16.50 Lecture 18

Subject: Aircraft Engine Modeling; Turbojet engine.

All aircraft engines are Heat Engines, in that they use the thermal energy derived from combustion of fossil fuels to produce mechanical energy in the form of kinetic energy of an exhaust jet. The excess of momentum of the exhaust jet over that of the incoming airflow produces thrust.

In studying these devices we thus employ two types of modeling.

a) Thermodynamic, in which the production of mechanical energy from thermal is studied by the approaches of Thermodynamics. Here the change in thermodynamic state of the air as it passes through the engine is studied. The physical configuration of the engine is not identified. Rather the processes are specified, by pressure and temperature ratios.

b) Fluid mechanical, in which we relate the changes in pressure, temperature and velocity of the air, to the physical characteristics of the engine.

With these ideas in mind, let's first outline a general approach to the modeling of aircraft propulsion systems. Our general expression for thrust , in which we have a main interest, is

$$F = \dot{m}_e u_e - \dot{m}_o u_o + A_e(p_e - p_o) \qquad (1)$$

where $\dot{m}_e = (1 + f)\,\dot{m}_o$; $f = \dfrac{\dot{m}_f}{\dot{m}_o}$

We write this more conveniently in dimensionless form as

$$\frac{F}{\dot{m}_o u_o} = (1 + f)\frac{u_e}{u_o} - 1 + \frac{A_e p_o}{\dot{m}_o u_o}\left(\frac{p_e}{p_o} - 1\right) \qquad (2)$$

In our modeling of the aircraft engine we will often assume $p_e = p_o$, and usually take $f<<1$, so, using the flight Mach number $M_0=u_0/a_0$, this expression becomes simply

$$\frac{F}{\dot{m}_o a_o} = M_o\left[\frac{u_e}{u_o} - 1\right] \qquad (3)$$

but it should be recalled that just as for the rocket engine, the behavior of the nozzle can be somewhat more complex. In practice the deviation from ideal expansion becomes important for supersonic flight. In particular, there can be sonic or supersonic underexpansion, with an exhaust pressure $p_e>p_0$. This happens in particular with purely convergent nozzles, that are commonly used in subsonic engines, when they operate off-design. It can also happen in a variable-geometry supersonic nozzle that is not correctly adapted to the ambient pressure.

Our tasks in estimating F are then

a) to estimate $\dot{m}_o$

b) to estimate $\frac{u_e}{u_o}$.

Many of the engines we deal with will have 2 exhaust streams. In this case we apply (3) separately to each stream.

Let us begin with a Turbojet Engine, shown schematically below, and break the engine into a set of Components with functions as follows.

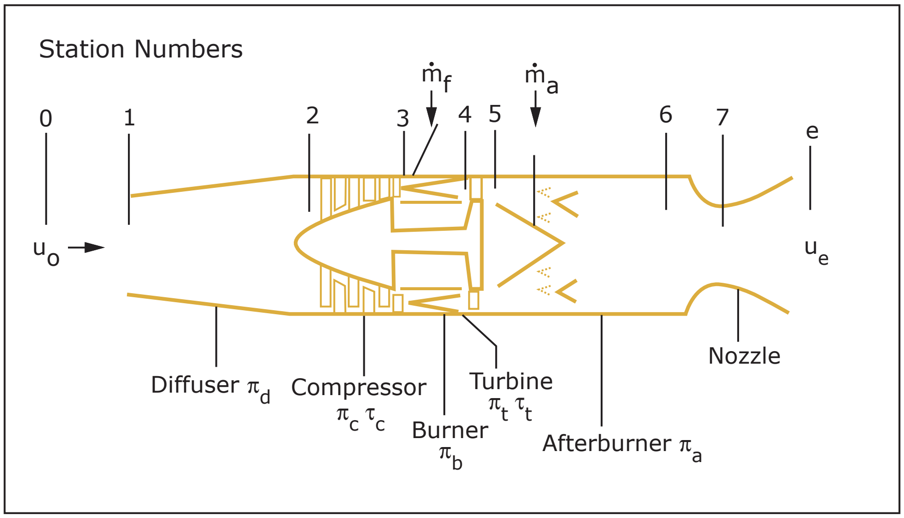

Image by MIT OpenCourseWare.
Adapted from Figure 1.4 Kerrebrock, Jack L. (1992).
Aircraft Engines and Gas Turbines (2nd Edition). MIT Press.

Diffuser (d) - Brings airflow from the flight Mach number M_o, to the axial Mach number M_2, required by the compressor.

Compressor (c) Raises temperature and pressure of airflow, as nearly isentropically as possible.

Combustor (b) Raises temperature, nearly at constant pressure.

Turbine (t) Drops temperature and pressure, as nearly isentropically as possible.

Afterburner (a) Heats air again, at nearly constant pressure (to a higher temperature than the turbine can tolerate without cooling).

Nozzle (n) Expands hot gases to produce a high-velocity jet. Station 7 denotes the sonic throat.

We first note that for this engine,

$$\frac{u_e}{u_o} = \frac{M_e}{M_o}\sqrt{\frac{Te}{T_o}}$$

It is most efficient to find the exit Mach number and temperature by keeping track of the stagnation temperatures and pressures through the several components. The following procedure works for all aircraft engines, so it's worth your paying some attention to the procedure itself, as well as the result. We employ the defining relations for the stagnation properties:

$$T_t = T\left(1+\frac{\gamma-1}{2}M^2\right)$$

$$p_t = p\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}}$$

It is very helpful to define a set of symbols that represent explicitly ratios of these stagnation properties and distinguish them from the static or thermodynamic properties of the gas, because in general it is the stagnation properties that most conveniently represent the effect of the components on the fluid as it flows through the engine. Thus,

A ratio of p_t's will be denoted by the symbol π;

A ratio of T_t's will be denoted by the symbol τ.

A ratio of a stagnation temperature to the ambient static temperature T_o will be denoted θ and

A ratio of a stagnation pressure to the ambient static pressure p_o will be denoted δ.

So for the flow upstream of the engine,

$$\frac{T_{to}}{T_o} = 1+\frac{\gamma-1}{2}M_o^2 = \theta_o$$

$$\frac{p_{to}}{p_o} = (1+\frac{\gamma-1}{2}M_o^2)^{\frac{\gamma}{\gamma-1}} = \delta_o$$

The turbine-inlet temperature is represented by

$$\frac{T_{t4}}{T_o} = \theta_t$$

or, alternatively, by

$$\theta = \frac{T_{t4}}{T_{t2}} = \frac{T_{t4}}{T_{t0}} = \frac{\theta_t}{\theta_0}$$

which is more convenient for scaling purposes, since it relates two engine total temperatures, a ratio that is often independent of ambient conditions.

For the compressor and for the turbine (both ideal by assumption),

$$\frac{p_{t3}}{p_{t2}} = \pi_c, \quad \frac{T_{t3}}{T_{t2}} = \tau_c; \quad \pi_c = \tau_c^{\frac{\gamma}{\gamma-1}} \qquad\qquad \frac{p_{t5}}{p_{t4}} = \pi_t, \quad \frac{T_{t5}}{T_{t4}} = \tau_t; \quad \pi_t = \tau_t^{\frac{\gamma}{\gamma-1}}$$

Now let us use this system of notation to develop expressions for the Thrust and Specific Impulse of the Turbojet Engine. We begin by tracking the changes of stagnation temperature and pressure through the engine.

Temperature accounting:

$$T_{te} = T_e(1 + \tfrac{\gamma-1}{2} M_e{}^2) = T_o\theta_o\tau_c\tau_b\tau_t = T_o\theta_t\tau_t \tag{4}$$

Pressure accounting:

$$p_{te} = p_e(1 + \tfrac{\gamma-1}{2} M_e^2)^{\frac{\gamma}{\gamma-1}} = p_o\delta_o\pi_c\pi_b\pi_t \tag{5}$$

From (5), if $p_e = p_o$(ideally expanded nozzle) and if $\pi_b \approx 1$

$$(1 + \tfrac{\gamma-1}{2} M_e^2) = (\delta_o\pi_c\pi_b\pi_t)^{\frac{\gamma-1}{\gamma}} = \theta_o\tau_c\tau_t \tag{5b}$$

where the second equality assumes that the compression and expansion processes are reversible adiabatics. From this we find an expression for the exit Mach number,

$$M_e^2 = \frac{2}{\gamma-1}[\theta_o\tau_c\tau_t - 1] \tag{6}$$

It is very important to realize that <u>although this expression for the exit Mach number is written in terms of temperature ratios, it comes from the pressure changes in the engine. This is a general result, namely that the exit Mach number depends on the ratio of jet stagnation pressure to the ambient pressure, not at all on the temperature.</u>

If the exhaust is to be choked ($M_e \geq 1$), we must have, from (6),

$$\frac{2}{\gamma-1}(\theta_0\tau_c\tau_t - 1) \geq 1 \tag{6b}$$

which may not be satisfied at low power and/or low Mach number.

From (4)

$$\frac{T_e}{T_o} = \frac{\theta_t\tau_t}{1 + \frac{\gamma-1}{2} M_e^2} = \frac{\theta_t}{\theta_o\tau_c} = \frac{\theta}{\tau_c}$$

So far these are quite general expressions applicable to any gas stream. substituting them in our expressions for the velocity ratio and the thrust we have

$$\frac{u_e}{u_0} = \frac{\sqrt{\frac{2}{\gamma-1}[\theta_o\tau_c\tau_t - 1](\frac{\theta_t}{\theta_o\tau_c})}}{M_o} \tag{7}$$

Finally, the thrust per unit of mass flow (times the speed of sound to make it dimensionless) is

$$\frac{F}{\dot{m}_0 a_0} = \sqrt{\frac{2}{\gamma-1}[\theta_o\tau_c\tau_t - 1](\frac{\theta_t}{\theta_o\tau_c})} - M_0 \qquad (8)$$

So far we have not made this peculiar to the turbojet engine, because we have not included the relationship between the compressor and turbine. The fact that distinguishes the turbojet engine from other engines we may consider later is that the turbine power equals the compressor power, so

$$T_{to}(\tau_c - 1) = T_{t4}(1 - \tau_t)$$

$$\tau_t = 1 - \frac{\theta_o}{\theta_t}(\tau_c - 1) \qquad (9)$$

So finally for the Turbojet Engine

$$\frac{F}{\dot{m}_o a_o} = \sqrt{\frac{2}{\gamma-1}[\theta_t - \theta_o(\tau_c - 1) - \frac{\theta_t}{\theta_o\tau_c}]} - M_o \qquad (10)$$

We are also interested in the fuel consumption. We get $\dot{m}_f$ from a combustor heat balance,

$$\dot{m}_f = \dot{m}_o \frac{C_p T_o}{h}(\theta_t - \theta_o\tau_c) \qquad (10b)$$

so that the fuel-specific impulse, $I = F/(\dot{m}_f g)$ is

$$I = (\frac{h a_o}{g C_p T_o})\frac{(F/\dot{m}_o a_o)}{(\theta_t - \theta_o\tau_c)} \qquad (11)$$

<u>Discussion on nozzle choking</u> Eq. (6b) was the condition for the exhaust to be at least sonic, with $p_e \geq p_0$ or for the throat to be sonic. It involves both the compressor and the turbine temperature ratios, but we can eliminate the turbine ratio using the shaft balance (Eq. (9)), so that the condition is now

$$\theta_0\tau_c(1 - \frac{\tau_c - 1}{\theta}) \geq \frac{\gamma+1}{2} \qquad (12)$$

When the equal sign applies, we have M_e=1 while still p_e =p_0. This limit can be rearranged into a quadratic equation for τ_c :

$$\tau_c^{\,2} - (\theta+1)\tau_c + \frac{\gamma+1}{2}\frac{\theta}{\theta_0} = 0$$

with the two solutions

$$\tau_c^{+,-} = \frac{\theta+1}{2} \pm \sqrt{(\frac{\theta+1}{2})^2 - \frac{\gamma+1}{2}\frac{\theta}{\theta_0}}$$

It can be verified that τ_c must be between these two roots to ensure $M_e \geq 1$. The τ_c^+ is normally very high, so the relevant condition is $\tau > \tau_c^-$. Values of τ_c^- are tabulated below:

	θ=4	θ=6	θ=8
M_0=0	1.296	1.253	1.234
M_0=0.85	1.066	1.059	1.056

These are fairly low compressor ratios, even for stationary engine conditions, so the assumption of a choked nozzle is a good one in general. Whether or not the nozzle is also matched is a different question, as noted before.

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