

16.50 Lecture 14

Subjects: Heat Transfer and Cooling

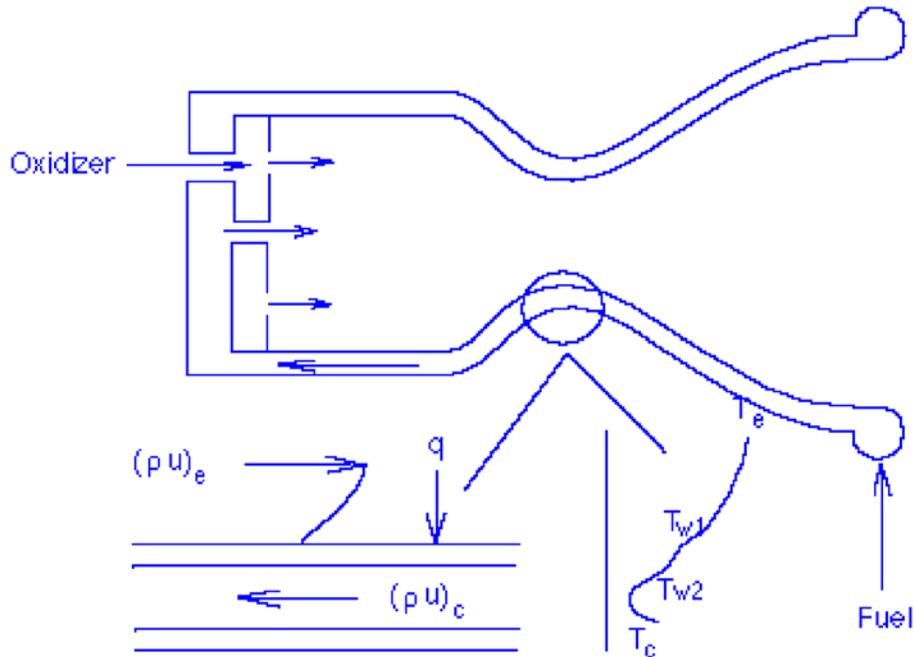
Because the combustion temperatures in most rocket engines are far beyond the levels tolerable by most common structural metals, the walls of the combustion chambers and nozzles must be cooled. The high pressures in the combustion chambers exacerbate the cooling problem by causing very high heat transfer rates, as we will see. For these reasons cooling is one of the critical technologies of rocket engines.

There are four ways of cooling rocket chambers and nozzles:

- a) Convective cooling by fuel or oxidizer
- b) Ablative cooling
- c) Heat sink cooling
- d) Radiation cooling

We will discuss each of these briefly, then return to a discussion of the physical mechanisms of importance in each, with the major emphasis on the first two because they are of the greatest practical importance.

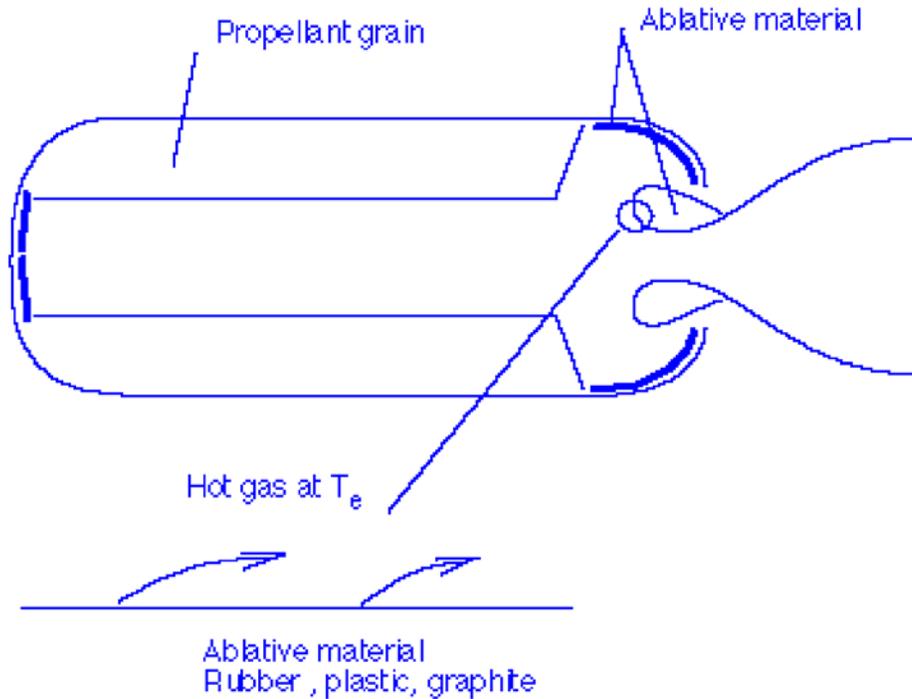
a) Convectively Cooled Chambers and Nozzles



There are 3 rather distinct heat transfer processes involved in such cooling:

- 1) Boundary layer heat transfer to the wall from the nozzle flow.
- 2) Conduction through the wall.
- 3) Convective heat transfer to the fluid in the cooling passages.

b) Ablatively Cooled Chambers and Nozzles



In this type of cooling, the important mechanisms are

- 1) Boundary layer heat transfer to the wall from the nozzle flow.
- 2) Decomposition and evaporation of the material of the wall, which generates a buffer gas layer, reducing the heat transfer rate.

c) Heat Sink Cooling

Here the wall temperature is maintained at a sufficiently low level by conduction of the heat into the material of the wall, so the mechanisms are

- 1) Boundary layer heat transfer to the wall from the nozzle flow.
- 2) Conduction of the heat into the wall, resulting in a temperature rise with time.

d) Radiative Cooling

For small rocket engines used in space applications, for example for station keeping, it may be possible to cool the nozzle by thermal radiation, eliminating the requirement for cooling passages, or for erosion of the nozzle as in ablative cooling.

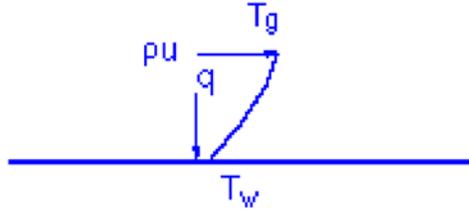
Since all of these cooling schemes have in common the heat transfer through a boundary layer to the wall, this aspect of them will be discussed first.

Heat transfer from a flowing fluid to a surface.

For this process it is conventional to define a film coefficient h by:

$$q = h(T_g - T_w)$$

where T_g is the gas' stagnation temperature and T_w is the wall temperature.



A more detailed way to represent this is:

$$q = \rho u c_p (T_g - T_w) St$$

where St is the Stanton Number, with the physical meaning

$$St = \frac{\text{heat flux to wall}}{\text{energy flux in flow relative to wall}}$$

Equating the two relations for the heat flux,

$$h = \rho u c_p St$$

and in dimensionless form we can write

$$\frac{hL}{k_f} = \frac{\rho u L}{\mu} \left(\frac{c_p \mu}{k_f} \right) St$$

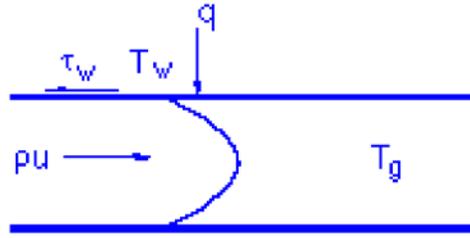
where the group on the left is called the Nusselt number, and the group involving just fluid properties is the Prandtl number:

$$\begin{array}{ccccccc} Nu & = & Re & Pr & St \\ \text{Nusselt} & & \text{Reynolds} & \text{Prandtl} & \text{Stanton} \end{array}$$

Here L is the length scale of the flow, for which we have differing interpretations depending on the flow geometry.

Heat Transfer to Passage Flows and Frictional Pressure Drop

Consider first the heat transfer to fluid flowing through a long narrow passage, which may represent the cooling passages in the nozzle.



If the length L of the passage is such the $\frac{L}{d} > 20$, the flow becomes Fully Developed, meaning that it has velocity and temperature profiles independent of x . The heat balance on a fluid element of length dx is

$$(\rho u A) dT_g = S q dx$$

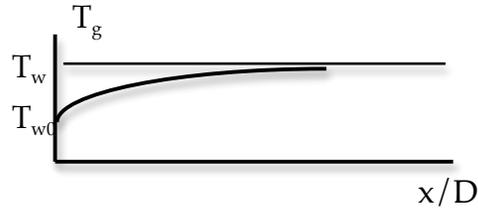
where A is the cross sectional area of the passage and S is the surface area per unit length (perimeter). For a circular passage, $S = \pi D$, $A = \frac{\pi D^2}{4}$, $\frac{A}{S} = \frac{D}{4}$; more generally, D here is defined as the "hydraulic diameter", $D = 4A/S$.

Writing $q_w = \rho u c_p S_t (T_w - T_g)$, we then have

$$\frac{dT_g}{dx} = \frac{4S_t}{D} (T_w - T_g)$$

If T_w and S_t can be taken as constant, this integrates to

$$T_w - T_g = (T_w - T_{g0}) e^{-4S_t \frac{x}{D}}$$



The total heat transferred to the fluid in a length L is then

$$Q = \dot{m} c_p [T_g(L) - T_{g0}] = \dot{m} c_p (T_w - T_{g0}) (1 - e^{-4S_t \frac{L}{D}})$$

But any time there is boundary layer heat conduction to a wall, there is also wall friction, and in the case of a duct, pressure drop. To see this quantitatively, take now a force balance on the same fluid element:

$$A \frac{dp}{dx} dx = -\tau_w S dx = -c_f \frac{\rho u^2}{2} S dx$$

$$\frac{dp}{dx} = -c_f \frac{S}{A} \frac{\rho u^2}{2} = -\frac{4}{D} c_f \frac{\rho u^2}{2}$$

and if compressibility is ignored, the pressure drop $\Delta p = p_0 - p$ in a pipe length L is

$$\Delta p = \frac{4c_f}{D} \frac{\rho u^2}{2} L$$

It is interesting to calculate the ratio of the non-dimensional pressure drop and the heat added. In particular, for small values of $4S_t L/D$, so that the expression for Q can be expanded to first order, we find

$$\frac{(\Delta p / \rho u^2)}{(Q / \dot{m} c_p (T_w - T_{g0}))} \approx \frac{c_f / 2}{S_t}$$

which is a quantity we will shortly find to be of order unity.

Sometimes we define a "friction factor" $f \equiv 4C_f$. We then find that f or c_f can be

represented as a function of only the Reynolds Number, defined as $Re \equiv \frac{\rho u D_h}{\mu}$

where ρu is the mass flow per unit area of the passage and D_h is the hydraulic diameter, defined as above. All fluid mechanics texts present the variation of C_f or f , and a sample is shown at the end of this lecture. Note that in that graph the quantity h_f on the left scale is $\Delta p / \rho$, and the scale on the left is e_s / d , where e_s is the "equivalent sand roughness" (for a hydraulically smooth surface, $e_s \ll d$).

Although there is some dependence of this relationship on Mach number for Mach numbers approaching 1, for most of our purposes we can take this relationship for the friction coefficient to be valid. For incompressible flow, the pressure drop is then directly calculable from the definition of the friction coefficient. For compressible flow we must account for the acceleration of the flow that occurs because of decreasing density as the fluid is heated.

Reynolds Analogy

To transfer our understanding of the viscous pressure drop to heat transfer, we observe that both in turbulent flow and in laminar flow the same mechanism transports momentum and energy to the wall (flow eddies in turbulent flow, molecular motion in laminar flow). Thus we would expect the ratio of heat flux into the wall to heat flux along the wall to equal the ratio of momentum flux into the wall divided by momentum flux along the wall.

$$\frac{q}{\rho u c_p (T - T_w)} = \frac{\tau_w}{\rho u^2} = \frac{c_f \frac{\rho u^2}{2}}{\rho u^2}$$

This results in

$$S_t = \frac{c_f}{2}$$

More detailed analysis shows that the thermal conductivity matters slightly, and we find (Hill/Peterson, pg. 129)

$$S_t Pr^{.67} = \frac{c_f}{2}$$

where $Pr = \frac{c_p \mu}{k}$ is the Prandtl Number.

Now we can use the empirical information on pressure drop to estimate heat transfer. Thus for turbulent flow in a smooth pipe,

$$S_t \approx \frac{c_f}{2} \text{Pr}^{-.67} = .023 \text{Re}^{-\frac{1}{5}} \text{Pr}^{-.67}$$

Heat Transfer Through Boundary Layers

On the gas side of the rocket nozzle wall we have a nearly inviscid flow, with a thin viscous layer near the wall. In rocket nozzles, the flow is strongly influenced by a streamwise pressure gradient. The "favorable" pressure gradient produced by the pressure drop in the flow direction accelerates the flow in the boundary layer, so the Boundary Layer tends to be thinner than it would be on a flat plate or on a pipe with fully developed flow. It is thinnest at the nozzle throat, where the pressure gradient is the strongest. For this reason and because pu is largest there, the heat flux is largest at the nozzle throat.

Much of the early progress in quantifying the heat transfer rate at the throat came from using data from passage flows, with Boundary Layer Thickness \approx

$\frac{\text{Tube diameter}}{2}$ ($\delta \approx \frac{D_h}{2}$). One then applies various techniques to estimate δ , and applies the passage flow data in the form

$$\text{St} = .023 \text{Re}_\delta^{-1/5} \text{Pr}^{-.67}$$

where $\text{Re}_\delta \equiv \frac{(\rho u)_\infty 2\delta}{\mu}$. But it is hard to accurately predict δ , so in practice one resorts to empirical results. Bartz (See HP pg. 549) recommended for the nozzle throat

$$\text{St} = \frac{h}{G_{Cp}} = .023 (\text{Re}_D)^{-1/5} \text{Pr}^{-.67}$$

where D is the throat diameter.

With these elements you are now in a position to analyze the most critical processes in the cooling system for a convectively cooled rocket engine.

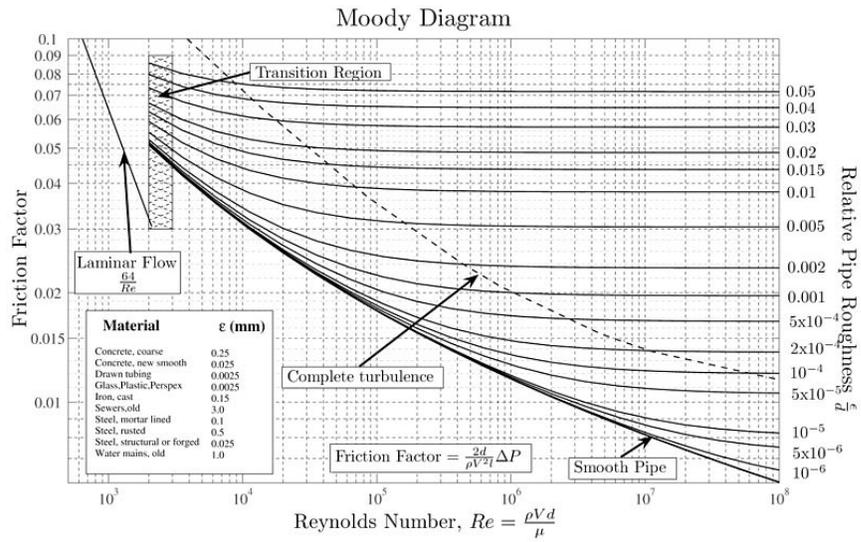


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