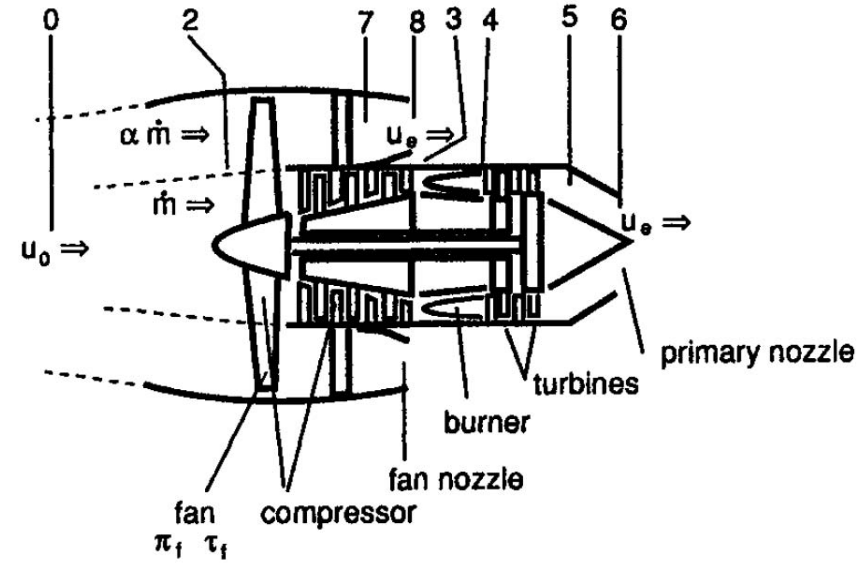
16.50 Lecture 21

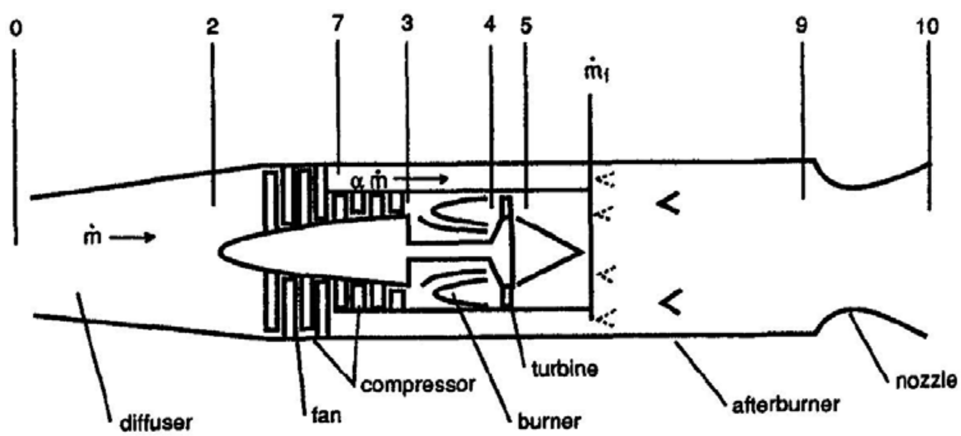
Subject: Turbofan Engines

In the lecture 19 we saw that for low M_o, η_p is not very high for turbojets. In essence there is too much kinetic energy in the exhaust jet (per unit mass). This is the main reason for use of the Turbofan engine. If it is designed for subsonic cruise flight it looks like:



From Kerrebrock, Jack L. *Aircraft Engines and Gas Turbines*. 2nd edition. MIT Press, 1992. © Massachusetts Institute of Technology. Used with permission.

If designed for both subsonic cruise and for supersonic flight with afterburning it looks more like:



From Kerrebrock, Jack L. *Aircraft Engines and Gas Turbines*. 2nd edition. MIT Press, 1992. © Massachusetts Institute of Technology. Used with permission.

In both of these diagrams the inlet has been greatly simplified, of course.

Let us now see how we can model these engines thermodynamically. In the first (cruise engine) we have 2 jets. Up until we evaluated τ_t, our argument for finding the jet velocity applies to either jet. Thus from equation (8) of Lecture 18 we get for the core jet:

$$\frac{F_c}{\dot{m}a_o}=\sqrt{\frac{2}{\gamma-1}\left(\frac{\theta_t}{\theta_o\tau_c}\right)[\theta_o\tau_c\tau_t-1]}-M_o$$

For the fan stream, there is no turbine, so let us repeat the argument.

$$T_{t8}=T_8\left(1+\frac{\gamma-1}{2}M_8^2\right)=T_o\theta_o\tau_f$$

and

$$p_{t8}=p_8\left(1+\frac{\gamma-1}{2}M_8^2\right)^{\frac{\gamma}{\gamma-1}}=p_o\,\delta_o\,\pi_f=p_o\,(\theta_o\tau_f)^{\frac{\gamma}{\gamma-1}}$$

So for $p_8=p_o$

$$1+\frac{\gamma-1}{2}M_8^2=\theta_o\tau_f$$

$$M_8=\sqrt{\frac{2}{\gamma-1}(\theta_o\tau_f-1)}$$

As in the turbojet, a fixed convergent nozzle is more likely to be sonic and under-expanded, namely, to have $M_8=1$ and $p_8>p_0$, but we will here ignore the differences this entails.

We also have

$$\frac{T_8}{T_o}=\frac{\theta_o\tau_f}{\theta_o\tau_f}=1$$

so

$$\frac{F_{BP}}{\alpha\dot{m}a_0}=M_0\left(\frac{u_8}{u_0}-1\right)=M_0\left[\frac{\sqrt{\frac{2}{\gamma-1}\left(\theta_0\tau_f-1\right)}}{M_0}-1\right]$$

adding this thrust to the thrust of the core jet, we find the total thrust:

$$\frac{F}{\dot{m}a_o}=\left\{\sqrt{\tfrac{2}{\gamma-1}\left(\tfrac{\theta_t}{\theta_o\tau_c}\right)[\theta_o\tau_c\tau_t-1]}-M_o\right\}+\alpha\left\{\sqrt{\tfrac{2}{\gamma-1}(\theta_o\tau_f-1)}-M_o\right\} \qquad (14)$$

Now we need τ_t. We will see later that most engines have two shafts, but for now we lump their power together; at off-design conditions, this would have to be modified. From a work balance between the compressor, fan and turbine, (both shafts),

$$\dot{m}c_p(T_{t4}-T_{t5})=\dot{m}c_p(T_{t3}-T_{t2})+\alpha\dot{m}c_p(T_{t7}-T_{t2})$$

$$\theta_t(1-\tau_t)=\theta_o(\tau_c-1)+\alpha\theta_o(\tau_f-1)$$

$$\tau_t = 1 - \frac{\theta_o}{\theta_t}[(\tau_c - 1) + \alpha(\tau_f - 1)] \tag{15}$$

Substituting this gives us our result for the thrust of the turbofan. It doesn't really help to carry out the substitution at this point. Instead let us think about how to simplify the expressions to make them more easily understandable.

For this engine there are more parameters than for the turbojet:

θ_t, τ_c - as before

α, τ_f - characterizing the fan flow.

We can relate some of the parameters to others by noting that the highest propulsive efficiency is realized when $u_8 = u_6$, since this maximizes the ratio of $\frac{\text{momentum}}{\text{energy}}$ in the jets. For this situation, the two $\sqrt{\ }$'s in the thrust equation are equal, and this requires that:

$$\left(\frac{\theta_t}{\theta_0 \tau_c}\right)[\theta_o \tau_c \tau_t - 1] = \theta_o \tau_f - 1 \tag{16}$$

$$(u_8 = u_6)$$

Solving for τ_f as a function of α using (15),

$$\tfrac{\theta_o \tau_c}{\theta_t}(\theta_o \tau_f - 1) + 1 = \theta_o \tau_c - \frac{\theta_o{}^2 \tau_c}{\theta_t}[(\tau_c - 1) + \alpha(\tau_f - 1)]$$

$$\theta_o \tau_f - 1 + \frac{\theta_t}{\theta_o \tau_c} = \theta_t - \theta_o[(\tau_c - 1) + \alpha(\tau_f - 1)]$$

$$\tau_f = \frac{1 + \theta_t - \frac{\theta_t}{\theta_o \tau_c} - \theta_o(\tau_c - 1) + \theta_o \alpha}{\theta_o + \theta_o \alpha}$$

$$\tau_f = \frac{1 + \theta_t + \theta_o(1 + \alpha - \tau_c) - \frac{\theta_t}{\theta_o \tau_c}}{\theta_o(1 + \alpha)} \tag{17}$$

When this equation is satisfied, the thrust equation becomes simply

$$\frac{F}{\dot{m} a_o} = (1 + \alpha)\left[\sqrt{\tfrac{2}{\gamma - 1}(\theta_o \tau_f - 1)} - M_0\right]$$

Now as for the turbojet there is still the choice of the compression ratio. Generally we want to choose it for maximum power, which in this case for given turbine inlet temperature θ_t and bypass ratio α means maximum thrust. To maximize F, we choose τ_c to maximize τ_f, since F increases monotonically with τ_f. From the expression for τ_f

$$\frac{\delta \tau_f}{\delta \tau_c} = -\theta_o + \frac{\theta_t}{\theta_o \tau_c{}^2} = 0$$

$$\tau_c^{\ 2} = \frac{\theta_t}{\theta_o^{\ 2}} \qquad\qquad (\tau_c)_{F\max} = \frac{\sqrt{\theta_t}}{\theta_0}$$

Notice that this is precisely the same result as for the Turbojet. Substituting in the expression for τ_f

$$(\tau_f)_{\max F} = \frac{1+\theta_t+\theta_o(1+\alpha)-\sqrt{\theta_t}-\sqrt{\theta_t}}{\theta_o(1+\alpha)}$$

$$(\tau_f)_{\max F} = \frac{(\sqrt{\theta_t}-1)^2}{\theta_o(1+\alpha)}+1 \qquad (17b)$$

$$\left(\frac{F}{\dot{m}a_0}\right)_{\max} = (1+\alpha)\left[\sqrt{\frac{2}{\gamma-1}\left[\frac{(\sqrt{\theta_t}-1)^2}{1+\alpha}+\theta_o-1\right]}-M_o\right] \qquad (14b)$$

Now we have 3 parameters,

θ_t - which we set at the maximum feasible value

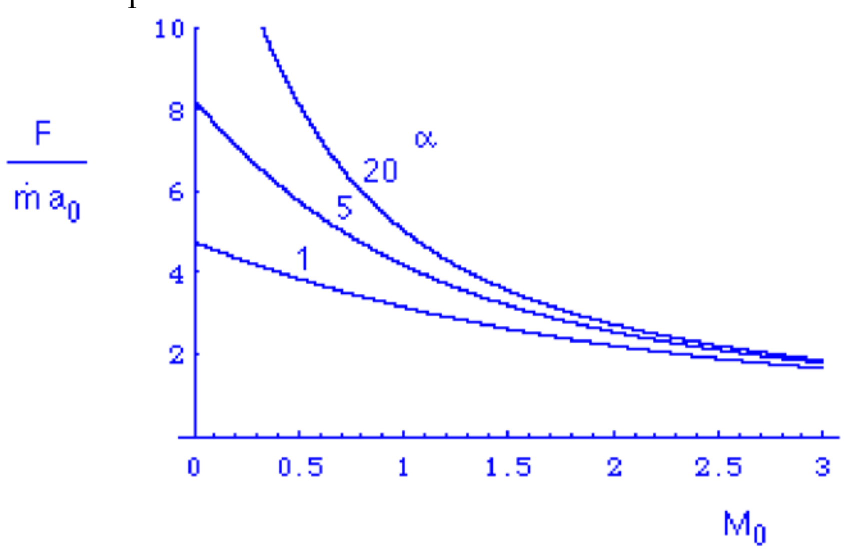
M_o - flight speed

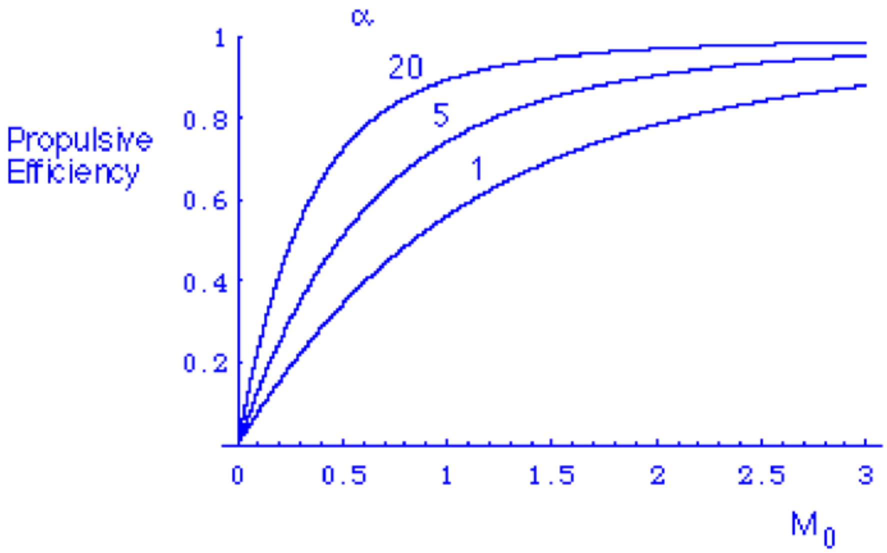
α - the prime variable distinguishing the turbofan

As before

$$\eta_{propulsive} = \frac{2}{\frac{u_6}{u_0}+1} = \frac{2}{\frac{F/\dot{m}a_0}{M_0(1+\alpha)}+2}$$

The variation of F/ma_0 and η_p with M_0 and α is shown in the figures below for θ_t=6.25. Of course for the higher bypass ratios we are really only interested in the range of M_0<1, but the lower bypass and higher M_0 range may be of interest for a supersonic transport.





α
20
5
1
Propulsive Efficiency
1
0.8
0.6
0.4
0.2
0
0.5
1
1.5
2
2.5
3
M0