

16.50 Lecture 19

Subject: Turbojet engines (continued); Design parameters; Effect of mass flow on thrust.

In this chapter we examine the question of how to choose the key parameters of the engine to obtain some specified performance at the design conditions, and how the performance varies if these parameters are changed, still at the design conditions. Later we will look at a complementary question, namely, how the performance of a particular design changes when conditions are different from design conditions.

With the results that we worked out last time for the Turbojet engine, let us look at the dependence of $F/\dot{m} a_o$ on the principal parameters, M_o , τ_c and θ_t . We can view them this way

$\tau_c \rightarrow$ design choice (compressor pressure ratio)

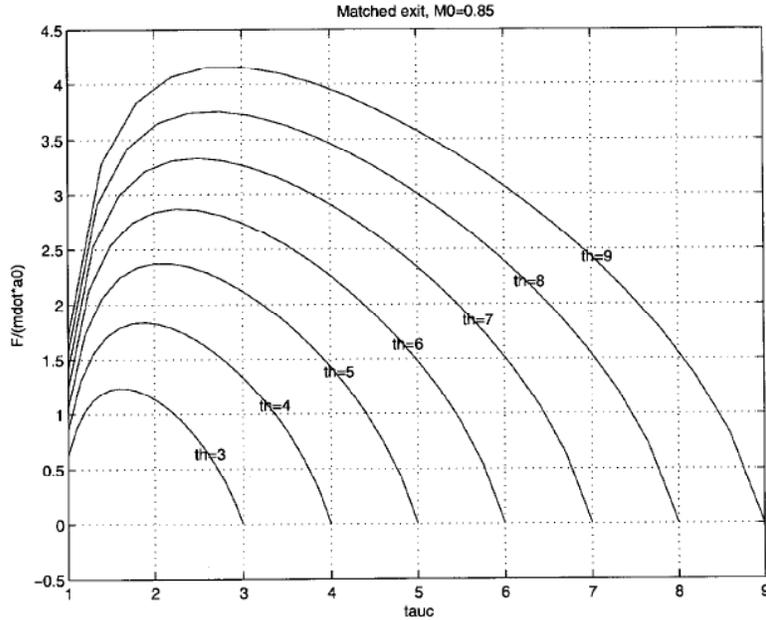
$M_o \rightarrow$ flight speed

θ_t or $\theta \rightarrow$ combustor outlet temperature, an operating variable, limited by turbine materials to some maximum value.

Assuming the exhaust is matched, we can re-write Eq. (10) of the previous lecture as

$$\frac{F}{\dot{m}_o a_o} = \sqrt{\frac{2}{\gamma - 1} \left[\theta_t \left(1 - \frac{1}{\theta_o \tau_c} \right) - \theta_o (\tau_c - 1) \right] - M_o^2}$$

we can see that since $\theta_o \tau_c > 1$, $\frac{F}{\dot{m} a_o}$ always increases with θ_t . This relationship is displayed below for a subsonic case:



For a given θ_t , what is the variation with τ_c ? By inspection we see there is a maximum at the maximum of the bracketed quantity, so at the value of τ_c that satisfies

$$\frac{\partial}{\partial \tau_c} \left[\theta_t \left(1 - \frac{1}{\theta_o \tau_c} \right) - \theta_o (\tau_c - 1) \right] = \frac{\theta_t}{\theta_o} \frac{1}{\tau_c^2} - \theta_o = 0$$

This value is

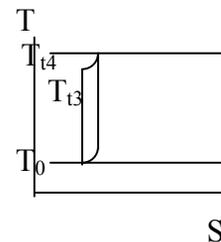
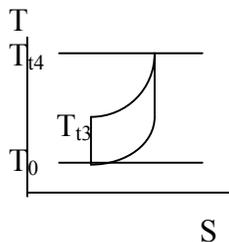
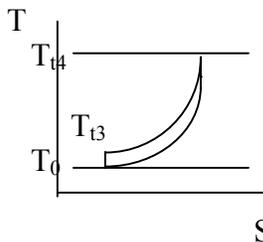
$$(\tau_c)_{\max} = \frac{\sqrt{\theta_t}}{\theta_o}$$

This result can be seen to be equivalent to $T_{t3} = \sqrt{T_0 T_{t4}}$, namely, the compressor exhaust should be at the geometrical mean of the ambient and combustor exhaust temperatures. If it were much lower or much higher, the T-S diagram of the equivalent Brayton cycle would be too “skinny”, and enclose too little area (too little work per unit mass):

(a) Too little compression

(b) Optimal compr.

(c) Too much compr.



Whether this power is utilized as jet kinetic energy, as in the turbojet, or as shaft power in a turboprop, is immaterial. Also, as far as this argument, the compression T_{t3}/T_0 can be arbitrarily divided between ram compression (θ_0) and mechanical compression (τ_c). What is the meaning of this for the turbojet? Putting this value into (10) and the corresponding expression for the specific impulse, we have the thrust and I for engines optimized for thrust per unit of airflow:

$$\left(\frac{F}{\dot{m}a_o}\right)_{\max F} = \sqrt{\frac{2}{\gamma-1}(\sqrt{\theta_t}-1)^2 + M_o^2} - M_o \quad (11)$$

$$(I)_{\max F} = \left(\frac{a_o h}{C_p T_o g}\right) \frac{(F/\dot{m}a_o)}{(\theta_t - \sqrt{\theta_t})}$$

As an example take: $\theta_t = 6.25$ $\sqrt{\theta_t} = 2.5$, $\gamma = 1.4$

$$\left(\frac{F}{\dot{m}a_o}\right)_{\max F} = \sqrt{5(2.25) + M_o^2} - M_o$$

$$(I)_{\max F} = \left(\frac{a_o h}{C_p T_o g}\right) \frac{F/\dot{m}a_o}{3.75}$$

$$\frac{a_o h}{C_p T_o g} = \frac{(283 \text{ m/s})(4.3 \times 10^7 \text{ J/kg})}{(1004 \text{ J/kgK})(200 \text{ K})(9.81 \text{ m/s}^2)} = 6178 \text{ s}$$

There is an upper limit on M_o reached when the compressor outlet temperature equals the turbine inlet temperature, so no fuel can be added. That is we must have $\theta_o \tau_c < \theta_t$. Since θ_o increases with Mach number, the theoretical limit is reached when $\tau_c = 1$ and $\theta_o = \theta_t$, i.e. when there is no compressor and we have a ramjet. But for $\tau_c = \frac{\sqrt{\theta_t}}{\theta_o}$ the real limit is reached when $\theta_o = \sqrt{\theta_t}$, i.e., all the compression is due once again to the ram effect, but we are allowing a good margin for heat addition in the burner, so this ramjet does produce maximum thrust. For a currently practical value of $\theta_t = 9$, $\theta_o < 3$ or $M_o < 3.9$

Propulsive Efficiency

The Turbojet engine is attractive for its simplicity and its good thrust behavior at high Mach numbers. Unfortunately it is not very efficient at low Mach numbers, because its jet velocity is too high. To see this we consider the Propulsive Efficiency, defined as

$$\eta_{\text{propulsive}} \equiv \frac{\text{power - to - airplane}}{\text{power - in - jet}}$$

$$= \frac{F u_o}{\dot{m} \left(\frac{u_e^2}{2} - \frac{u_o^2}{2} \right)} = \frac{2 \dot{m} (u_e - u_o) u_o}{\dot{m} (u_e - u_o) (u_e + u_o)} = \frac{2 u_o}{(u_e + u_o)}$$

From this we see that there is a direct conflict between the desire for high jet velocity to give high thrust, and jet velocity near the flight velocity, to maximize the propulsive efficiency.

In terms of our expression for thrust, since

$$\frac{F}{\dot{m}a_o} = M_o \left(\frac{u_e}{u_o} - 1 \right)$$

$$\frac{u_e}{u_o} = (F / \dot{m}a_o) / M_o + 1$$

and we can write the expression for the propulsive efficiency in terms of our expression for thrust

$$\eta_{propulsive} = \frac{2}{\frac{F}{\dot{m}a_o M_o} + 2}$$

Since $F/\dot{m} a_o \sim 2$ to 3 for low M_o , η_{prop} is not good for the turbojet at low Mach numbers. We will see later how this deficiency is remedied by adding a fan to the engine to produce a Turbofan.

Thermal and Overall Efficiencies

The Thermal Efficiency is define for the Turbojet Engine as

$$\eta_{thermal} \equiv \frac{\text{power} - \text{in} - \text{jet}}{\text{power} - \text{in} - \text{fuel} - \text{flow}} = \frac{\dot{m} \left(\frac{u_e^2}{2} - \frac{u_o^2}{2} \right)}{\dot{m}_f h}$$

Finally we can define an Overall Efficiency as

$$\eta_{overall} \equiv \frac{\text{power} - \text{to} - \text{airplane}}{\text{power} - \text{in} - \text{fuel} - \text{flow}} = \frac{Fu_o}{\dot{m}_f h}$$

We see that

$$\eta_{overall} = \eta_{thermal} \eta_{propulsive}$$

It is also important that the overall efficiency is directly related to the specific impulse:

$$\eta_{overall} = \frac{Fu_o}{\dot{m}_f h} = \frac{F}{\dot{m}_f g} \frac{gu_o}{h} = \frac{gu_o}{h} I$$

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