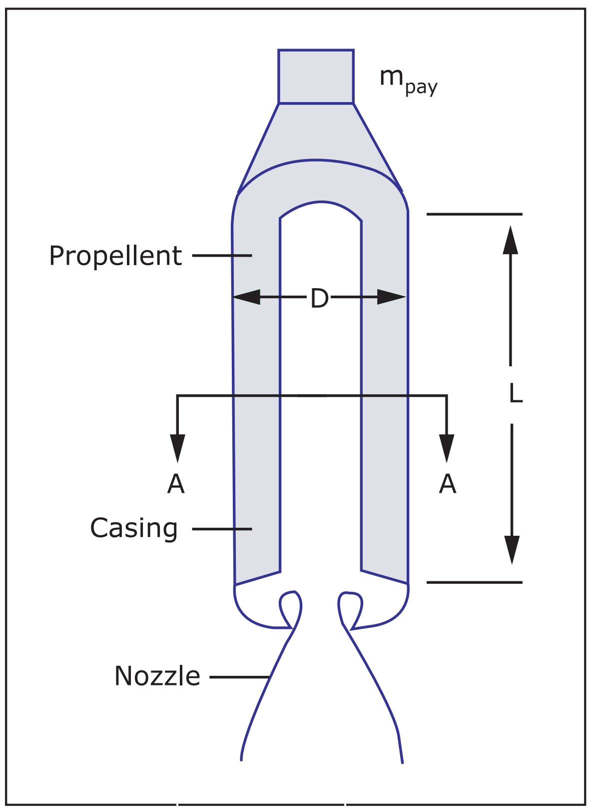
16.50 Lecture 13

Subject: Rocket casing design; Structural modeling

Thus far all our modeling has dealt with the fluid mechanics and thermodynamics of rockets. This is appropriate because it is these features that set rockets apart from most other devices. On the other hand it is not possible to understand the characteristics and limitations of rockets as systems without at least a rudimentary understanding of their structural characteristics which determine their mass, durability etc. To aid such understanding we must develop some simple Structural Models. The first step is to understand the Loads that the rocket's structure must withstand. Let us begin with a Solid Propellant Rocket

1) Loads

We model the rocket case as a sphere-cylinder full of fuel, acted on by the thrust F and the payload reaction (a+g)(M_{pay}) where a is the acceleration and g the gravitational acceleration.



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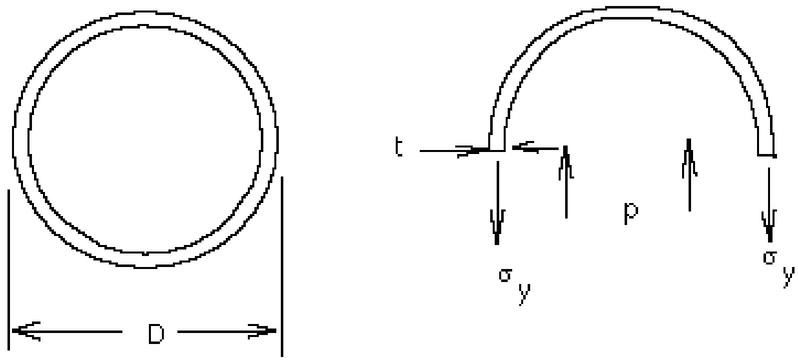
The casing is subjected to the following:

a) internal pressure, p_c

b) shear loads from the propellant grain, which is bonded to the case

c) compressive loads

To find the forces in the casing due to the pressure, we make free-body diagrams. First consider section A-A.

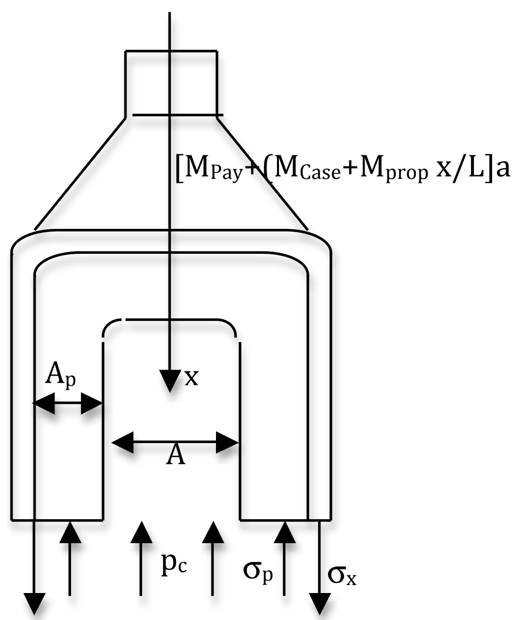


Section A-A

From this free body diagram, the circumferential stress σ_y is

$$\sigma_y = \frac{pD}{2t}$$

There is also an axial load generated by the combination of the internal pressure, the compressive load from the payload and the compressive stress in the propellant. Constructing another free-body diagram,



$$[M_{pay} + (M_{case} + M_{prop})\frac{x}{L}](a+g) + \sigma_x \pi D t = p_c A + \sigma_p A_p$$

$$\sigma_x = \frac{1}{\pi D t}\ \{p_c A + \sigma_p A_p - [M_{pay} + (M_{case} + M_{prop})\frac{x}{L}\ ]\ (a+g)\}$$

This stress is most positive (tension) at x=0:

$$(\sigma_x)_{Max} = \frac{(p_c A + \sigma_p A_p)}{\pi D t} \simeq \frac{p_c A_{case}}{\pi D t} = \frac{p_c D}{4t}$$

where we have assumed hydrostatic grain equilibrium, i.e., $\sigma_p = p_c$.

At the other end, x=L, we have either least tension or possibly compression of the casing wall; this last possibility would imply buckling problems. Putting x=L,

$$(\sigma_x)_{\min} = \frac{p_c A_{case} - (M_0 - M_{nozzle})(a+g)}{\pi D t}$$

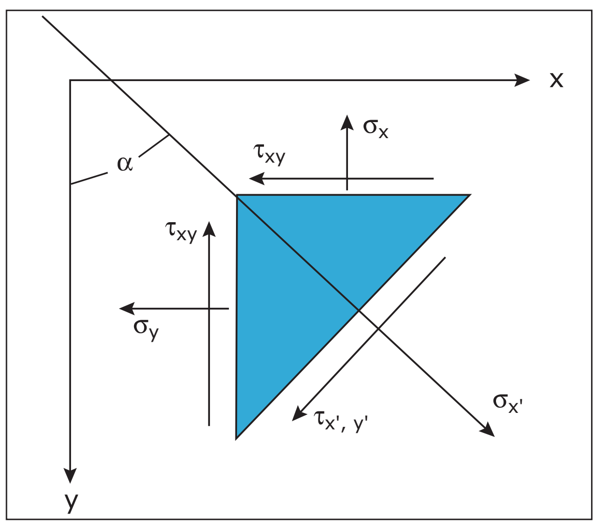
and recalling that $a + g = F / M_0 = c_F p_c A_t / M_0$,

$$(\sigma_x)_{\min} = \frac{p_c}{\pi D t}[A_{case} - c_F(1 - \frac{M_{nozzle}}{M_0})A_t]$$

So, unless A_t is less than about A_{case}/c_F, there will still be tension at the casing's base.

2) State of Stress

With this representation of the loads we can now deduce the State of Stress in the casing wall, subject to some simplifying assumptions. We assume t << D, so the stress may be described as a state of plane stress in x-y coordinates, where x is axial and y tangential. In these coordinates imagine a small triangular element. The stresses on the element in general are tensions σ_x, σ_y and shear $\tau_{xy} = \tau_{yx}$:



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Now consider the stresses on a plane perpendicular to a line making the angle α with the x axis. In treatments of stress and strain it is shown that

$$\sigma_{x'} = \sigma_x \cos^2\alpha + \sigma_y \sin^2\alpha + 2\tau_{xy} \sin\alpha \cos\alpha$$

$$\tau_{x'y'} = -(\sigma_x - \sigma_y) \sin\alpha \cos\alpha + \tau_{xy} (\cos^2\alpha - \sin^2\alpha)$$

It is then shown that the maximum (or minimum) of $\sigma_{x'}$ occurs at an angle α such that

$$\tan 2\alpha = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

and that these stresses are:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\ 2}}$$

In our case $\tau_{xy} = 0$, so σ_{max} is simply the larger of σ_x or σ_y. and usually $\sigma_y > \sigma_x$:

$$\sigma_y = \frac{p_c D}{2t}; \qquad (\sigma_x)_{Max} = \frac{p_c D}{4t} = \frac{1}{2}\sigma_y$$

So the casing must be designed to withstand the hoop stresses:

$$t = D\frac{p_c}{\sigma_{ult}} \times (a\ safety\ factor)$$

3) Case Mass

Again assuming constant t<<D and hemispherical caps,

$$M_{case} = (\pi D^2 + \pi DL)t\ \ \rho_{case} = \pi D^3\left(1 + \frac{L}{D}\right)\frac{p_c}{2\sigma_y}\ \ \rho_{case}$$

The mass of the propellant is

$$M_{prop} = \left(\frac{\pi D^3}{6} + \frac{\pi D^2 L}{4}\right)\varepsilon\,\rho_{prop} = \pi D^3\left(\frac{1}{6} + \frac{1}{4}\frac{L}{D}\right)\varepsilon\,\rho_{prop}$$

where ε is the fraction of case volume filled by propellant.

$$\frac{M_{case}}{M_{prop}} = \frac{3\left(1 + \frac{L}{D}\right)\frac{p_c}{\sigma}\rho_{case}}{\left(1 + \frac{3}{2}\frac{L}{D}\right)\varepsilon\,\rho_{prop}}$$

Typically, for p_c = 50 atm= 750 psi , $\sigma_y \approx$ 150,000 psi (about 100kg/mm^2 in European notation), and $\rho_{case} = .3\,\frac{lb}{ln^3}$ (steel) , $\rho_{prop} = .06\,\frac{lb}{ln^3}$

$$\frac{M_{case}}{M_{prop}} \approx .093\,\frac{1 + \frac{L}{D}}{1 + \frac{3}{2}\frac{L}{D}}$$

and for $\frac{L}{D} = 3$, $\quad \frac{M_{case}}{M_{prop}} \approx .068.$

Is this a reasonable estimate? For the Minuteman, from Sutton & Ross

M_{case} = 2557 lb $\qquad\qquad$ M_{prop} = 45,831

$$\frac{M_{case}}{M_{prop}} = 0.056$$

It seems we have been somewhat conservative in our design parameters.

4) What is left out of our estimate?

a) Attachments
b) End bells have 1/2 the stress - make 1/2 as thick?
c) Shear loads on propellant grain

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