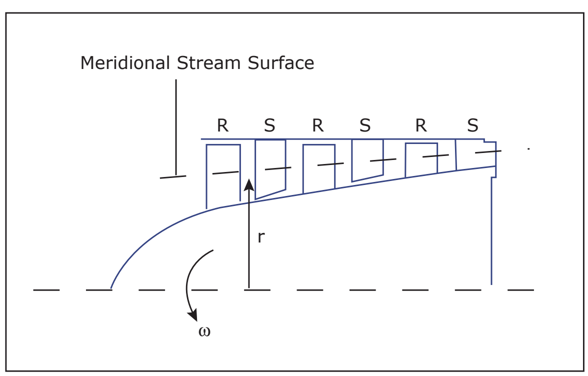
Large aircraft engine compressors or fans are mainly of the axial-flow type, with rotating rows of rotor blades, separated by stator blades. A cross-sectional view through the axis looks like

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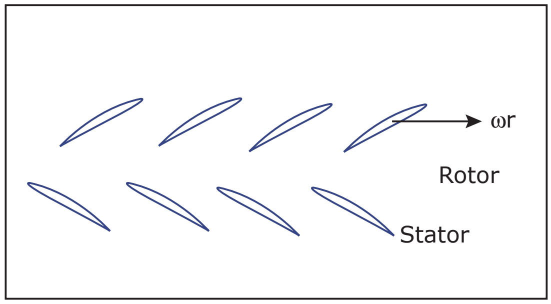
Both rows of blades function to deflect the flow in the tangential direction, the rotors adding angular momentum in the direction of the rotation, the stators removing it. Because of their motion, the rotating blades can do work on the flow, increasing its energy. The stator blades in contrast only diffuse the flow, exchanging momentum for pressure rise.

In the sketch the surface of rotation defined by the dashed line is termed a meridional stream surface. On this stream surface, the cross section of the blades looks like:

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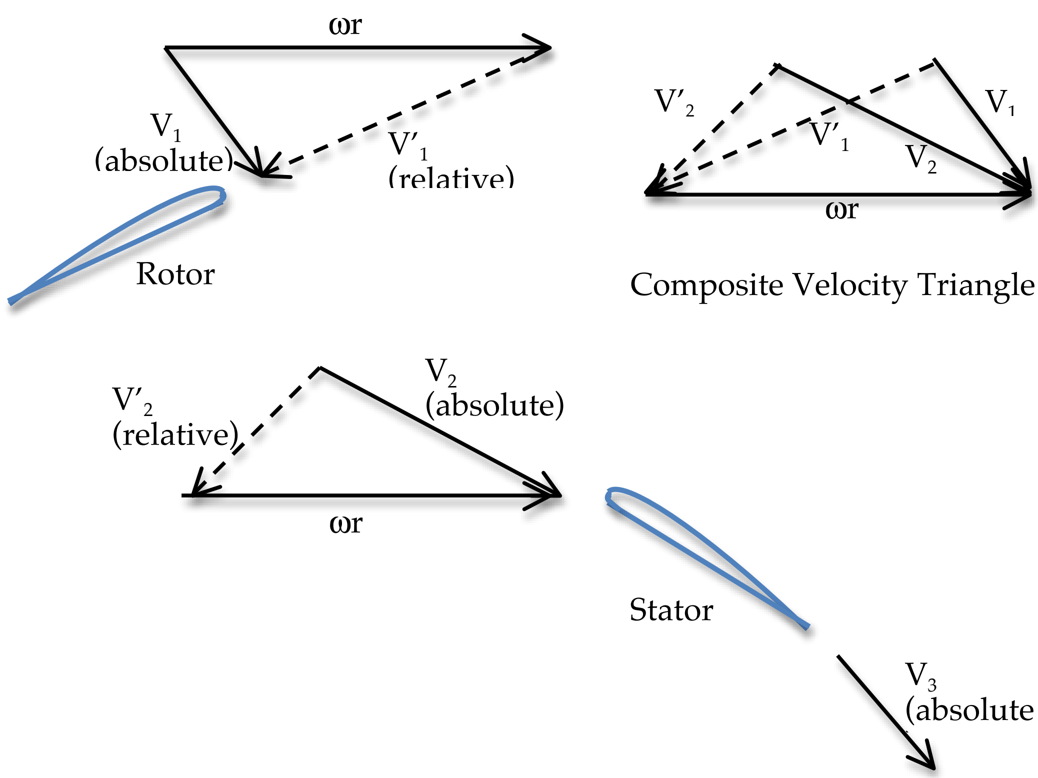
To simulate the rotational symmetry of the actual blade rows we think of a cascade, extending to infinity in both directions. In this limit the rotor blades then have a constant velocity and the stators are stationary.

To describe the effect of the blades, Velocity Diagrams are used, which show how the velocities change across the rotor and stator blade rows, and in particular the effects of the relative motion of the two:




If the velocities at 3 are the same as at 1 then we can put another identical stage after this one.

The energy exchange all takes place in the rotor blade rows. The force on a blade F is equal to the rate of change of tangential momentum per blade:

$$F = (\dot{m} \text{ per blade}) (v_2 - v_1)$$

where v is the tangential component of **V** at any point.

The power delivered to the fluid by a blade is the force times the blade velocity and this must equal the change in fluid energy per blade, across the rotor, so

$$\text{Power} = F\, \omega r$$

$$\text{Power per blade} = (\omega r) (\dot{m} \text{ per blade}) (v_2 - v_1)$$

$$= (\dot{m} \text{ per blade})\, c_p\, (T_{t2} - T_{t1})$$

$$c_p\, (T_{t2} - T_{t1}) = \omega r\, (v_2 - v_1)$$

This is the famous (and important) Euler Turbine Equation.

The pressure generally rises across both rotor and stator blade rows. If the flow relative to the blades is subsonic we can see this easily from Bernoulli's equation:

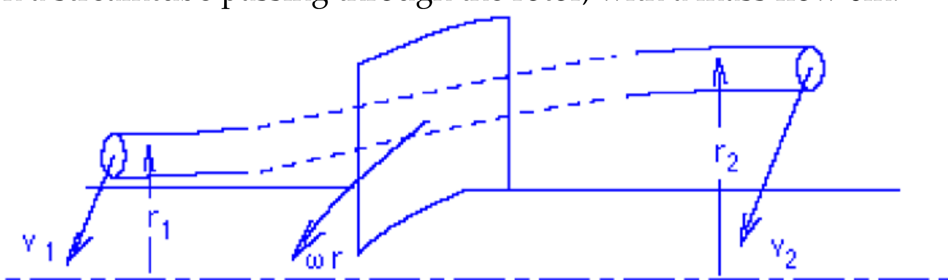
$A_2 > A_1$ (because of the turn towards axial)

$V_2 < V_1$

$$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2$$

$p_2 > p_1$

The energy exchange across the rotor is most generally understood as follows. We focus on a streamtube passing through the rotor, with a mass flow $\delta m$:




The torque on the rotor due to this stream tube is

$$\delta T = \delta \dot{m} (r_2 v_2 - r_1 v_1)$$

and the power is

$$\delta\text{Power} = \omega \delta T = \delta \dot{m}\, \omega (r_2 v_2 - r_1 v_1)$$

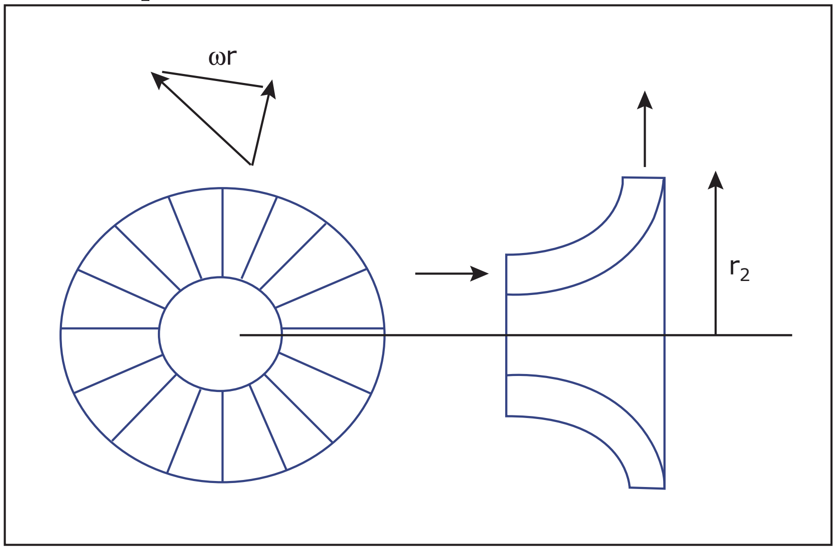
Equating this to the total enthalpy rise of the fluid

$$\delta\text{Power} = \delta \dot{m}\; c_p\, (T_{t2} - T_{t1})$$

$$c_p\, (T_{t2} - T_{t1}) = \omega (r_2 v_2 - r_1 v_1)$$

This is the Euler Turbine Equation in a general form, although the left hand side is even more generally the total enthalpy rise in the streamtube across the rotor, whether or not the gas can be modeled as being ideal.

The Euler Turbine Equation applies to other geometries than the axial flow one, e.g. the centrifugal or radial compressor

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In this case $v_1 \approx 0$ and if the vanes are radial, $v_2 \approx \omega r$ so we have

$$c_p\,(T_{t2} - T_{t1}) = (\omega r_2)^2$$

For the axial compressor, $r_2 \approx r_1$, and we have

$$c_p\,(T_{t2} - T_{t1}) \approx \omega r(v_2 - v_1)$$

$$\frac{T_{t2}}{T_{t1}} - 1 \approx \frac{(\omega r)^2}{c_p T_{t1}}\left(\frac{v_2 - v_1}{\omega r}\right)$$

$$= \frac{\gamma R}{c_p}\frac{T_1}{T_{t1}}\frac{(\omega r)^2}{\gamma R T_1}\left(\frac{v_2 - v_1}{\omega r}\right)$$

$$\frac{T_{t2}}{T_{t1}} - 1 = (\gamma - 1)\frac{T_1}{T_{t1}} M_T^{\,2}\left(\frac{v_2 - v_1}{\omega r}\right)$$